$$4\pi r^2 E_3 = \frac{1}{\varepsilon_0}\Big[4\pi a^2\left(-\frac{k}{a}\right) + 4\pi b^2\left(+\frac{k}{b}\right) + \int_a^b 4\pi \cancel{r'^2}\left(-\frac{k}{\cancel{r'^2}}\right)dr'\Big]$$

$$= \frac{1}{\varepsilon_0}\Big[-4\cancel{\pi k}a + 4\cancel{\pi k}b - 4\cancel{\pi k}(b-\cancel{a})\Big]$$

$$= 0$$

$$\boxed{\therefore \vec{E}_3 = 0}$$

b) $\oint \vec{D}\cdot d\vec{A} = Q_{f_{enc}}$ $\quad \vec{D} = \varepsilon_0 \vec{E} + \vec{P}$

$r < a$ $\quad \vec{D}_1 = D_1(\vec{r})\hat{r}$ by symmetry

$d\vec{A} = r^2\, d\cos\theta\, d\phi\, \hat{r}$

$$\oint D_1 \hat{r}\cdot(r^2\, d\cos\theta\, d\phi\, \hat{r}) = 0$$

$$4\pi r^2 D_1 = 0$$

$$\boxed{\therefore \vec{D}_1 = 0 \quad \vec{E} = \frac{\vec{D}-\vec{P}}{\varepsilon_0} = 0}$$

$a \le r \le b$

$\vec{D}_2 = D_2(r)\hat{r}$ by symmetry

$d\vec{A} = r^2\, d\cos\theta\, d\phi\, \hat{r}$

$$\int D_2 \hat{r} \cdot (r^2 \, d\cos\theta \, d\phi \, \hat{r}) = 0$$

$$4\pi r^2 D_2 = 0$$

$$\therefore \vec{D}_2 = 0 \qquad \vec{E} = \frac{\vec{D} - \vec{P}}{\epsilon_0}$$

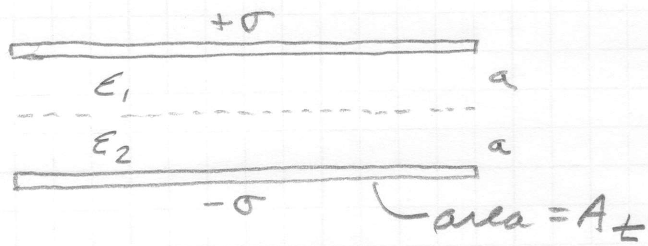
$$\boxed{\vec{E} = -\frac{k}{\epsilon_0 r}\hat{r}}$$

$r > b$

$\vec{D}_3 = 0$ as before

$$\boxed{\vec{E} = \frac{\vec{D} - \vec{P}}{\epsilon_0} = 0}$$

4-18)



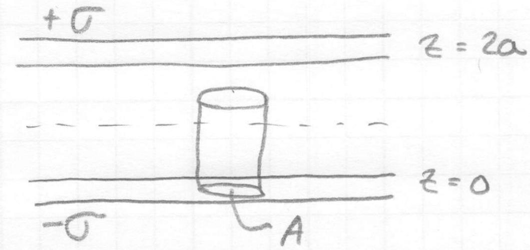
$\varepsilon_1 = 2 \quad \varepsilon_2 = 1.5$

a.

$$\nabla \cdot \vec{D} = \rho_f$$

$$\int \nabla \cdot \vec{D}\, d\tau = \int \rho_f\, d\tau$$

$$\oint \vec{D} \cdot d\vec{A} = -\sigma A$$



$\vec{D} = D\hat{z}$ by symmetry

$\vec{D} = 0$ inside the plate.

$$D \cdot \cancel{A} = -\sigma \cancel{A}$$

$$D = -\sigma$$

$$\therefore \vec{D} = -\sigma \hat{z}$$

b.

$$\vec{D} = \varepsilon \vec{E}$$

$$\vec{E} = \frac{\vec{D}}{\varepsilon}$$

$$\vec{E}_2 = \frac{\vec{D}}{\varepsilon_0 \varepsilon_2} = -\frac{\sigma}{\varepsilon_0 \varepsilon_2}\hat{z} \qquad 0 \le z \le a$$

$$\vec{E}_1 = \frac{\vec{D}}{\varepsilon_0 \varepsilon_1} = -\frac{\sigma}{\varepsilon_0 \varepsilon_1}\hat{z} \qquad a < z \le 2a$$

c.

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

$$\vec{P} = \vec{D} - \varepsilon_0 \vec{E}$$

$$\vec{P}_1 = -\sigma \hat{z} + \varepsilon_0 \left(\frac{+\sigma}{\varepsilon_0 \varepsilon_1} \hat{z} \right) \qquad a < z < 2a$$

$$\vec{P}_1 = \sigma \left(\frac{1}{\varepsilon_1} - 1 \right) \hat{z}$$

$$\vec{P}_2 = -\sigma\hat{z} - \varepsilon_0\left(-\frac{\sigma}{\varepsilon_0\varepsilon_2}\hat{z}\right) \qquad 0 \le a \le 2a$$

$$\vec{P}_2 = \sigma\left(\frac{1}{\varepsilon_2} - 1\right)\hat{z}$$

d. $$V = -\int_0^{2a} \vec{E}\cdot d\vec{l} \qquad d\vec{l} = dz\,\hat{z}$$

$$= -\left[\int_0^a \vec{E}_2 \cdot dz\,\hat{z} + \int_a^{2a} \vec{E}_1 \cdot dz\,\hat{z}\right]$$

$$= -\left[\int_0^a \left(-\frac{\sigma}{\varepsilon_0\varepsilon_2}\hat{z}\right)\cdot\left(dz\,\hat{z}\right) + \int_a^{2a}\left(-\frac{\sigma}{\varepsilon_0\varepsilon_1}\hat{z}\right)\cdot\left(dz\,\hat{z}\right)\right]$$

$$= +\left[\frac{\sigma}{\varepsilon_0\varepsilon_2}(a) + \frac{\sigma}{\varepsilon_0\varepsilon_1}(2a-a)\right]$$

$$\boxed{V = \frac{\sigma a}{\varepsilon_0}\left(\frac{1}{\varepsilon_2} + \frac{1}{\varepsilon_1}\right)}$$

$$= \frac{\sigma a}{\varepsilon_0}\left(\frac{1}{2} + \frac{2}{3}\right)$$

$$= \frac{7\sigma a}{6\varepsilon_0}$$

e. at $z = 0$ $\quad \sigma_{21} = \vec{P}_2 \cdot (-\hat{z})$

$$= -\sigma\left(\frac{1}{\varepsilon_2} - 1\right)$$

$$= \sigma\left(1 - \frac{1}{\varepsilon_2}\right)$$

at $z = 2a$ $\quad \sigma_{11} = \vec{P}_1 \cdot (\hat{z})$

$$= +\sigma\left(\frac{1}{\varepsilon_1} - 1\right)$$

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for $0 < z < a$ $\quad \rho_2 = -\nabla \cdot \vec{P}_2 = \underline{0}$

$a < z < 2a$ $\quad \rho_1 = -\nabla \cdot \vec{P}_1 = \underline{0}$

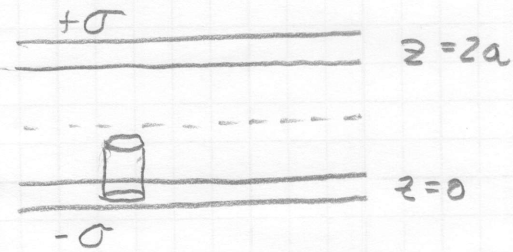
at $z = a$ $\quad \sigma_{22} = \vec{P}_2 \cdot \hat{z} = \sigma\left(\frac{1}{\varepsilon_2} - 1\right)$

$\sigma_{12} = \vec{P}_1 \cdot (-\hat{z}) = \sigma\left(1 - \frac{1}{\varepsilon_1}\right)$

total charge $= \sigma_{21} A_t + \sigma_{22} A_t + \sigma_{11} A_t + \sigma_{12} A_t$

$\underline{= 0}$

f. $\nabla \cdot \vec{E} = \frac{\rho_f + \rho_b}{\varepsilon_0}$



$\int \nabla \cdot \vec{E}\, d\tau = \int \left(\frac{\rho_f + \rho_b}{\varepsilon_0}\right) d\tau$

$\oint \vec{E} \cdot d\vec{A} = \int \frac{\rho_f + \rho_b}{\varepsilon_0}\, d\tau$ $\qquad \vec{E} = 0$ inside the plate

$z < a$

$$E_2 \cancel{A} = \frac{\left(-\sigma \cancel{A} + \sigma_{21} \cancel{A}\right)}{\varepsilon_0}$$

$$= \frac{1}{\varepsilon_0}\left(-\sigma + \sigma\left(1 - \frac{1}{\varepsilon_2}\right)\right)$$

$$\vec{E}_2 = -\frac{\sigma}{\varepsilon_0 \varepsilon_2}\hat{z}$$

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$a < z < 2a$

$$E_1 \cancel{A} = \left(-\sigma \cancel{A} + \sigma_{21} \cancel{A} + \sigma_{22} \cancel{A} + \sigma_{12} \cancel{A} \right) \frac{1}{\varepsilon_0}$$

$$= \left[\cancel{-\sigma} + \sigma \cancel{\left(1 - \frac{1}{\varepsilon_2} \right)} + \sigma \cancel{\left(\frac{1}{\varepsilon_2} - 1 \right)} + \sigma \left(\cancel{1} - \frac{1}{\varepsilon_1} \right) \right] \frac{1}{\varepsilon_0}$$

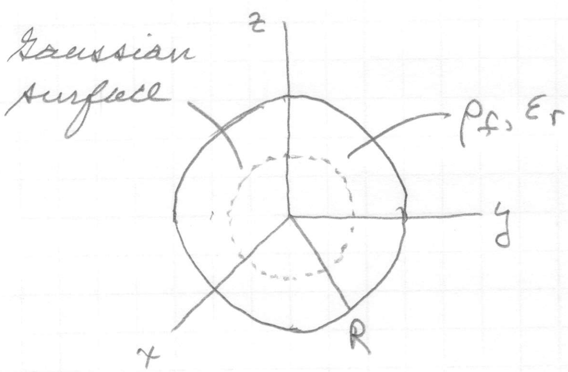
$$\vec{E}_1 = - \frac{\sigma}{\varepsilon_1 \varepsilon_0} \hat{z}$$

4-20)

$$V = -\int \vec{E}\cdot d\vec{l}$$

$$\vec{D} = \epsilon_0 \epsilon_r \vec{E}$$

$$\nabla\cdot\vec{D} = \rho_f$$



$$\int_0^r \nabla\cdot\vec{D}\, d\tau = \int_0^r \rho_f\, d\tau$$

$$\oint \vec{D}\cdot d\vec{A} = \int_0^r \rho_f\, r'^2 dr' d\cos\theta' d\phi'$$

$\vec{D} = D(r)\hat{r}$ by symmetry

$$\cancel{4\pi} r^{\cancel{2}} D = \rho_f \frac{\cancel{4}}{3}\cancel{\pi} r^{\cancel{3}} \quad r < R$$

$$D = \frac{\rho_f}{3} r$$

$$\vec{E} = \frac{\rho_f}{3\epsilon_0\epsilon_r}\vec{r} \longleftarrow \boxed{\vec{D} = \frac{\rho_f}{3}\vec{r}}$$

$$\cancel{4\pi} r^2 D = \rho_f \frac{\cancel{4}}{3}\cancel{\pi} R^3 \quad r > R$$

$$D = \frac{\rho_f R^3}{3}\frac{1}{r^2}$$

$$\vec{E} = \frac{\rho_f R^3}{3\epsilon_0\epsilon_r}\frac{\hat{r}}{r^2} \longleftarrow \boxed{\vec{D} = \frac{\rho_f R^3}{3}\frac{1}{r^2}\hat{r}}$$

$$V = -\int_\infty^r \vec{E}\cdot d\vec{l} \qquad d\vec{l} = \hat{r}\,dr$$

for $r > R$ $\quad V = -\int_\infty^r \frac{\rho_f R^3}{3\epsilon_0}\frac{\hat{r}}{r^2}\cdot(dr'\hat{r}')$

$$= + \frac{\rho_f R^3}{3\epsilon_0}\frac{r'^{-1}}{+1}\Big|_\infty^r$$

$$= \frac{\rho_f R^3}{3\epsilon_0}\left(\frac{1}{r} - \cancelto{0}{\frac{1}{\infty}}\right)$$

$$V = \frac{\rho_f R^3}{3\varepsilon_0} \frac{1}{r}$$

$$Q_f = \frac{4}{3}\pi R^3 \rho_f$$

$$\frac{Q}{4\pi} = \frac{\rho_f R^3}{3}$$

$$\boxed{V = \frac{Q}{4\pi\varepsilon_0} \frac{1}{r}}$$

for $r < R$ $\quad V = -\int_\infty^R \frac{\rho_f R^3}{3\varepsilon_0} \frac{\hat{r}'}{r'^2} \cdot (dr'\hat{r}) +$

$$- \int_R^r \frac{\rho_f}{3\varepsilon_0\varepsilon_r} r'\hat{r}' \cdot (dr'\hat{r})$$

$$V = + \frac{\rho_f R^3}{3\varepsilon_0} \left.\frac{r'^{-1}}{(+1)}\right|_\infty^R +$$

$$- \frac{\rho_f}{3\varepsilon_0\varepsilon_r} \left.\frac{r'^2}{2}\right|_R^r$$

$$= \frac{\rho_f R^3}{3\varepsilon_0} \frac{1}{R} - \frac{\rho_f}{3\varepsilon_0\varepsilon_r} \frac{1}{2}\left(r^2 - R^2\right)$$

$$= \frac{\rho_f R^3}{3\varepsilon_0}\left[\frac{1}{R} + \frac{1}{2\varepsilon_r}\left(\frac{1}{R} - \frac{r^2}{R^3}\right)\right]$$

$$\frac{Q}{4\pi} = \frac{\rho_f R^3}{3}$$

$$V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{R}\left(1 + \frac{1}{2\varepsilon_r}\right) - \frac{r^2}{2\varepsilon_r R^3}\right]$$

$$\boxed{V = \frac{Q}{4\pi\varepsilon_0} \frac{1}{R}\left[1 + \frac{1}{2\varepsilon_r} - \frac{r^2}{2\varepsilon_r R^3}\right]}$$

for $r = 0$ $\quad V = \frac{Q}{4\pi\varepsilon_0} \frac{1}{R}\left[1 + \frac{1}{2\varepsilon_r}\right]$

$$= \frac{\rho_f R^2}{3\varepsilon_0}\left(1 + \frac{1}{2\varepsilon_r}\right)$$

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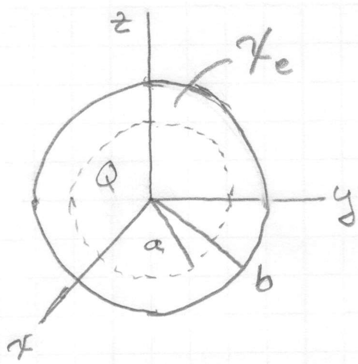
4-26) for $r < a$ $\quad \nabla \cdot \vec{D} = \rho_f \quad \varepsilon_r = 1$

$$\int \nabla \cdot \vec{D}\, d\tau' = \int \rho_f \, d\tau'$$

$$\rho_f = \frac{Q}{\frac{4}{3}\pi a^3}$$



$$\oint \vec{D} \cdot d\vec{A} = \rho_f \frac{4}{3}\pi r^3$$

$$D\, 4\pi \cancel{r^2} = \frac{Q}{\cancel{\frac{4}{3}\pi} a^3} \cancel{\frac{4}{3}\pi} r^{\cancel{3}\,1}$$

$\vec{D} = D(r)\hat{r}$ by symmetry

$$D = \frac{Q}{4\pi} \frac{r}{a^3}$$

$\vec{D} = \varepsilon \vec{E}$

$\chi_e = \varepsilon_r - 1$

$$\therefore \vec{E}_1 = \frac{Q}{4\pi\varepsilon_0} \frac{r}{a^3} \hat{r}$$

for $a < r < b$ $\quad \nabla \cdot \vec{D} = \rho_f$

$$\int \nabla \cdot \vec{D}\, d\tau' = \int \rho_f \, d\tau'$$

$$\oint \vec{D} \cdot d\vec{A}' = \rho_f \frac{4}{3}\pi a^3$$

$$D\, 4\pi r^2 = \cancel{\frac{Q}{\frac{4}{3}\pi a^3} \frac{4}{3}\pi a^3}$$

$$D = \frac{Q}{4\pi} \frac{1}{r^2}$$

$$\therefore \vec{E}_2 = \frac{Q}{4\pi\varepsilon_0} \frac{1}{\varepsilon_r} \frac{1}{r^2} \hat{r}$$

for $r > b$ use same reasoning as above with $\varepsilon_r = 1$ so

$$\vec{E}_3 = \frac{Q}{4\pi\varepsilon_0} \frac{1}{r^2} \hat{r}$$

$$\omega = \frac{1}{2}\int \vec{D}\cdot\vec{E}\,d\tau$$

$$= \frac{1}{2}\int \varepsilon\,\vec{E}\cdot\vec{E}\,d\tau$$

$$= \frac{1}{2}\int \varepsilon E^2\,d\tau$$

$$= \frac{1}{2}\left[\int_0^a \varepsilon_0 E_1^2\,d\tau + \int_a^b \varepsilon_0\varepsilon_r E_2^2\,d\tau + \int_b^\infty \varepsilon_0 E_3^2\,d\tau\right]$$

$$d\tau = r^2\,d\cos\theta\,d\phi\,dr$$

$$= \frac{4\pi}{2}\left[\varepsilon_0\left(\frac{Q}{4\pi a^3\varepsilon_0}\right)^2 \frac{r^5}{5}\Big|_0^a + \varepsilon_0\cancel{\varepsilon_r}\left(\frac{Q}{4\pi\varepsilon_0}\right)^2 \frac{1}{\varepsilon_r^{\cancel{2}1}}\frac{r^{-1}}{(-1)}\Big|_a^b + \varepsilon_0\left(\frac{Q}{4\pi\varepsilon_0}\right)^2\frac{r^{-1}}{(-1)}\Big|_b^\infty\right]$$

$$= \cancel{4}\pi\cancel{\varepsilon_0}\,\frac{Q^2}{\underset{8}{\cancel{16}}\pi^2\varepsilon_0^{\cancel{2}1}}\left[\frac{1}{5a^{\cancel{6}1}}\left(a^{\cancel{5}} - 0\right) + \frac{1}{\varepsilon_r}\left(\frac{1}{a} - \frac{1}{b}\right) + \left(\frac{1}{b} - \frac{1}{\infty}\right)\right]$$

$$= \frac{Q^2}{8\pi^2\varepsilon_0}\left[\frac{1}{5a} + \frac{1}{\varepsilon_r a} - \frac{1}{\varepsilon_r b} + \frac{1}{b}\right]$$

$$\omega = \frac{Q^2}{8\pi^2\varepsilon_0}\left(\frac{1}{5a} + \frac{1}{b} + \frac{1}{\varepsilon_r}\left(\frac{1}{a} - \frac{1}{b}\right)\right)$$

$$\varepsilon_r = \chi_e + 1$$