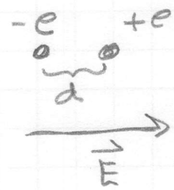
4-1) $r = 0.5\ \text{Å}$ $\qquad R = 0.5\ \text{Å}$

$a = 1\ mm$

$V = 500V$

$d = ?$



$$F_E = F_e$$

$$eE = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{d^2} \qquad E = \frac{V}{a}$$

$$\frac{\cancel{e}V}{a} = \frac{1}{4\pi\varepsilon_0}\frac{e^{\cancel{2}}}{d^2}$$

$$d^2 = \frac{1}{4\pi\varepsilon_0}\frac{ea}{V}$$

$$d = \sqrt{\frac{1}{4\pi\varepsilon_0}\frac{ea}{V}}$$

$$= \left[\left(8.99\times10^{-11}\ \frac{Nm^2}{C^2}\right)\frac{(1.6\times10^{-9}\ C)(10^{-3}\ m)}{500\ N\text{-}m/C}\right]^{\frac{1}{2}}$$

$$= 3.4\times10^{-8}\ m$$

$$\boxed{d = 540\ \text{Å}} \qquad \text{too big}$$

another way

use $\frac{\alpha}{4\pi\varepsilon_0} = 0.667\times10^{-30}\ m^3$

$$\vec{p} = \alpha\vec{E} = ed$$

$$\therefore\ d = \frac{\alpha}{e}E = \frac{\alpha}{e}\frac{V}{a}$$

$$= \frac{0.667\times10^{-30}m^3\ 4\pi\, 8.85\times10^{-12}\frac{C^2}{Nm^2}\times 500\frac{Nm}{C}}{1.6\times10^{-19}C\times10^{-3}m}$$

$$= 2.3\times10^{-16}\ m$$

$$\frac{d}{R} = \frac{2.3\times10^{-16}\,m}{0.5\times10^{-10}\,m} = 4.6\times10^{-6}$$ very small part of the atom.

---

to ionize set $d = R$ and solve for V

$$d = \frac{\alpha}{e}\frac{V}{a} = R$$

$$V = \frac{a e R}{\alpha}$$

$$= \frac{(10^{-3}\,m)(1.6\times10^{-19}\,c)(0.5\times10^{-10}\,m)}{0.667\times10^{-30}\,m^3 \times 4\pi \times 8.85\times10^{-12}\,\frac{C^2}{Nm^2}}$$

$$= 1.1\times10^{8}\,V$$

$$= 100\ MV$$

4-2) $\rho(r) = \frac{q}{\pi a^3} e^{-2r/a}$ $a = 0.5 \text{Å}$ Bohr radius

$q = e$

$\alpha = ?$

$\vec{p} = \alpha \vec{E}$

Get the field 'inside' the atom.

$\nabla \cdot \vec{E}_{in} = \frac{\rho}{\varepsilon_0}$ $\vec{E} = E(r)\hat{r}$ by symmetry

$\int \nabla \cdot \vec{E}_{in}\, d\tau = \frac{1}{\varepsilon_0} \int \rho\, d\tau$

$\oint \vec{E}_{in} \cdot d\vec{A} = \frac{1}{\varepsilon_0} \int_0^r \frac{e}{\pi a^3} e^{-2r'/a} r'^2 dr' d\cos\theta\, d\phi$

$E_{in} \cancel{4\pi} r^2 = \frac{e}{\pi \varepsilon_0 a^3} \cancel{4\pi} \int_0^r e^{-2r'/a} r'^2 dr'$

$E_{in} r^2 = \frac{e}{\pi \varepsilon_0 a^{\cancel{3}2}} \left[\frac{\cancel{a}}{2}\left(a^2 - e^{-\frac{2r}{a}}\left(2r^2 + 2ar + a^2\right)\right)\right]$

$E_{in} = \frac{e}{2\pi\varepsilon_0} \frac{1}{r^2} \left[\frac{a^2}{a^2} - e^{-\frac{2r}{a}} \left(\frac{2r^2}{a^2} + \frac{2ar}{a^2} + \frac{a^2}{a^2} \right) \right]$

$= \frac{e}{2\pi\varepsilon_0} \frac{1}{r^2} \left[1 - e^{-2r/a} \left(1 + 2\frac{r}{a} + 2\left(\frac{r}{a}\right)^2 \right) \right]$

$e^{-\frac{2r}{a}} = 1 - \frac{2r}{a} + \frac{4r^2}{2a^2} - \frac{8r^3}{6a^3} + \dots$

$E_{in} = \frac{e}{2\pi\varepsilon_0} \frac{1}{r^2} \left[1 - \left(1 - \frac{2r}{a} + 2\frac{r^2}{a^2} - \frac{4}{3}\frac{r^3}{a^3} \right) \times \left(1 + \frac{2r}{a} + \frac{2r^2}{a^2} \right) \right]$

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$$E_{in} = \frac{e}{2\pi\varepsilon_0}\frac{1}{r^2}\Big[\cancel{1} - \cancel{1} - \cancel{\frac{2r}{a}} - \cancel{2\left(\frac{r}{a}\right)^2}$$

$$+ \cancel{\frac{2r}{a}} + \cancel{4\frac{r^2}{a^2}} + \cancel{4\left(\frac{r}{a}\right)^3}$$

$$- \cancel{2\left(\frac{r}{a}\right)^2} - \cancel{4\left(\frac{r}{a}\right)^3} - 4\left(\frac{r}{a}\right)^4$$

$$+ \frac{4}{3}\frac{r^3}{a^3} + \frac{8}{3}\frac{r^4}{a^4} + \frac{8}{3}\frac{r^5}{a^5}\Big]$$

Keep terms out to $\left(\frac{r}{a}\right)^3$

$$E_{in} = \frac{e}{2\pi\varepsilon_0}\frac{1}{r^2}\left[\frac{4}{3}\frac{r^3}{a^3}\right]$$

$$E_{in} = \frac{e}{\cancel{2}\pi\varepsilon_0}\frac{\cancel{4}}{3}\frac{r}{a^3}$$

at equilibrium, $r = d$

$\therefore\ p = ed \qquad E_e = \dfrac{2e}{3\pi\varepsilon_0}\dfrac{d}{a^3}$

$$d = \frac{3\pi\varepsilon_0 a^3}{2e}E_e$$

$$p = \cancel{e}\left(\frac{3\pi\varepsilon_0 a^3 E_e}{2\cancel{e}}\right)$$

$$= \frac{3\pi\varepsilon_0 a^3}{2}E_e$$

$$= \alpha E_e$$

$$\therefore \alpha = \frac{3\pi \varepsilon_0 a^3}{2}$$

$$= \frac{3\pi}{2}\left(8.85 \times 10^{-12} \frac{C^2}{Nm^2}\right)\left(0.5 \times 10^{-10} m\right)^3$$

$$\alpha = 5.2 \times 10^{-42} \frac{C^2\text{-}m}{N}$$

Unit: $\frac{C^2}{Nm^2} m^3 = \frac{C^2 m}{N}$

Comparison with experimental value

$$\frac{\alpha}{4\pi\varepsilon_0} = 0.667 \times 10^{-30} m^3$$

For me

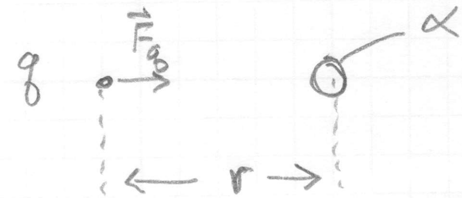
$$\frac{\alpha}{4\pi\varepsilon_0} = \frac{\frac{3\cancel{\pi}\cancel{\varepsilon_0} a^3}{2}}{4\cancel{\pi}\cancel{\varepsilon_0}} = \frac{3a^3}{8}$$

$$= \frac{3(0.5 \times 10^{-10} m)^3}{8}$$

$$= 0.047 \times 10^{-30} m^3$$

My calculated value is a factor too small by a factor of 14.

4-4)



$$p = \alpha E$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

$$\therefore p = \frac{\alpha}{4\pi\epsilon_0}\frac{q}{r^2}$$

$$\vec{E}_{atom} = \frac{p}{4\pi\epsilon_0}\frac{1}{r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right)$$

$$\theta = 0$$

$$\therefore \text{answer} = \vec{F}_q = q\,\vec{E}_{atom}(r, \theta=0)$$

$$= \frac{qp}{\cancel{4}\pi\epsilon_0}\frac{1}{r^3}(2)\,\hat{r}$$

along the line joining the centers

$$= \frac{qp}{2\pi\epsilon_0 r^3}\hat{r}$$

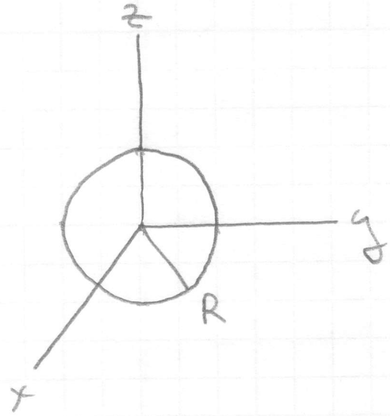
$$= \frac{q}{2\pi\epsilon_0 r^3}\frac{\alpha}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

$$\vec{F}_q = 2\alpha\left(\frac{q}{4\pi\epsilon_0}\right)^2\frac{1}{r^5}\hat{r}$$

units $\frac{C\text{-}m}{N/C}\, C^2\left(\frac{Nm^2}{C^2}\right)^2\frac{1}{m^5} =$

$$\frac{\cancel{C^4}\,\cancel{m}}{\cancel{N}}\;\frac{N^{\cancel{2}\,1}\,m^{\cancel{4}}}{\cancel{C^4}}\;\frac{1}{\cancel{m^5}} = N$$

4-10) $\vec{P}(\vec{r}) = k\vec{r} = kr\hat{r}$



a $\sigma_b = \vec{P} \cdot \hat{r}$

$= k\vec{r} \cdot \hat{r}$

$$\boxed{\sigma_b = kr = kR}$$

$\rho_b = -\nabla \cdot \vec{P}$

$= -\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 kr)$

$= -\frac{1}{r^2}\frac{\partial}{\partial r}(kr^3)$

$$\boxed{\rho_b = -3k}$$

b. for $r < R$ $\vec{E} = \vec{E}_\sigma + \vec{E}_\rho$

$\vec{E}_\sigma = 0$ see problem 2-11

$\vec{E}_\rho = \frac{\rho r}{3\varepsilon_0}\hat{r}$ see 2-12

$\vec{E} = \frac{\rho_b r}{3\varepsilon_0}\hat{r} = -\frac{kr}{\varepsilon_0}\hat{r}$

for $r > R$ $\vec{E} = \vec{E}_\sigma + \vec{E}_\rho$

$\vec{E}_\sigma = \frac{1}{4\pi\varepsilon_0}\frac{4\pi R^2\sigma}{r^2}\hat{r} = \frac{\sigma R^2}{\varepsilon_0}\frac{1}{r^2}\hat{r}$ see 2-11

$\vec{E}_\rho = \frac{1}{4\pi\varepsilon_0}\frac{\frac{4}{3}\pi R^3\rho}{r^2}\hat{r} = \frac{\rho R^3}{3\varepsilon_0}\frac{1}{r^2}\hat{r}$ see 2-8

$\vec{E} = \frac{R^2}{\varepsilon_0}\left(\sigma + \frac{\rho R}{3}\right)\frac{1}{r^2}\hat{r} = \frac{R^2}{\varepsilon_0}(kR - kR)\frac{1}{r^2}\hat{r} = 0$

4-14) $\sigma_b = \vec{P} \cdot \hat{n}$ $\rho_b = -\nabla \cdot \vec{P}$

$$\text{total charge} = \oint_s \sigma_b \, dA + \int_V \rho_b \, d\tau$$

$$= \oint_s \vec{P} \cdot \hat{n} \, dA + \int_V (-\nabla \cdot \vec{P}) \, d\tau$$

$\hat{n} \, dA = d\vec{A}$

the first integral is over the closed surface surrounding the volume charge density.

use the divergence theorem on the first term.

$$= \int_V \nabla \cdot \vec{P} \, d\tau - \int_V \nabla \cdot \vec{P} \, d\tau$$

$$= 0$$

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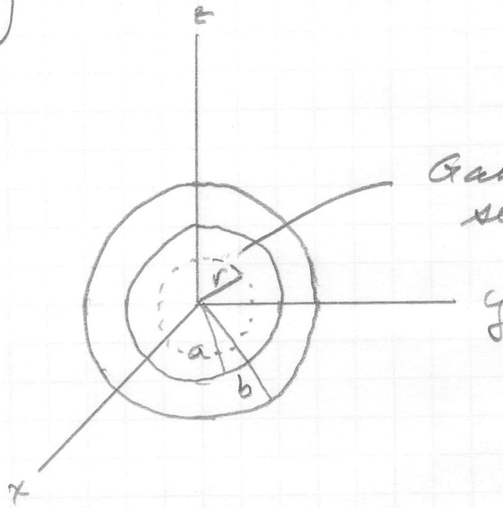
4-15)



$$\vec{P}(\vec{r}) = \frac{k}{r}\hat{r}$$

a) $$\oint_S \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

$r < a$ $\quad \vec{E}_1 = E_1(r)\,\hat{r}$ by symmetry

$$d\vec{A} = r^2\, d\cos\theta\, d\phi\, \hat{r}$$

$$\therefore \quad \oint E_1(r)\,\hat{r}\cdot r^2\, d\cos\theta\, d\phi\, \hat{r} = 0$$

$$E_1(4\pi r^2) = 0$$

$$\boxed{E_1 = 0}$$

$a \le r \le b$

$$\sigma_a = \vec{P}\cdot\hat{n} = -\vec{P}\cdot\hat{r} = -\frac{k}{a}$$

$\vec{E}_2 = E_2(r)\,\hat{r}$ by symmetry

$$\rho_a = -\nabla\cdot\vec{P}$$

$$= \frac{-1}{r^2}\frac{\partial}{\partial r}\left(r^{\cancel{2}}\frac{k}{\cancel{r}}\right)$$

$$\rho_a = \frac{-k}{r^2}$$

$$\oint E_2(r)\,\hat{r}\cdot\left(r^2\,d\cos\theta\,d\phi\,\hat{r}\right) =$$

$$= \frac{1}{\varepsilon_0}\left[4\pi a^2\sigma_a + \int_a^r 4\pi r'^2\rho_a\,dr'\right]$$

$$4\pi r^2 E_2 = \frac{1}{\varepsilon_0}\left[4\pi a^{\cancel{2}}\left(-\frac{k}{\cancel{a}}\right) + \right.$$

$$\left. - \int_a^r 4\pi \cancel{r'^2}\frac{k}{\cancel{r'^2}}\,dr'\right]$$

$$= \frac{1}{\varepsilon_0}\left[-4\pi k a - 4\pi k(r-a)\right]$$

$$\cancel{4\pi} r^2 E_2 = -\frac{1}{\varepsilon_0}\,\cancel{4\pi} k r$$

$$\boxed{\vec{E}_2 = -\frac{k}{\varepsilon_0 r}\,\hat{r}}$$

$r > b$

$\sigma_b = \vec{P}\cdot\hat{n} = +\vec{P}\cdot\hat{r} = \frac{k}{b}$

$\vec{E}_3 = E_3\,\hat{r}$ by symmetry

$\rho_b = -\nabla\cdot\vec{P} = -\frac{k}{r^2}$ as before.

$$\oint E_3(r)\,\hat{r}\cdot\left(r^2\,d\cos\theta\,d\phi\,\hat{r}\right) =$$

$$\frac{1}{\varepsilon_0}\left[4\pi a^2\sigma_a + 4\pi b^2\sigma_b + \right.$$

$$\left.\int_a^b 4\pi r'^2\rho_a\,dr'\right]$$