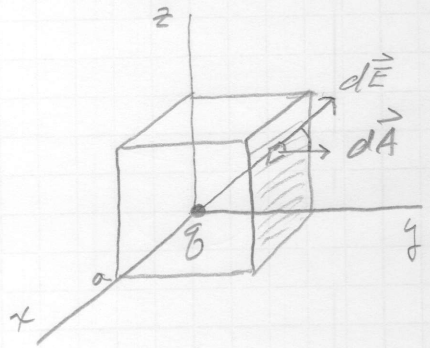
2-10)



$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

$$\Phi = \int \vec{E}\cdot d\vec{A}$$

$$d\vec{A} = dx\,dz\,\hat{y}$$

$$\hat{r}_x = \sin\theta\cos\phi$$

$$\hat{r}_y = \sin\theta\sin\phi$$

$$\hat{r}_z = \cos\theta$$

$$\hat{r} = \hat{r}_x\,\hat{x} + \hat{r}_y\,\hat{y} + \hat{r}_z\,\hat{z}$$

$$\Phi = \int \vec{E}\cdot d\vec{A}$$

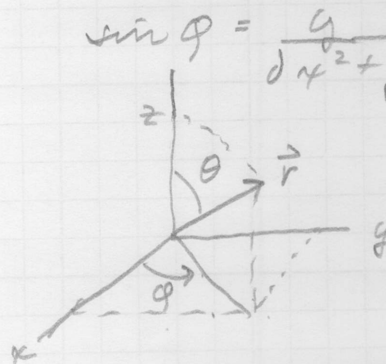
$$= \int_0^a\int_0^a \frac{1}{4\pi\varepsilon_0}\frac{q}{(x^2+y^2+z^2)}\left(\hat{r}_x\,\hat{x} + \hat{r}_y\,\hat{y} + \hat{r}_z\,\hat{z}\right)\cdot dx\,dz\,\hat{y}$$

$$= q\int_0^a\int_0^a \frac{1}{4\pi\varepsilon_0}\frac{1}{(x^2+y^2+z^2)}\hat{r}_y\,dx\,dz$$

$$y = a$$

$$= q\int_0^a\int_0^a \frac{1}{4\pi\varepsilon_0}\frac{\sin\theta\sin\phi}{(x^2+a^2+z^2)}\,dx\,dz$$

$$\sin\phi = \frac{y}{\sqrt{x^2+y^2}} = \frac{a}{\sqrt{x^2+a^2}}$$



$$\sin\theta = \frac{\sqrt{x^2+a^2}}{\sqrt{x^2+a^2+z^2}}$$

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$$\Phi = q\int_0^a \int_0^a \frac{1}{4\pi\varepsilon_0} \frac{\cancel{\sqrt{x^2+a^2}}}{\sqrt{x^2+a^2+z^2}} \frac{a}{\cancel{\sqrt{x^2+a^2}}} \frac{dx\,dz}{(x^2+a^2+z^2)}$$

$$= \frac{1}{4\pi\varepsilon_0} aq \int_0^a \int_0^a \frac{1}{(x^2+a^2+z^2)^{3/2}} dx\, dz$$

$$= \frac{aq}{4\pi\varepsilon_0} \int_0^a \frac{x}{(a^2+z^2)(a^2+z^2+x^2)^{1/2}} \Big|_0^a dz$$

$$= \frac{aq}{4\pi\varepsilon_0} \int_0^a \frac{a}{(a^2+z^2)(a^2+z^2+a^2)^{1/2}} dz$$

$$= \frac{q}{4\pi\varepsilon_0} a^2 \int_0^a \frac{1}{(a^2+z^2)(2a^2+z^2)^{1/2}} dz$$

$$= \frac{q}{4\pi\varepsilon_0} \cancel{a^2} \frac{\mathrm{ArcTan}\left(\frac{z}{\sqrt{2a^2+z^2}}\right)}{\cancel{a^2}} \Big|_0^a$$

$$= \frac{q}{4\pi\varepsilon_0} \left[\mathrm{ArcTan}\left(\frac{a}{\sqrt{2a^2+a^2}}\right) - \mathrm{ArcTan}(0)\right]$$

$$= \frac{q}{4\pi\varepsilon_0} \underbrace{\mathrm{ArcTan}\left(\frac{1}{\sqrt{3}}\right)}_{\frac{\pi}{6}}$$

$$= \frac{q}{4\pi\varepsilon_0} \frac{\pi}{6}$$

$$\boxed{\Phi = \frac{q}{24\varepsilon_0}}$$