2-9) $\vec{E} = kr^3\,\hat{r}$

a. $\rho = ?$ $\quad \nabla\cdot\vec{E} = \dfrac{\rho}{\varepsilon_0}$

$$\rho = \varepsilon_0\left(\nabla\cdot\vec{E}\right)$$

$$= \varepsilon_0\left[\frac{\partial}{\partial r}E_r + \frac{2}{r}E_r + \frac{1}{r\sin\varphi}\frac{\partial E_\theta}{\partial\theta} + \frac{1}{r}\frac{\partial E_\varphi}{\partial\varphi} + \frac{\cot\varphi}{r}E_\varphi\right]$$

$$E_\varphi = E_\theta = 0$$

$$\frac{\partial E_r}{\partial r} = 3kr^2$$

$$\rho = \varepsilon_0\left[3kr^2 + \frac{2}{\cancel{r}}kr^{\cancel{3}}\right]$$

$$= \varepsilon_0\left[3kr^2 + 2kr^2\right]$$

$$\boxed{\rho = 5k\varepsilon_0 r^2}$$

b. $Q = \displaystyle\int_0^R \rho\, d\tau \qquad d\tau = 4\pi r^2 dr$

$$= \int_0^R 5k\varepsilon_0 r^2\, 4\pi r^2 dr$$

$$= 20\pi k\varepsilon_0 \int_0^R r^4 dr$$

$$= \overset{4}{\cancel{20}}\pi k\varepsilon_0 \frac{r^5}{\cancel{5}}\Big|_0^R$$

$$\boxed{Q = 4\pi k\varepsilon_0 R^5}$$

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0} \qquad d\vec{A} = r^2\, d\cos\theta\, d\varphi\,\hat{r}$$

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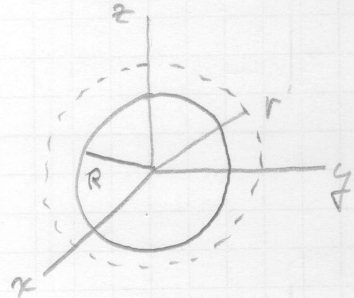

$$\oint k r^3 \hat{r} \cdot r^2 \, d\cos\theta \, d\phi \, \hat{r} = \frac{Q}{\varepsilon_0}$$

$$k \int_0^{2\pi}\int_{-1}^{1} r^5 \, d\cos\theta \, d\phi = \frac{Q}{\varepsilon_0}$$

$$k r^5 \, 4\pi = \frac{Q}{\varepsilon_0} \qquad \boxed{r = R}$$

Use the field at the surface.

$$\boxed{Q = 4\pi \varepsilon_0 k R^5}$$