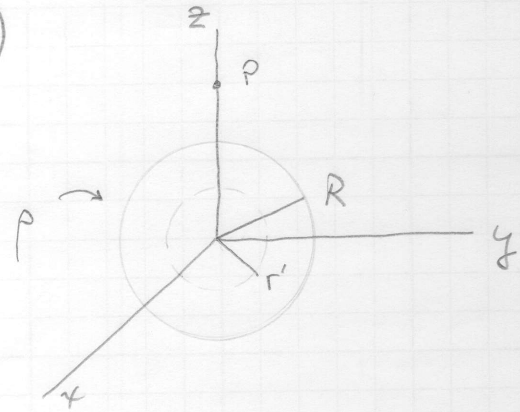
2-8)



$\vec{E} = \int d\vec{E}$

$d\vec{E}$ is the field of a spherical shell

$d\vec{E} = 0 \quad r < r'$

$= \frac{1}{4\pi\varepsilon_0} \frac{dq}{z^2}$

charge in the shell $= 4\pi r'^2 dr' \rho$

for $z > R$

$$\vec{E}_{out} = \int_0^R \frac{1}{4\pi\varepsilon_0} \frac{4\pi r'^2 dr' \rho}{z^2} \hat{z}$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{1}{z^2} \rho \int_0^R 4\pi r'^2 dr' \hat{z}$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{1}{z^2} \underbrace{\rho \frac{4}{3}\pi R^3}_{\text{total charge } Q} \hat{z}$$

$$\vec{E}_{out} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{z^2} \hat{z}$$

By symmetry $\boxed{\vec{E}_{out} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \hat{r}}$

for $z < R$

$$\vec{E}_{in} = \int_0^R d\vec{E}$$

$$= \int_0^z d\vec{E} + \underbrace{\cancel{\int_z^R d\vec{E}}}_{0 \text{ see } 2\text{-}7}$$

$$= \int_0^z \frac{1}{4\pi\varepsilon_0} \frac{4\pi r'^2 dr' \rho}{z^2} \hat{z}$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{1}{z^2} \rho \int_0^z 4\pi r'^2 dr' \hat{z}$$

$$\vec{E}_{in} = \frac{1}{4\pi\epsilon_0} \frac{1}{z^2} \rho \frac{4}{3}\pi r'^3 \Big|_0^z \hat{z}$$

$$= \frac{1}{\cancel{4\pi}\epsilon_0} \frac{1}{\cancel{z^2}} \rho \frac{\cancel{4}}{3} \cancel{\pi} z^{\cancel{3}\,1} \hat{z}$$

$$\vec{E}_{in} = \frac{\rho z}{3\epsilon_0} \hat{z}$$

By symmetry $\boxed{\vec{E}_{in} = \frac{\rho z}{3\epsilon_0} \hat{z}}$

$$\rho = \frac{Q}{\frac{4}{3}\pi R^3}$$

$$\therefore \quad \vec{E}_{in} = \frac{Q}{\frac{4}{\cancel{3}}\pi R^3} \frac{z}{\cancel{3}\epsilon_0} \hat{z}$$

$$= \frac{Q}{4\pi R^3} \frac{z}{\epsilon_0} \hat{z}$$

$$= \frac{1}{4\pi\epsilon_0} Q \frac{z}{R^3} \hat{z}$$