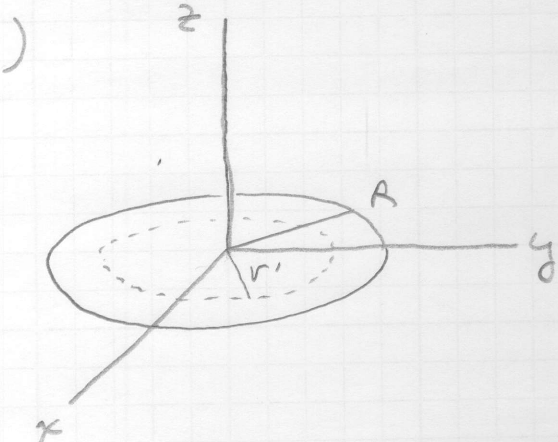
2-5) see notes $\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q z}{(z^2+r^2)^{3/2}} \hat{z}$

2-6)



Use 2-5 result

$$d\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{\sigma\, 2\pi r' dr' z}{(z^2+r'^2)^{3/2}} \hat{z}$$

$$\vec{E} = \int_0^R d\vec{E}$$

$$= \int_0^R \frac{1}{4\pi\epsilon_0} \frac{2\pi\sigma r' dr' z}{(z^2+r'^2)^{3/2}} \hat{z}$$

$$= \frac{1}{4\pi\epsilon_0} 2\pi\sigma z \int_0^R \frac{r'\, dr'}{(z^2+r'^2)^{3/2}} \hat{z}$$

$$= \frac{1}{4\pi\epsilon_0} 2\pi\sigma z \left[-\frac{1}{\sqrt{z^2+r'^2}} \right]_0^R \hat{z}$$

$$= \frac{1}{4\pi\epsilon_0} 2\pi\sigma z \left[-\frac{1}{\sqrt{z^2+R^2}} - \left(-\frac{1}{\sqrt{z^2+0^2}} \right) \right] \hat{z}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} 2\pi\sigma z \left[\frac{1}{z} - \frac{1}{\sqrt{z^2+R^2}} \right] \hat{z}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} 2\pi\sigma\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)\hat{z}$$

for $z >> R$

$$\vec{E} \cong \frac{1}{4\pi\varepsilon_0} 2\pi\sigma\left(1 - \frac{z}{z}\right)\hat{z} = 0 \quad ?!$$

try using a Taylor series

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} 2\pi\sigma\left(1 - \frac{z}{z\left(1+\frac{R^2}{z^2}\right)^{1/2}}\right)\hat{z}$$

$$= \frac{1}{4\pi\varepsilon_0} 2\pi\sigma\left(1 - \left(1+\frac{R^2}{z^2}\right)^{-\frac{1}{2}}\right)\hat{z}$$

$$\left(1+\frac{R^2}{z^2}\right)^{-1/2} \sim 1 + \left(-\frac{1}{2}\right)\frac{R^2}{z^2}$$

$$\cong \frac{1}{4\pi\varepsilon_0} 2\pi\sigma\left(\cancel{1} - \cancel{1} + \frac{R^2}{2z^2}\right)\hat{z}$$

$$\vec{E} \cong \frac{1}{4\pi\varepsilon_0} \frac{\pi R^2\sigma}{z^2}$$ → total charge

Looks like a point charge.

for $z << R$

$$\vec{E} \sim \frac{1}{4\pi\varepsilon_0} 2\pi\sigma\left(1 - \cancelto{0}{\frac{z}{R}}\right)\hat{z}$$

$$= \frac{\sigma}{2\varepsilon_0}\hat{z}$$ constant field above an infinite plane.