

$$\begin{aligned}
 2-23) \quad \vec{E} &= \frac{k}{\epsilon_0} (b-a) \frac{\hat{r}}{r^2} & r > b \\
 &= \frac{k}{\epsilon_0} \left(\frac{1}{r} - \frac{a}{r^2} \right) \hat{r} & a \leq r \leq b \\
 &= 0 & r < a
 \end{aligned}$$

$$V = - \int_{\infty}^r \vec{E} \cdot d\vec{l}$$

$$d\vec{l} = dr \hat{r} + r d\theta \hat{\theta} + r \sin\theta d\phi \hat{\phi}$$

$$= - \int_{\infty}^r E dr \quad \text{since } \vec{E} = E(r) \hat{r} \text{ which eliminates the last two terms in } d\vec{l} \text{ by orthogonality.}$$

for $r < a$

$$V(r) = - \int_{\infty}^r E dr$$

$$\begin{aligned}
 &= - \int_{\infty}^b \frac{k}{\epsilon_0} (b-a) r^{-2} dr' \\
 &\quad - \int_b^a \frac{k}{\epsilon_0} (r' - a r'^{-2}) dr' \\
 &\quad - \int_a^r 0 dr'
 \end{aligned}$$

$$\begin{aligned}
 &= + \frac{k}{\epsilon_0} (b-a) \left(\frac{r'^{-1}}{+1} \right) \Big|_{\infty}^b \\
 &\quad - \frac{k}{\epsilon_0} \left[\ln r' + \frac{a r'^{-1}}{(-1)} \right]_b^a \\
 &\quad + 0
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{k}{\epsilon_0} \left[(b-a) \left(\frac{1}{b} - \cancel{\frac{1}{\infty}} \right) - \left(\ln a + \frac{a}{a} \right) \right. \\
 &\quad \left. + \left(\ln b + \frac{a}{b} \right) \right]
 \end{aligned}$$

$$\begin{aligned}
 V(r) &= \frac{\lambda}{\epsilon_0} \left[(b-a) \frac{1}{b} + \ln\left(\frac{b}{a}\right) - 1 + \frac{a}{b} \right] \\
 &= \frac{\lambda}{\epsilon_0} \left[\cancel{1} - \frac{a}{b} + \ln\left(\frac{b}{a}\right) - \cancel{1} + \frac{a}{b} \right]
 \end{aligned}$$

$$V(r) = \frac{\lambda}{\epsilon_0} \ln\left(\frac{b}{a}\right)$$