2-20) $\nabla \times \vec{E}_a = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_{ax} & E_{ay} & E_{az} \end{vmatrix}$ $\quad \vec{E}_a = k\left[xy\,\hat{x} + 2yz\,\hat{y} + 3xz\,\hat{z}\right]$

$$= \hat{x}\left(\frac{\partial E_{az}}{\partial y} - \frac{\partial E_{ay}}{\partial z}\right) - \hat{y}\left(\frac{\partial E_{az}}{\partial x} - \frac{\partial E_{ax}}{\partial z}\right) + \hat{z}\left(\frac{\partial E_{ay}}{\partial x} - \frac{\partial E_{ax}}{\partial y}\right)$$

$$= \hat{x}(0 - 2y) - \hat{y}(3z - 0) + \hat{z}(0 - x)$$

$$= -2y\hat{x} - 3z\hat{y} - x\hat{z}$$

$$\neq 0$$

$\nabla \times \vec{E}_b = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_{bx} & E_{by} & E_{bz} \end{vmatrix}$ $\quad \vec{E}_b = k\left[y^2\hat{x} + (2xy + z^2)\hat{y} + 2yz\,\hat{z}\right]$

$$= \hat{x}(2z - 2z) - \hat{y}(0 - 0) + \hat{z}(2y - 2y)$$

$$= 0$$

$\therefore$ $\vec{E}_a$ is impossible.

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for $\vec{E}_b$ $\quad V_b = -\int_{\mathcal{O}}^{\vec{r}} \vec{E}\cdot d\vec{l}$ $\quad \mathcal{O} \equiv$ origin

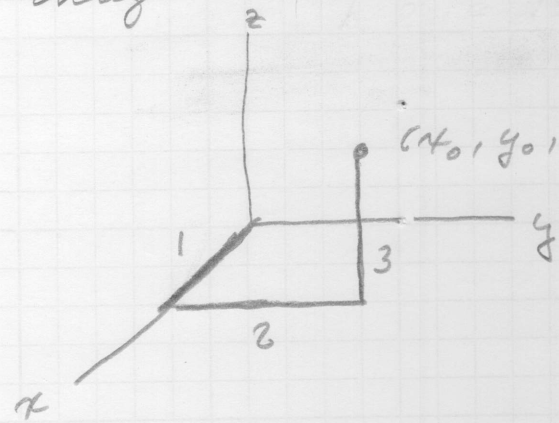
$$d\vec{l} = dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}$$

$$V_b(\vec{r}) = -\int_{\theta}^{\vec{r}} k\left[y^2\,dx + (2xy + z^2)\,dy + 2yz\,dz\right]$$

let $\vec{r} = x_0\hat{x} + y_0\hat{y} + z_0\hat{z}$

$\theta = \vec{\theta} = 0\hat{x} + 0\hat{y} + 0\hat{z}$

Pick a path to do the line integral



for step 1 $\quad y = z = 0 \quad 0 < x < x_0$

$$V_{b1}(\vec{r}) = -\int_0^{x_0,0,0} k(0\cdot dx + 0\cdot dy + 0\cdot dz)$$

$$= 0$$

for step 2 $\quad x = x_0 \quad z = 0 \quad 0 < y < y_0$

$dx = 0 \quad dz = 0 \quad dy \neq 0$

$$V_{b2}(\vec{r}) = -\int_{x_0,0,0}^{x_0,y_0,0} k\left[y^2\cdot 0 + (2x_0 y + 0)\,dy + 0\right]$$

$$= -\int_0^{y_0} k\,(2x_0 y)\,dy$$

$$= -2x_0 k \left.\frac{y^2}{2}\right|_0^{y_0}$$

$$V_{b2}(\vec{r}) = -\cancel{2} x_0 k \left(\frac{y_0^2}{\cancel{2}} - 0 \right)$$

$$= -k x_0 y_0^2$$

for step 3 $\quad x = x_0 \quad y = y_0 \quad 0 \le z \le z_0$

$\quad dx = 0 \quad dy = 0 \quad dz \neq 0$

$$V_{b3}(\vec{r}) = -\int_{x_0, y_0, 0}^{x_0, y_0, z_0} k \left[y_0^2 \cdot 0 + (2x_0 y_0 + z^2) \cdot 0 + 2 y_0 z \, dz \right]$$

$$= -k \int_0^{z_0} 2 y_0 z \, dz$$

$$= -k \cancel{2} y_0 \frac{z^2}{\cancel{2}} \Big|_0^{z_0}$$

$$= -k y_0 (z_0^2 - 0)$$

$$V_{b3}(\vec{r}) = -k y_0 z_0^2$$

$$\therefore V_b(\vec{r}) = V_{b1}(\vec{r}) + V_{b2}(\vec{r}) + V_{b3}(\vec{r})$$

$$= 0 + (-k x_0 y_0^2) + (-k y_0 z_0^2)$$

$$\boxed{V_b(\vec{r}) = -k(x y^2 + y z^2)}$$

Now compute $-\nabla V_b$

$$\vec{E}_b = -\nabla V_b = +\left(\frac{\partial}{\partial x} \hat{x} + \frac{\partial}{\partial y} \hat{y} + \frac{\partial}{\partial z} \hat{z} \right) \left[+ k (x y^2 + y z^2) \right]$$

$$= k \left[(y^2) \hat{x} + (2xy + z^2) \hat{y} + 2yz \hat{z} \right]$$

agrees with $\vec{E}$.