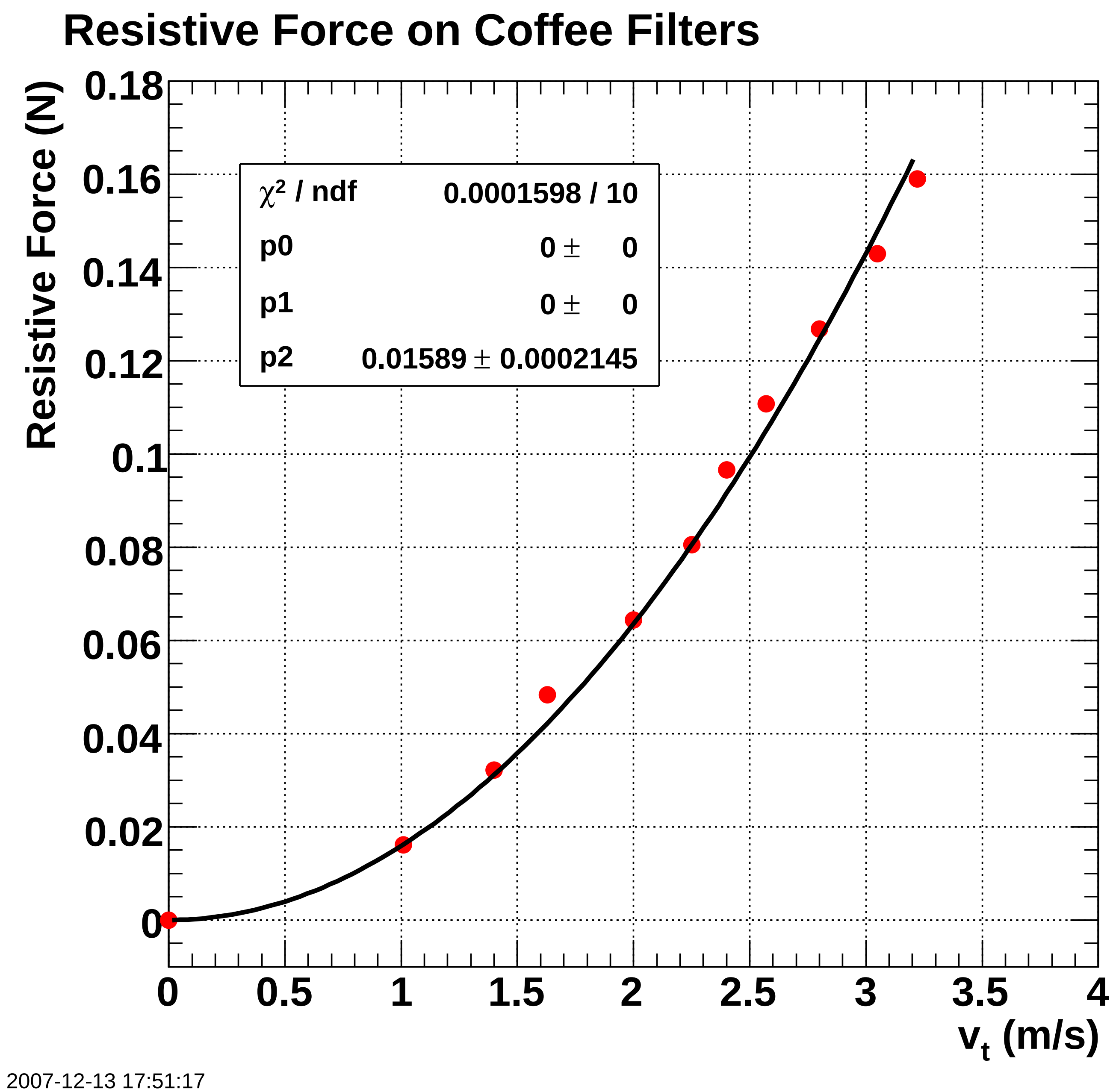
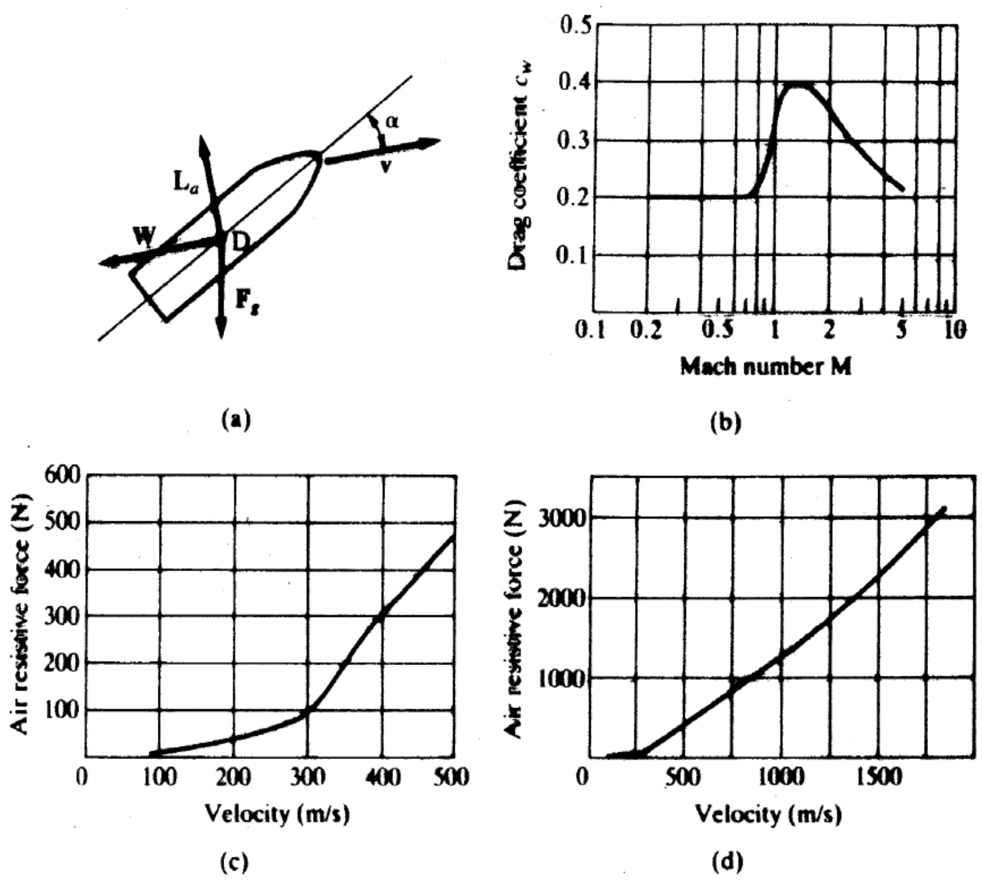
# EEEEKKKK!!!!

In January, 1942 a Soviet Ilyushin 4 flown by Lieutenant I.M.Chisov was badly damaged by German gunfire. At an altitude of 21,980 feet Lieutenant Chisov fell from the plane. Unfortunately, he did not have a parachute on when he fell. He landed on the slopes of a snow-covered ravine and slid to the bottom. He suffered a fractured pelvis and severe spinal damage, but lived. By 1974 he had become Lieutenant Colonel Chisov. How fast was Lieutenant Chisov moving when he hit the ravine? How long did his fall take?

# The Drag Force - 1



# The Drag Force - 2



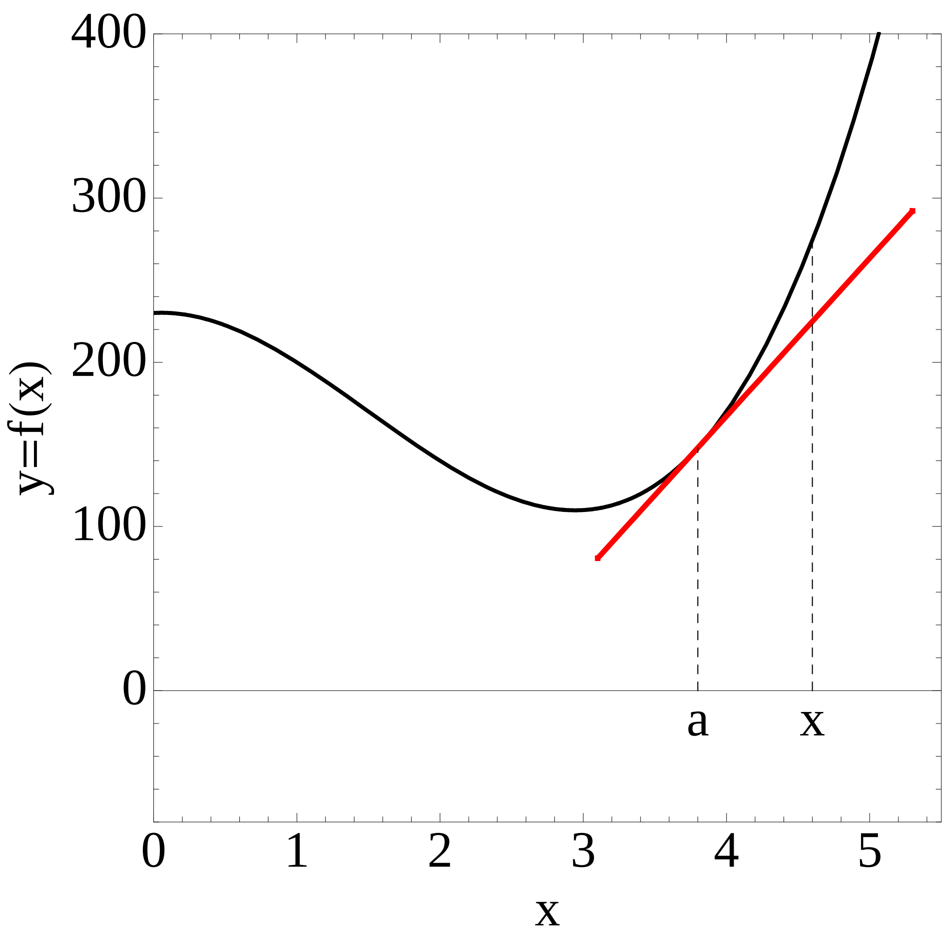

Aerodynamic forces acting on an artillery shell. The force $\vec{W}$ is the drag or air resistive force, $\vec{L}_a$ is the lift, $\vec{F}_g$ is gravity, and the point $D$ is the center of pressure. Note the change in the air resistive force at the speed of sound.

# Resistive Force on a Baseball

Jonathan Papelbon gives Derek Jeter some chin music at 90 mph (40 m/s). What is the drag coefficient of a baseball? What is the resistive force at that speed? The mass of a baseball is 0.145 kg, its radius is 3.7 cm, and its terminal velocity is measured to be 43 m/s.

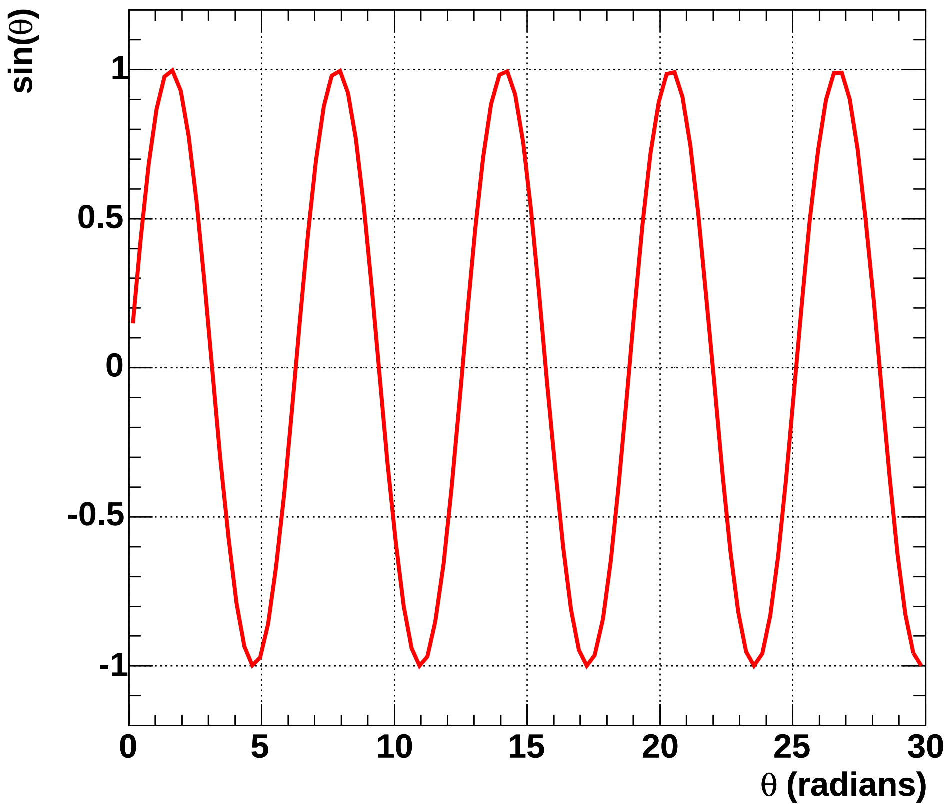
# Approximating a Function

The plot below shows an arbitrary curve (black) with the tangent curve (red) at one point.

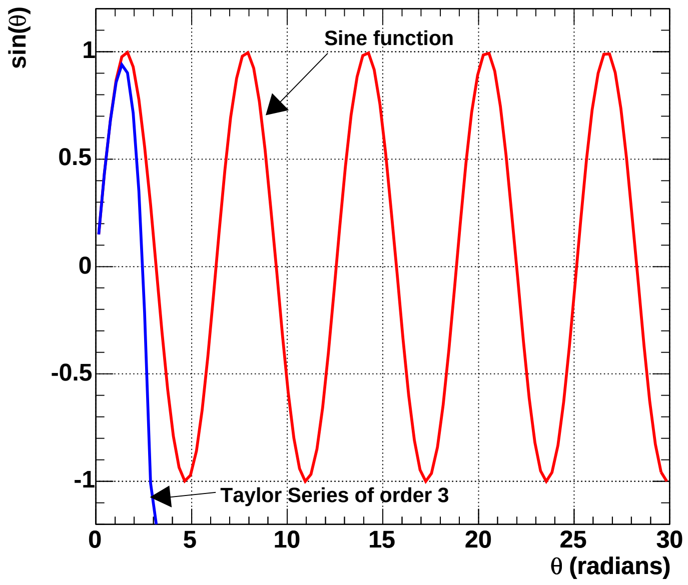


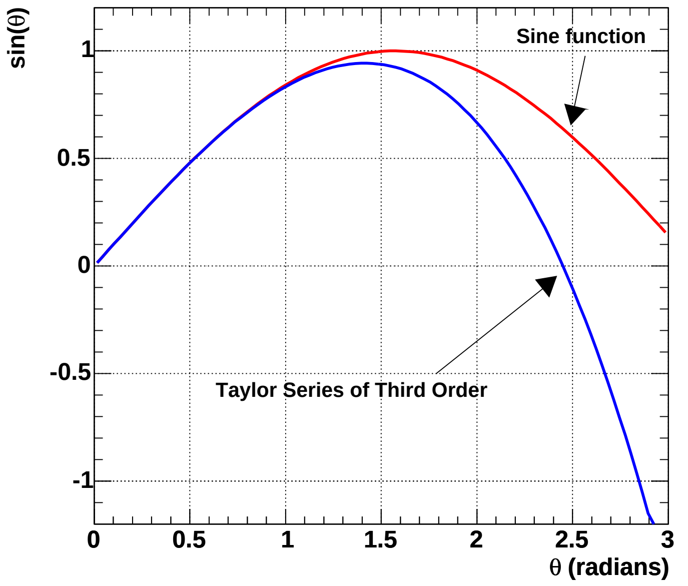
# Taylor Series for the Sine function

The plot below shows the sine function. What are the first two nonzero terms of the Taylor series for the sine function expanded about the point $\theta = 0$?



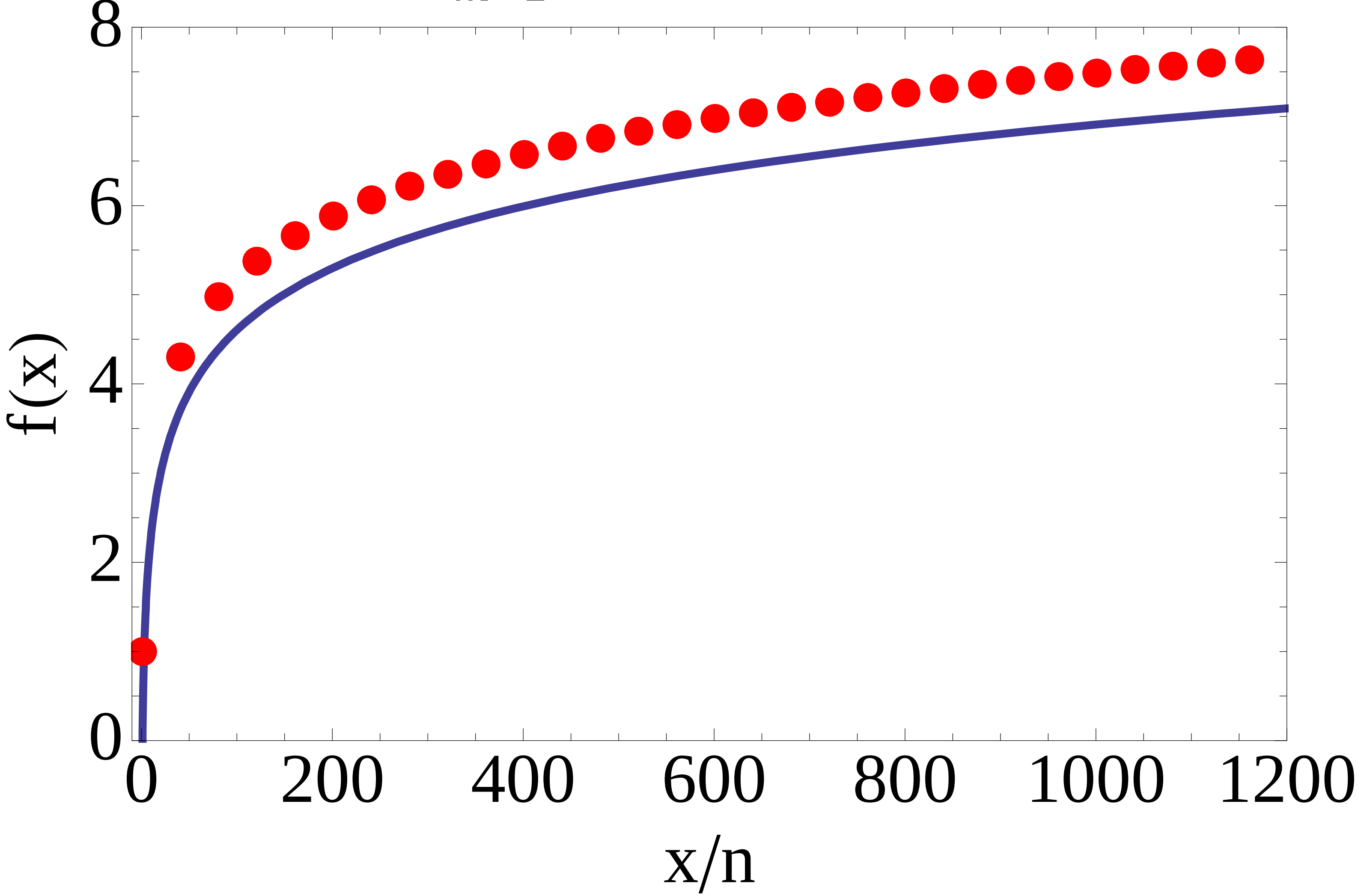
# Taylor Series for the Sine function



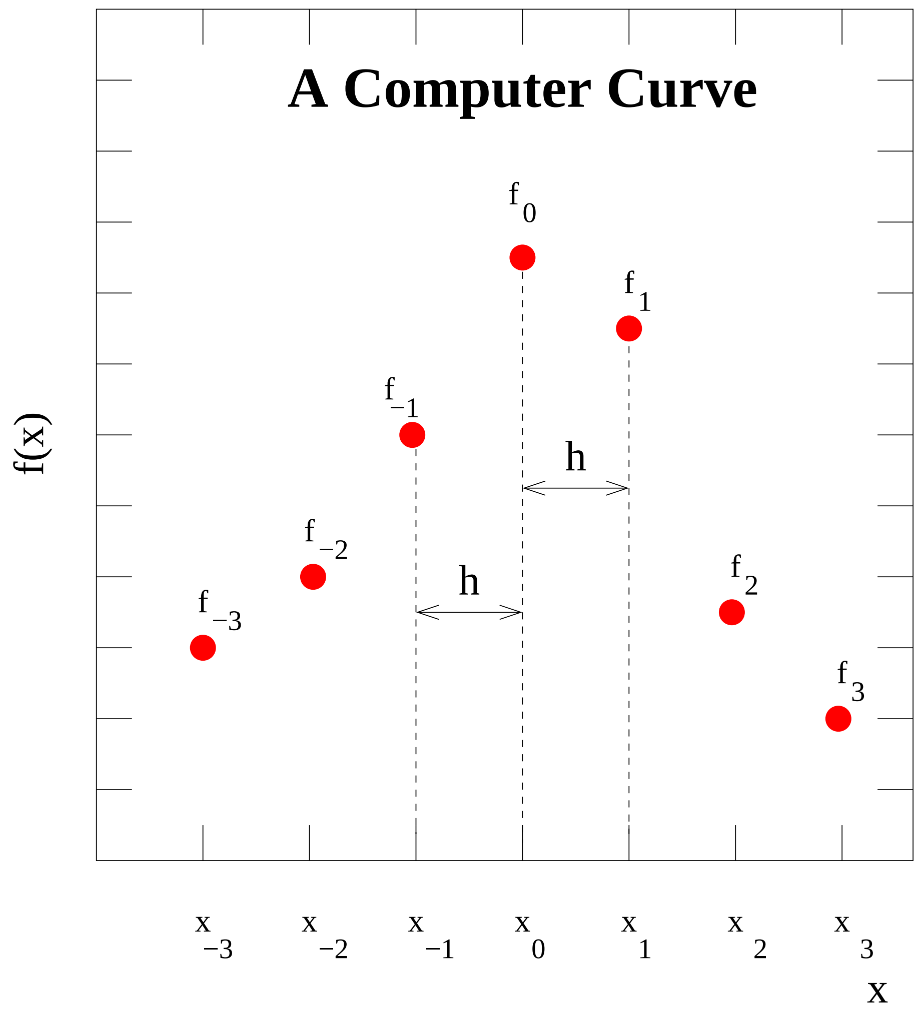



## The Integral Convergence Test

$$\sum_{m=1}^{n} \frac{1}{m} \text{ versus } \ln(x)$$



# Numerical Differentiation of a Curve



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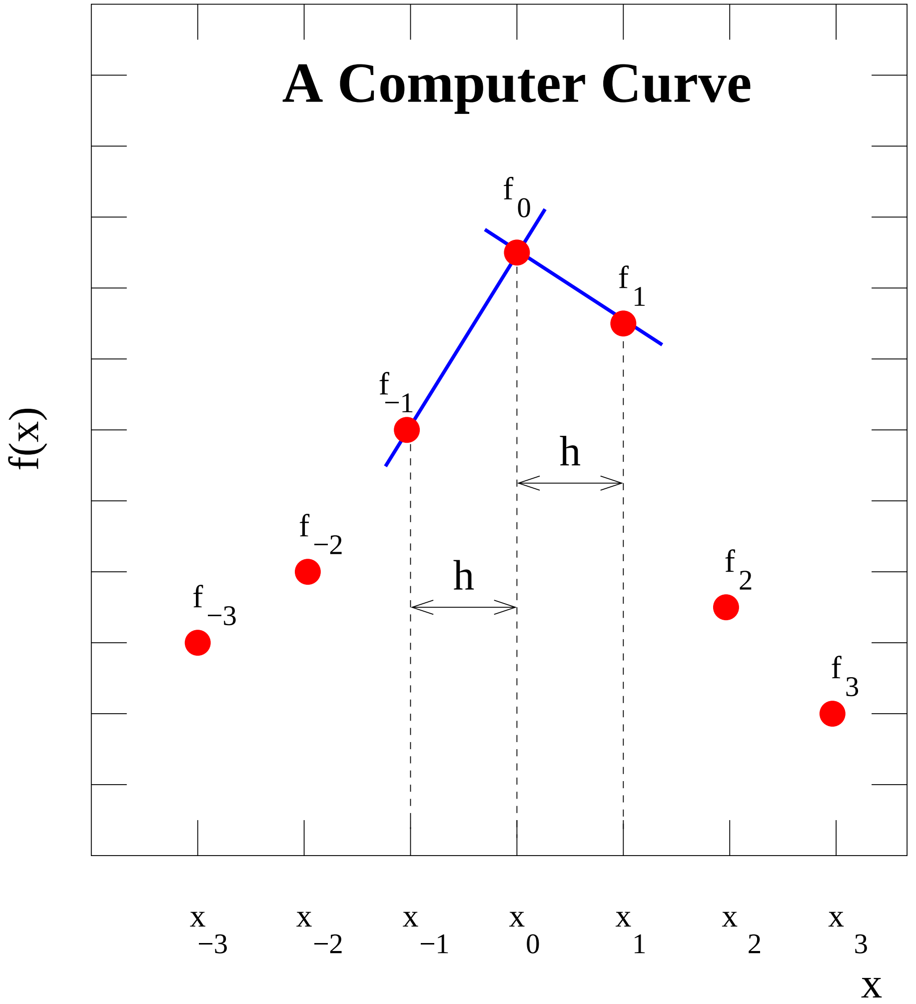
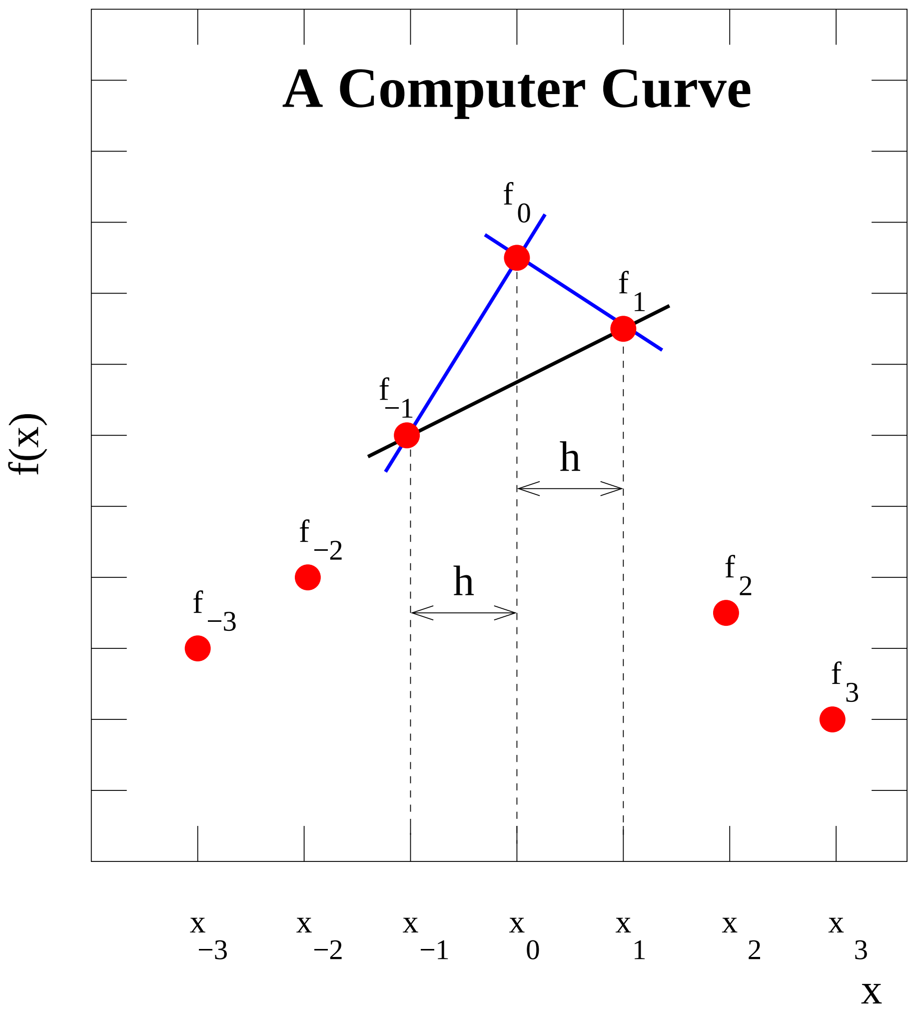


# Numerical Differentiation of a Curve



# The Limits of Accuracy

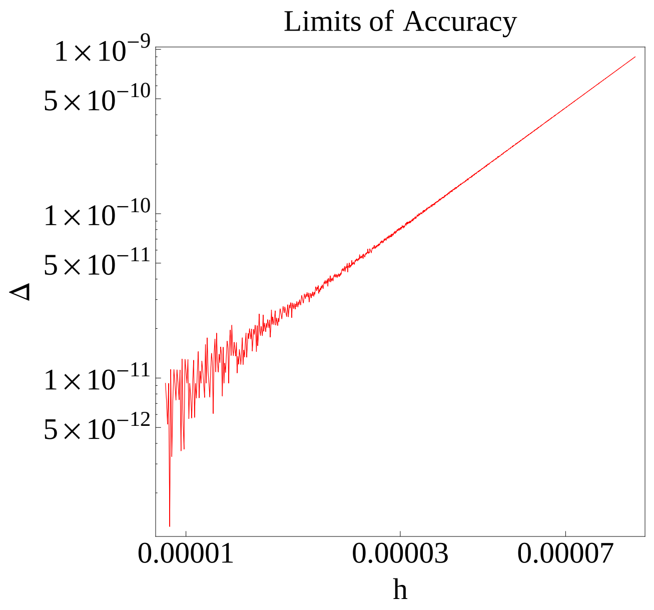
1. Consider the following code fragment. Why does $a \neq b$?

```
a = 1.0*10^17  + 1.0 - 1.0*10^17;
b = 1*10^17 + 1 - 1*10^17;
Print["a=", a, " b=", b]
a=0. b=1
```

2. Consider the following function.

$$\Delta = \cos\theta - \frac{\sin(\theta + h) - \sin(\theta - h)}{2h}$$

The plot shows the dependence of the function on the stepsize $h$.



# Life is a Differential Equation (DE)

Some definitions first

| | | |
|---|---|---|
| 1. | Ordinary DE | Only total derivatives. |
| 2. | Partial DE | More than one variable so use partial deviatives. |
| 3. | Order of the DE | Highest derivative in the DE. |
| 4. | Degree of the DE | Power of the higherst order derivative in the DE. |
| 5. | Linear DE | No multiplications among dependent variables and their derivatives. |
| 6. | Homogeneous | Every single term contains the dependent variables or their derivatives. |

# Strategy for Solving First-Order, Ordinary Differential Equations - 1

1. The differential equation and the initial conditions are given. For example
$$\frac{dy}{dx} = f(x, y) \quad \text{and} \quad y(x_0) = y_0$$
and $y(x_1)$ is unknown.

2. Divide the range $[x_0, x_1]$ into pieces.

3. Generate a recursion relationship between adjacent points.

4. Perform a step-by-step integration.

# Strategy for Solving First-Order, Ordinary Differential Equations - 2

Consider the following initial value problem.

$$\frac{dy}{dx} = -y \quad \text{and} \quad y(0) = 1.0$$

1. What is the analytical solution?
2. What is the recursion relationship?
3. Using $h = 0.1$, what is the value of the numerical solution for $x_1 = 0.4$?

# Nuclear Decay

The rate of radioactive decay of atomic nuclei is proportional to the number of nuclei $N$ in the sample so

$$\frac{dN}{dt} = -\lambda N$$

where $\lambda$ is a constant of proportionality (related to the half-life) and the negative sign means the number of nuclei is decreasing. The initial condition is $N(t=0) = N_0$. Generate an algorithm to solve this differential equation and apply it for the first three values of $N(t)$ 'by hand' for $h = \Delta t = 0.1\ s$ and $\lambda = 0.2\ s^{-1}$.

# Nuclear Decay Results

| t(s) | Calculation | Result | Analytic Result |
|---|---|---|---|
| 0 | $N_0 = N_0$ | 1000 | 1000 |
| 0.1 | $N_1 = N_0\ (1 - \lambda h)$ | 980. | 980.199 |
| 0.2 | $N_2 = N_1\ (1 - \lambda h)$ | 960.4 | 960.789 |
| 0.3 | $N_3 = N_2\ (1 - \lambda h)$ | 941.192 | 941.765 |
| 0.4 | $N_4 = N_3\ (1 - \lambda h)$ | 922.368 | 923.116 |
| 0.5 | $N_5 = N_4\ (1 - \lambda h)$ | 903.921 | 904.837 |
| 0.6 | $N_6 = N_5\ (1 - \lambda h)$ | 885.842 | 886.92 |
| 0.7 | $N_7 = N_6\ (1 - \lambda h)$ | 868.126 | 869.358 |
| 0.8 | $N_8 = N_7\ (1 - \lambda h)$ | 850.763 | 852.144 |
| 0.9 | $N_9 = N_8\ (1 - \lambda h)$ | 833.748 | 835.27 |
| 1. | $N_{10} = N_9\ (1 - \lambda h)$ | 817.073 | 818.731 |

# Solution of $\frac{dy}{dx} = -y$ using an Euler algorithm

```
(* initial parameter values *)

y0 = 1.0;
x0 = 0.0;
x1 = 0.4;
h = 0.1;

(* starting point of recursion relationship. *)
yn = y0;

(* make the table *)
t1 = Table[
   {x,
    yplus = yn*(1 - h);
    yn = yplus
    },
   {x, x0 + h, x1, h}];

(* stick the starting point at the front of the table. *)

t1 = Prepend[t1, {x0, y0}];
TableForm[t1]
```

| | |
|---|---|
| 0. | 1.0 |
| 0.1 | 0.9 |
| 0.2 | 0.81 |
| 0.3 | 0.729 |
| 0.4 | 0.6561 |

# Solution of $\frac{dy}{dx} = -y$ using an Euler algorithm

```
(* Solution of dy/dt = -y using an Euler relation so y(n+1) = (1-h)*yn.
 Initial Conditions. *)

y0 = 5.0 ; (* starting value of y *)
t0 = 0.0;  (* starting value of t *)

(* The algorithm - Use a right difference formula to generate a finite
 difference equation to solve the nuclear decay law.
Set up the starting point. *)
h = 0.1; (* stepsize *)
yn = y0;   (* \
initial value of y *)

(* the limits of the iterations. since we already
   have y(t=0) given, then the first value of
   y we calculate is for t=h.  *)
tmin = h; (* first calculated point *)

tmax = 10.0; (* maximum t *)
stepsize = h; (* step size *)

(* create a table t1 of ordered (t,y) *)
t2 = Table[
   {t,
     yplus = yn*(1 - stepsize); (* get the next value of y *)
         yn = yplus;   (*increment yn for the next step *)
    },
   {t, tmin, tmax, h}
  ];
```

```
(* the table contains points that start at t=h so we have to add the \
starting point to the table *)

t2 = Prepend[t2, {t0, y0 - (5*Exp[t0])}];

(* Plot the results. *)
ListPlot[t2,
  PlotRange -> {{t0, tmax}, {-0.15, 0.05}},
  PlotJoined -> True,
  Frame -> True,
  FrameLabel -> {"t", "\[CapitalDelta]",
            "Two-Point algorithm - Anaytical Result", ""},
  PlotStyle -> RGBColor[1, 0, 0]
  ]
```

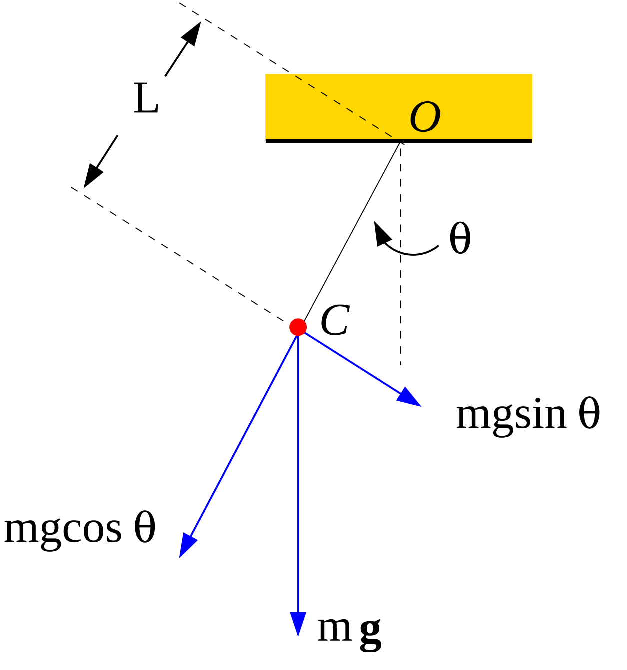
# The Pendulum - Stating the Problem

Hooke's Law states that when a spring or spring-like force is stretched or compressed then

$$F_s = -kx$$

where $F_s$ is the force exerted by the spring (the restoring force) and $x$ is the displacement from equilibrium where there is no net force acting on the mass.

1. What differential equation does $x$ satisfy?
2. What is the solution?
3. How would you prove the solution is correct?
4. What is the physical meaning of the constants in the solution?

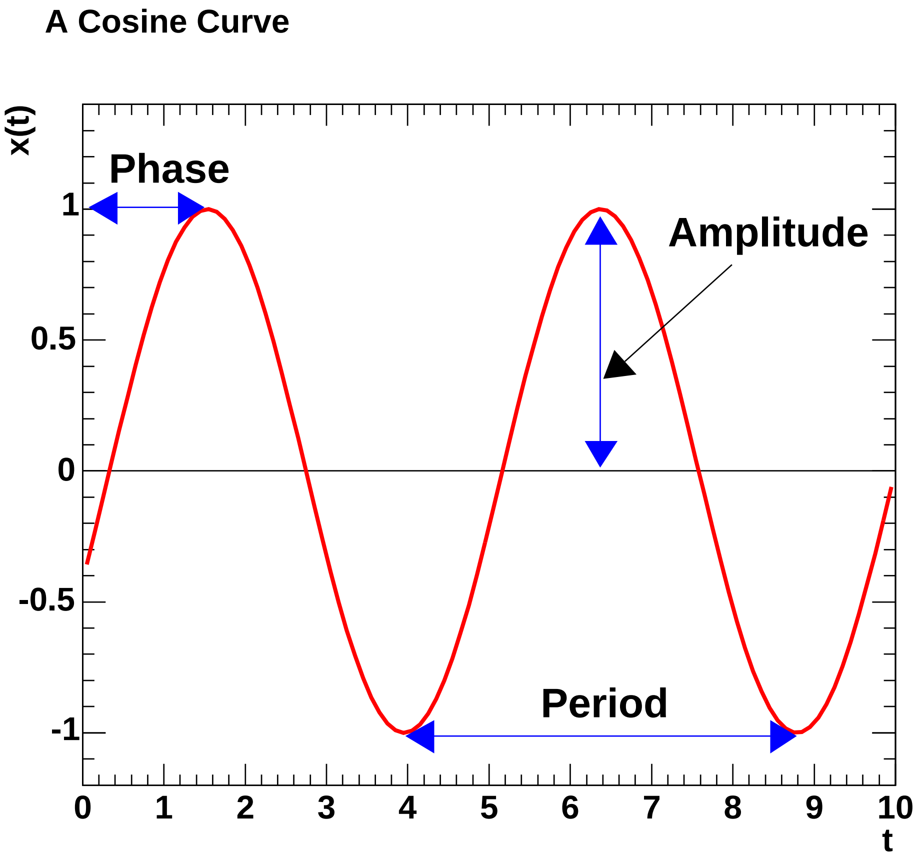


# The Simple Pendulum - The Solution

The solution for Hooke's Law is

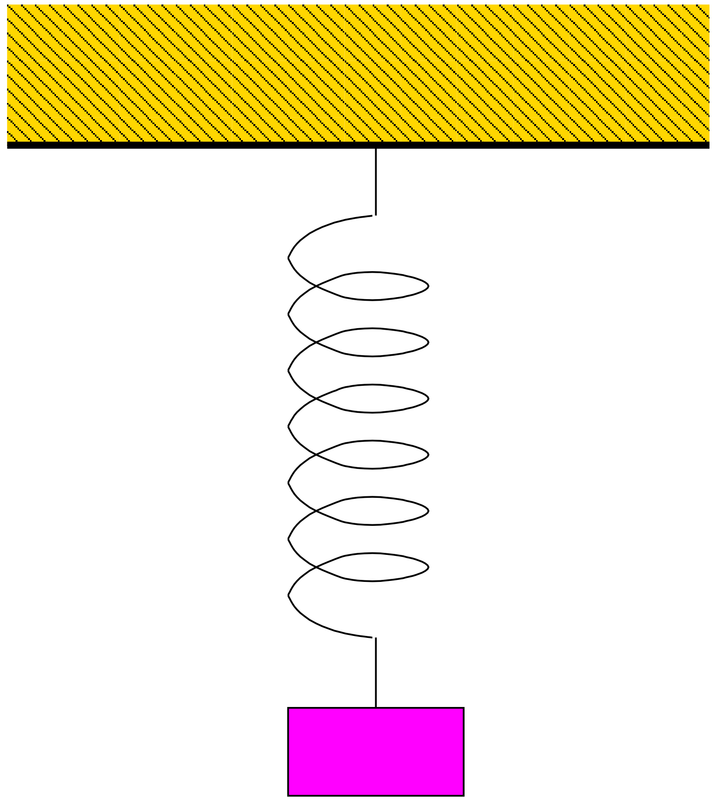
$$x(t) = A\cos(\omega t + \phi)$$

where $x(t)$ is the displacement from equilibrium.



# The Simple Pendulum - An Example

A harmonic oscillator consists of a block of mass $m = 0.33\ kg$ attached to a spring with spring constant $k = 400\ N/m$. See the figure below. At time $t = 0.0\ s$ the block's displacement from equilibrium and its velocity are $x = 0.100\, m$ and $v = -13.6\, m/s$. Find the particular solution for this oscillator. Use a centered derivative formula to generate an algorithm for solving the equation of motion.



# The Simple Pendulum with Friction

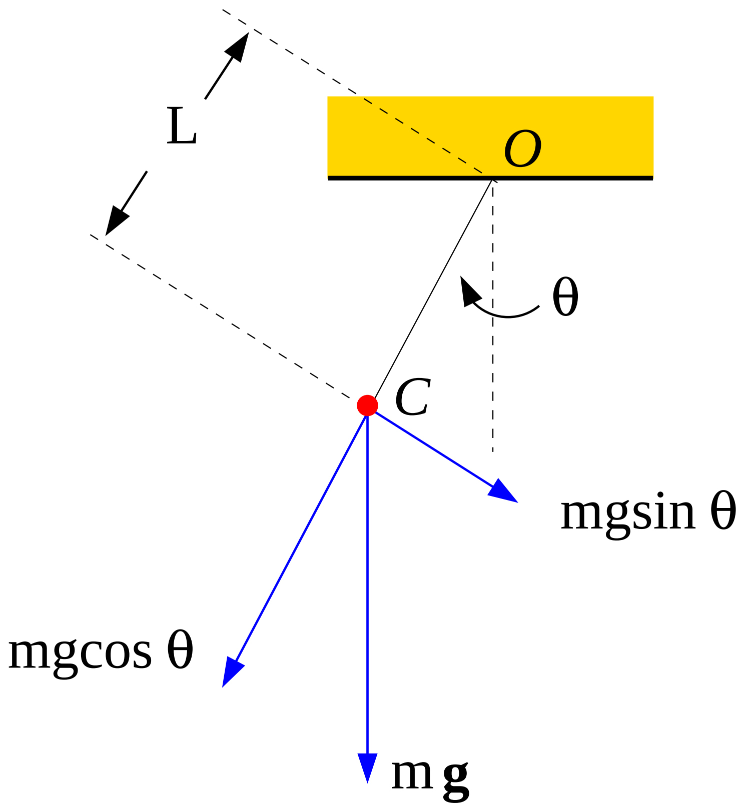
Consider the simple pendulum shown here. What is the differential equation describing the motion when the following forces are included in addition to gravity? For friction use

$$F_{friction} = -\frac{q}{L}v$$

where $q$ is a constant specific to a particular body. For the driving force use

$$F_{driving} = F_D \sin(\Omega t)$$

where $F_D$ is the magnitude of the driving force and $\Omega$ is its angular frequency.



# The Physical Pendulum

Consider the rod rotating about an end point in the figure. Starting from the definition of the torque

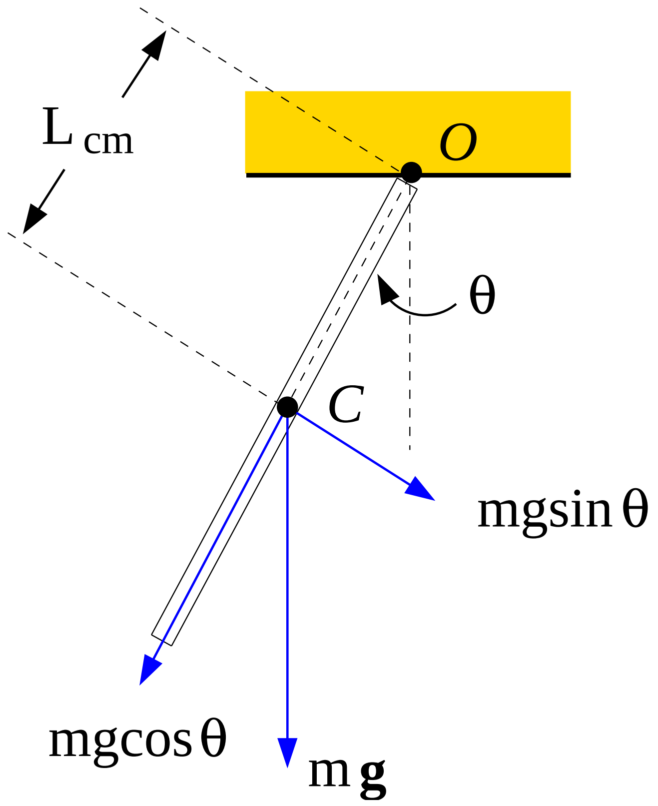
$$\vec{\tau} = \vec{r} \times \vec{F}$$

(1) derive the differential equation the angular position $\theta$ must satisfy.

(2) Derive a new differential equation if the pendulum is damped by a friction force $\vec{F}_f = -b\vec{v}$ where $b$ is some constant describing the the pendulum.

(3) Derive a final differential equation if the pendulum is now also driven by a force $\vec{F}_{drive} = F_D \sin(\Omega t)\hat{\theta}$.

(4) What does the phase space look like for each set of conditions if the initial conditions are $\theta_0 = 25^\circ$ and $\omega_0 = 0\ rad/s$?



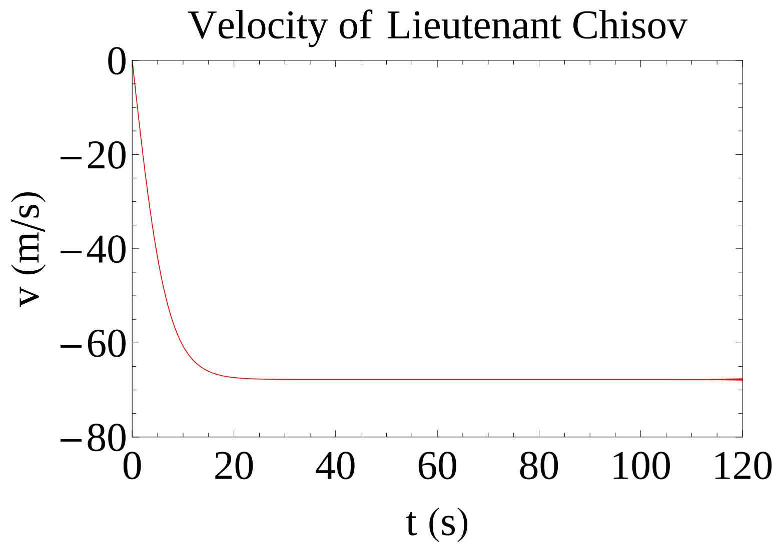
# Solution of Harmonic Oscillator Using Coupled Equations

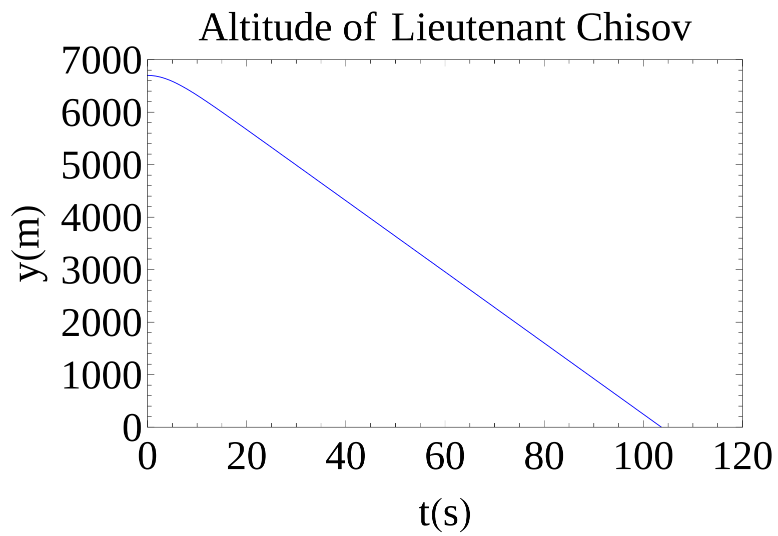
```
(* Solving the mass on a spring problem.
   Initial conditions and parameters *)
x0 = 0.0; (*   initial position in meters *)
v0 = 2.0; (* initial velocity in m/s *)
t0 = 0.0; (* initial time in seconds *)

(* set up the first two points.
   step size *)
step = 0.1;
t1 = t0 + step;
x1 = x0 + v0*step;
v1 = v0 - ( step*kspring*x0/mass);

xminus = x0;   (* initial value of x *)
vminus = v0;   (* initial value of v *)
xmid   = x1;
vmid   = v1;
mass = 0.33; (* the mass in kg *)
kspring = 0.5; (* spring constant in N/m *)

(* limits of the iterations. since we already
    have y(t=0) and we have calculated y(t=step),
    then the first value in the table will be
    for t=2*step. *)
tmin = 2*step;
tmax = 25.0;
```

```
(* create a table tpos of ordered (t,x). for each component the next value is calculated
   first and then the variables are incremented in preparation for the next interation. *)
tpos = Table[
    {t,
      vplus = vminus - (2*step*kspring/mass)*xmid;
      xplus =  xminus + (2*step*vmid);
      vminus = vmid;
      vmid  = vplus;
      xminus = xmid;
      xmid = xplus
     },
    {t, tmin, tmax, step}
   ];
```

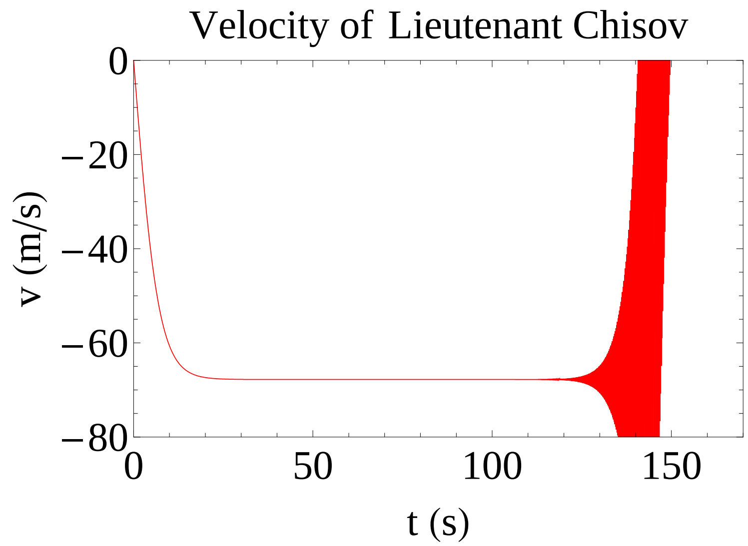
# PROBLEMS!!!!!!

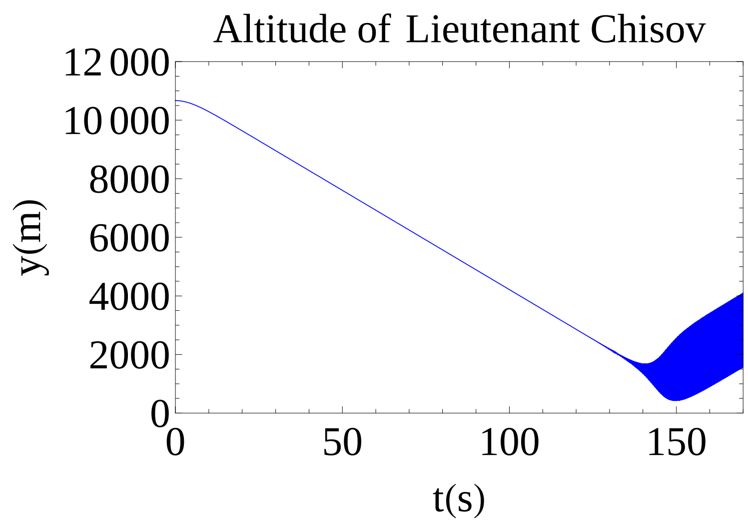
## The original set of initial conditions



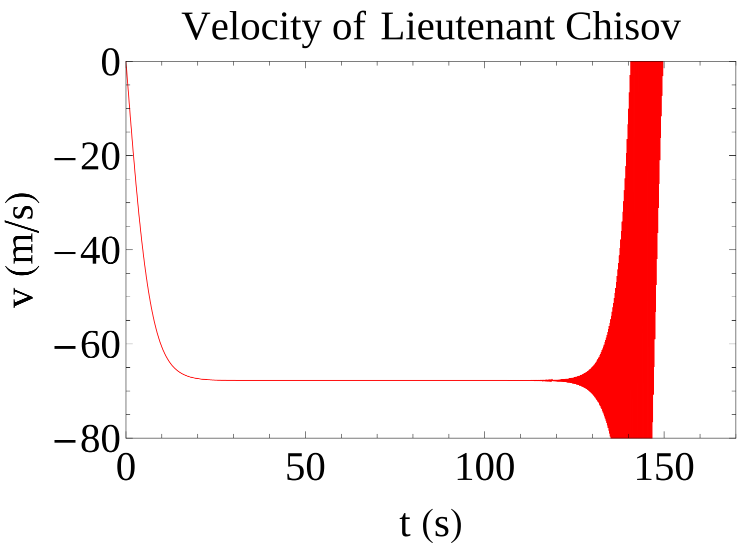

## Increase the initial altitude.

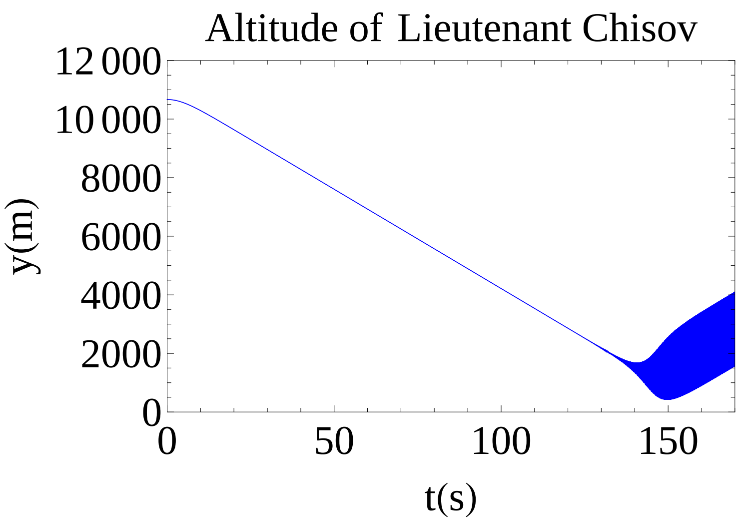


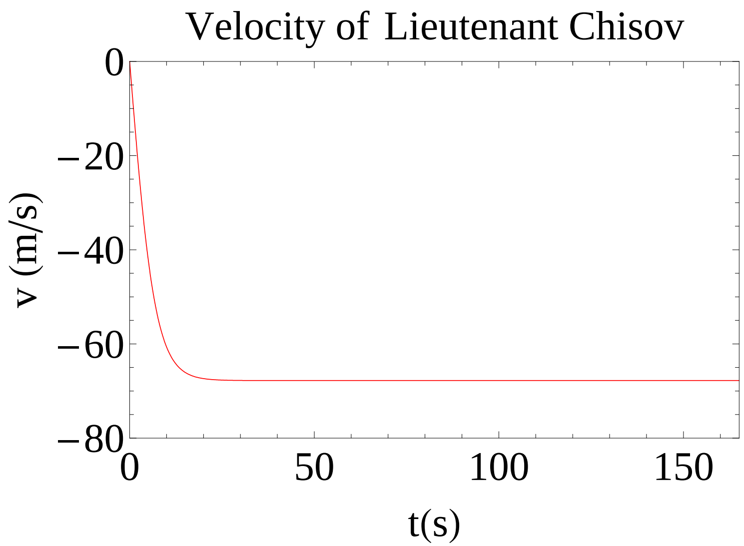
# SOLUTIONS!!!!!!

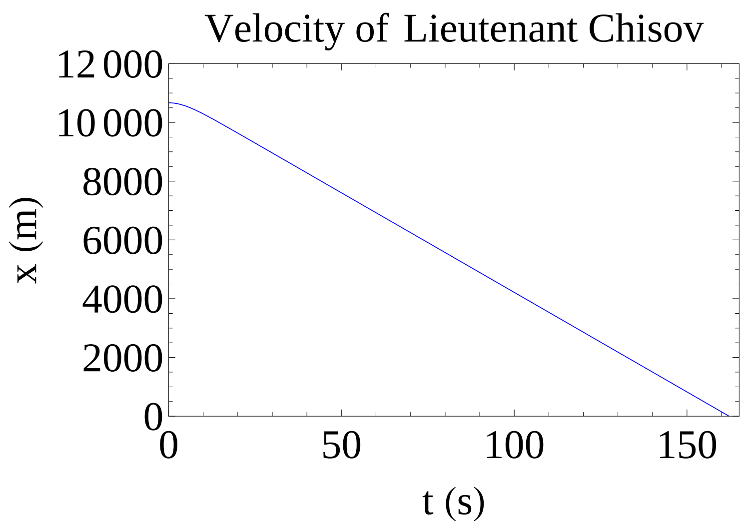
## Increase the initial altitude.




## The fix is in.



# The Stability Problem