EEEEKKKK!!!!

In January, 1942 a Soviet Ilyushin 4 flown by Lieutenant I.M.Chisov was badly damaged by German gunfire. At an altitude of 21,980 feet Lieutenant Chisov fell from the plane. Unfortunately, he did not have a parachute on when he fell. He landed on the slopes of a snow-covered ravine and slid to the bottom. He suffered a fractured pelvis and severe spinal damage, but lived. By 1974 he had become Lieutenant Colonel Chisov. How fast was Lieutenant Chisov moving when he hit the ravine? How long did his fall take?

The Drag Force

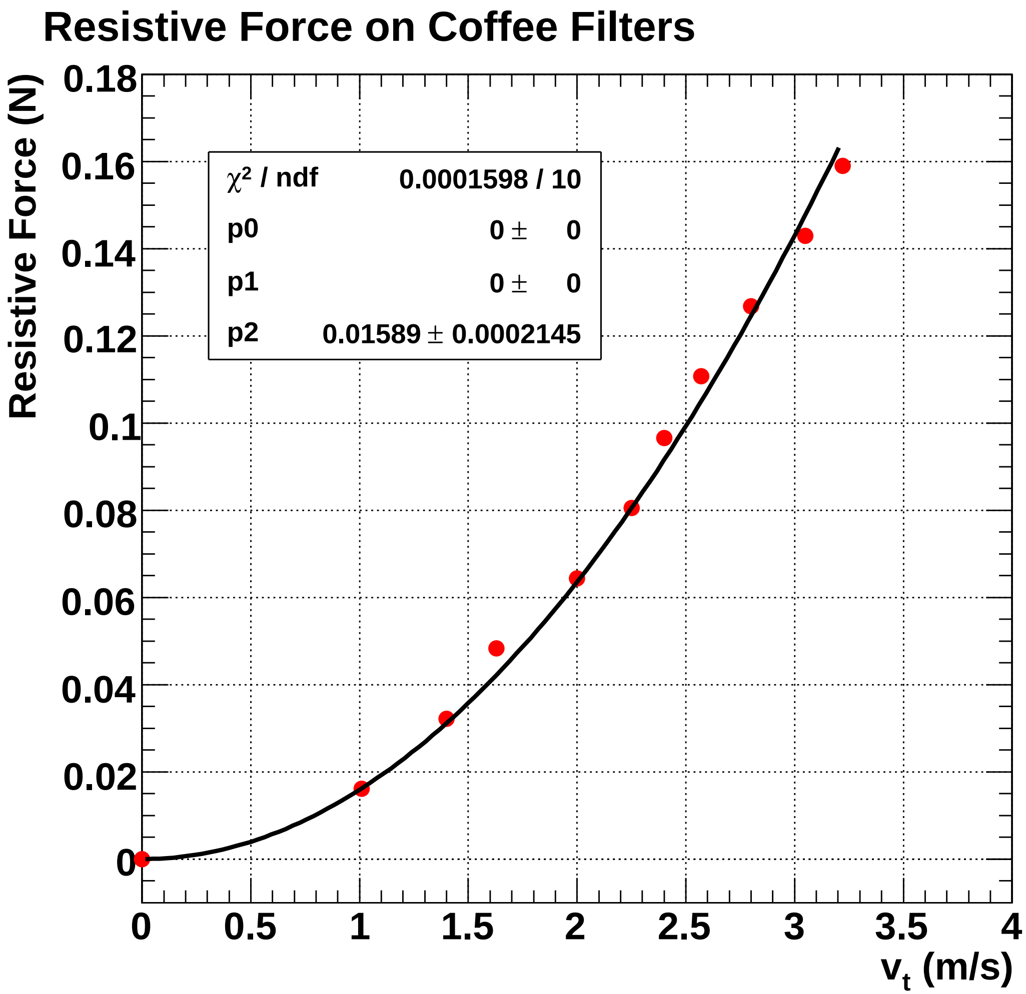
$$|\vec{F}_f| = \frac{1}{2} D\rho A v^2$$

D - drag coefficient (dimensionless).

ρ - air density (kg/m^3).

A - Cross sectional area (m^2).

v - speed (m/s).



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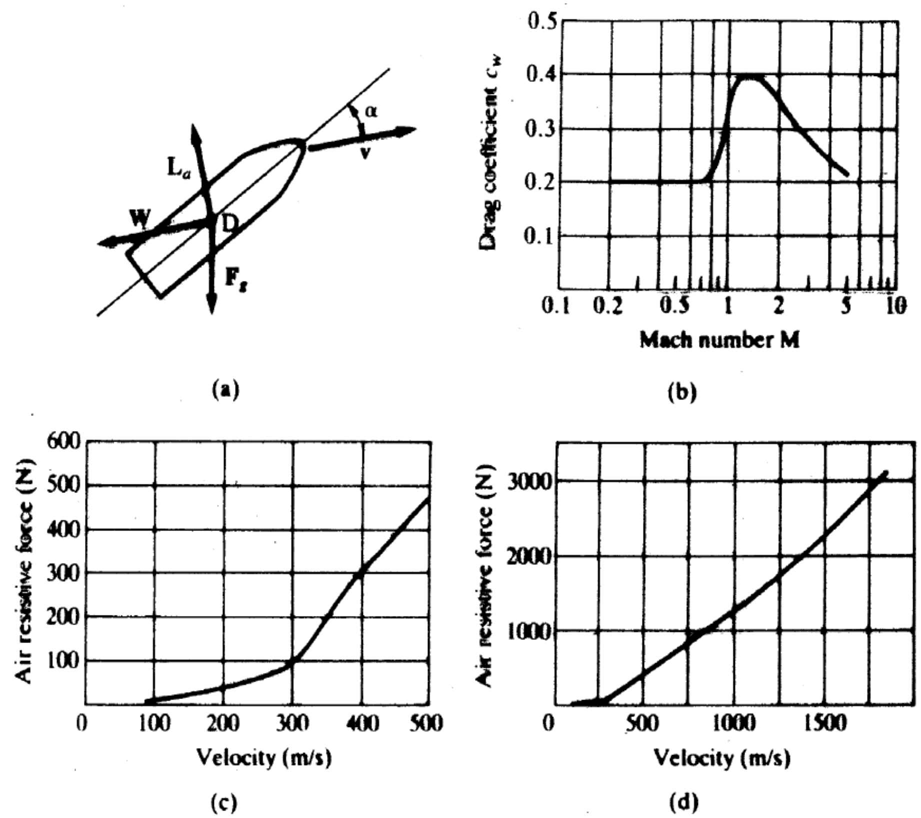
The Drag Force



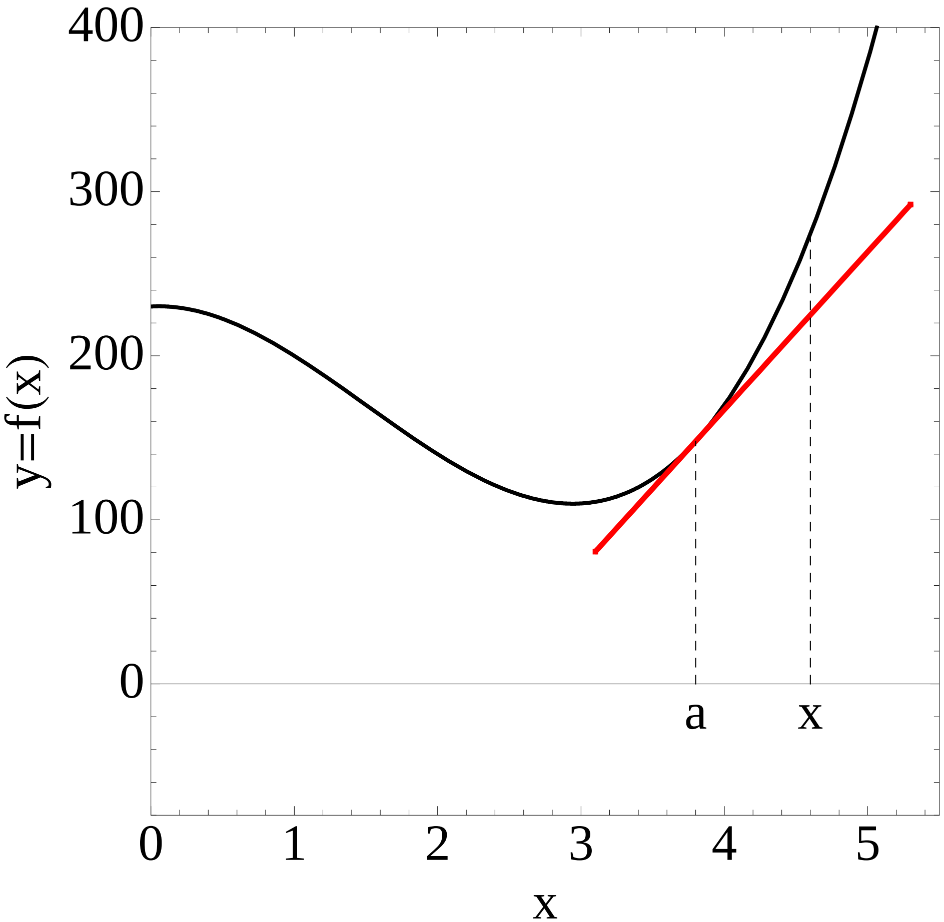
Aerodynamic forces acting on an artillery shell. The force $\vec{W}$ is the drag or air resistive force, $\vec{L}_a$ is the lift, $\vec{F}_g$ is gravity, and the point D is the center of pressure. Note the change in the air resistive force at the speed of sound.

Resistive Force on a Baseball

Jonathan Papelbon gives Derek Jeter some chin music at 90 mph (40 m/s). What is the drag coefficient of a baseball? What is the resistive force at that speed? The mass of a baseball is 0.145 kg, its radius is 3.7 cm, and its terminal velocity is measured to be 43 m/s.

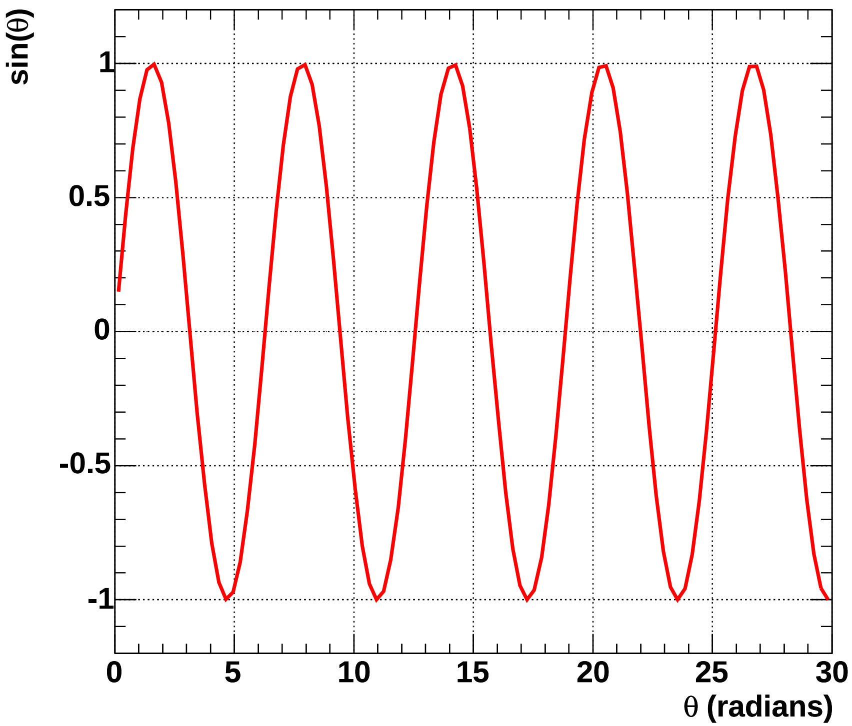
Approximating a Function

The plot below shows an arbitrary curve (black) with the tangent curve (red) at one point.



Taylor Series for the Sine function

The plot below shows the sine function. What are the first two nonzero terms of the Taylor series for the sine function expanded about the point $\theta = 0$? How close do they come to the sine function?



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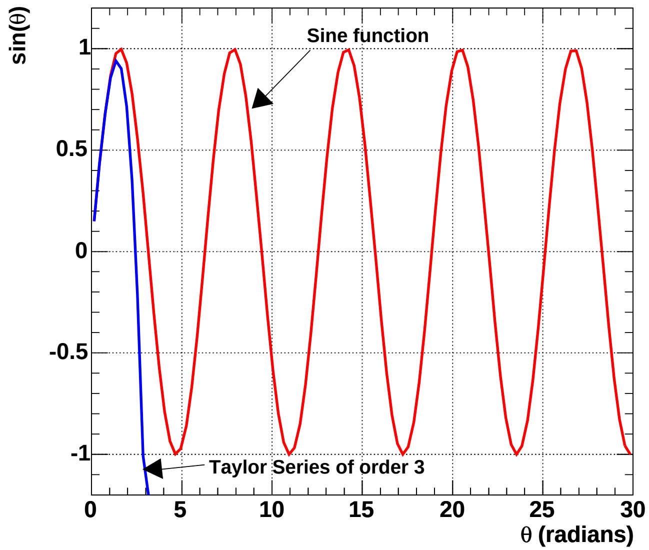
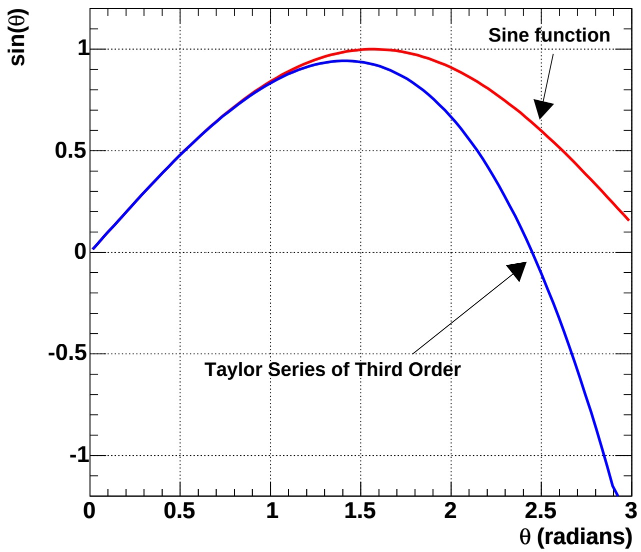
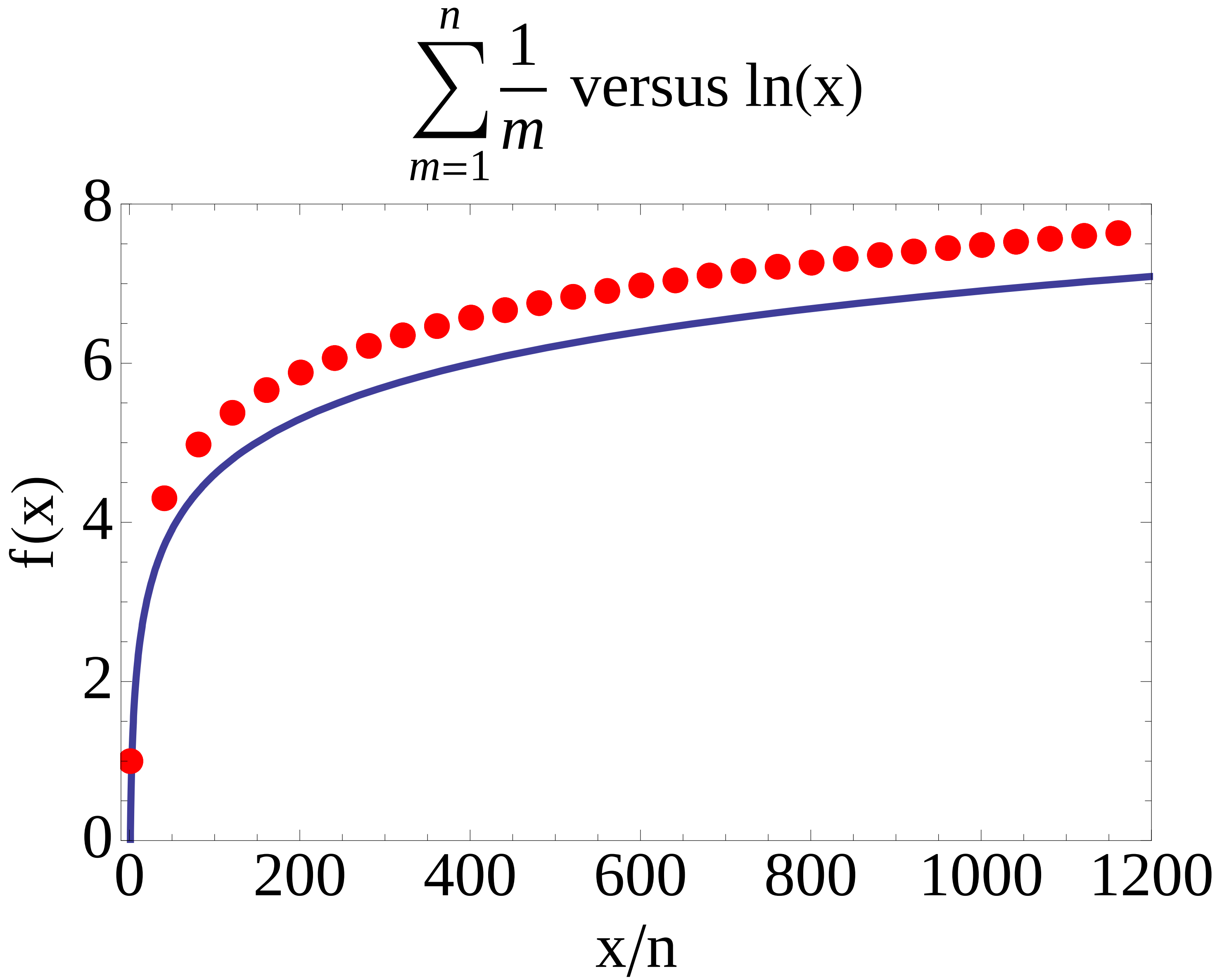
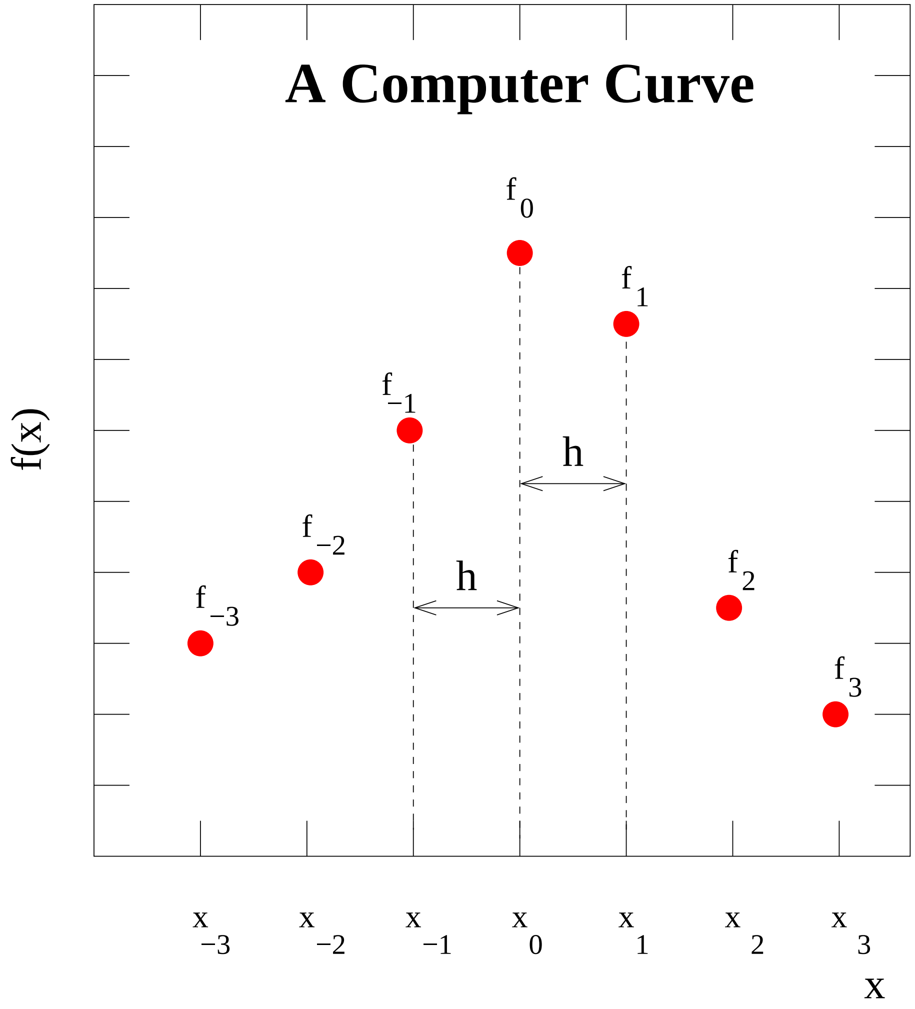




The Integral Convergence Test



Numerical Differentiation of a Curve



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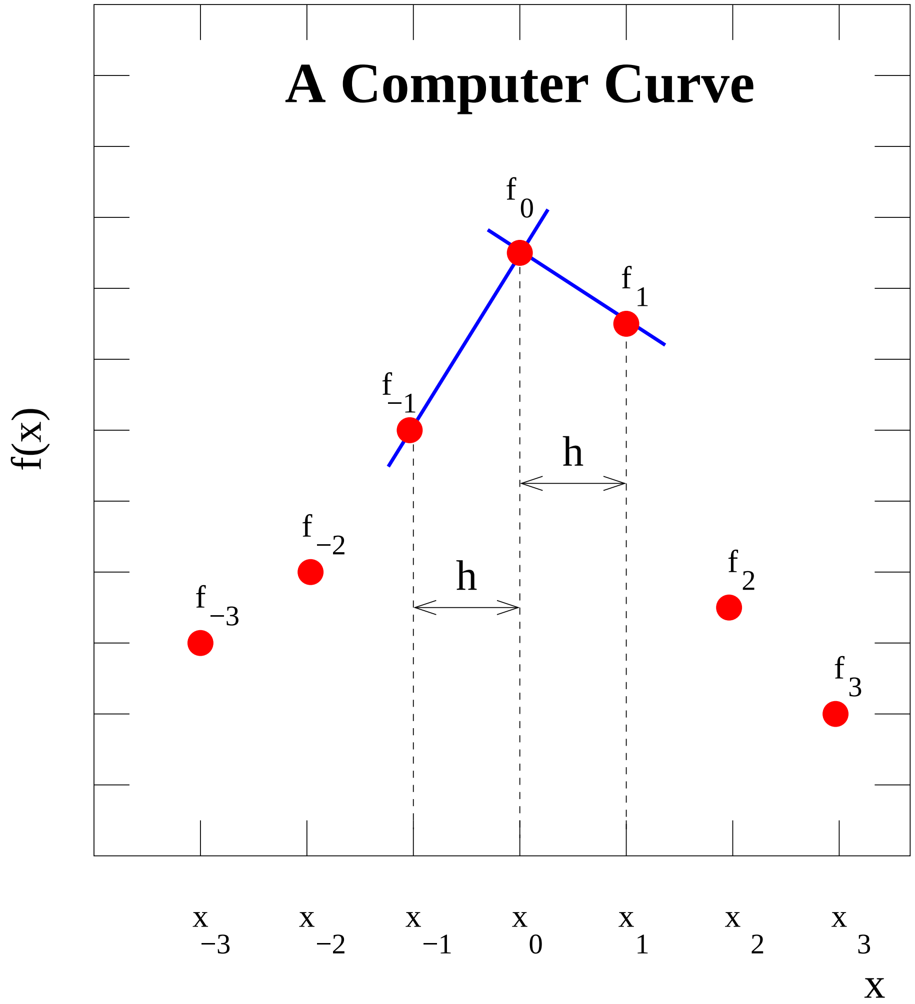

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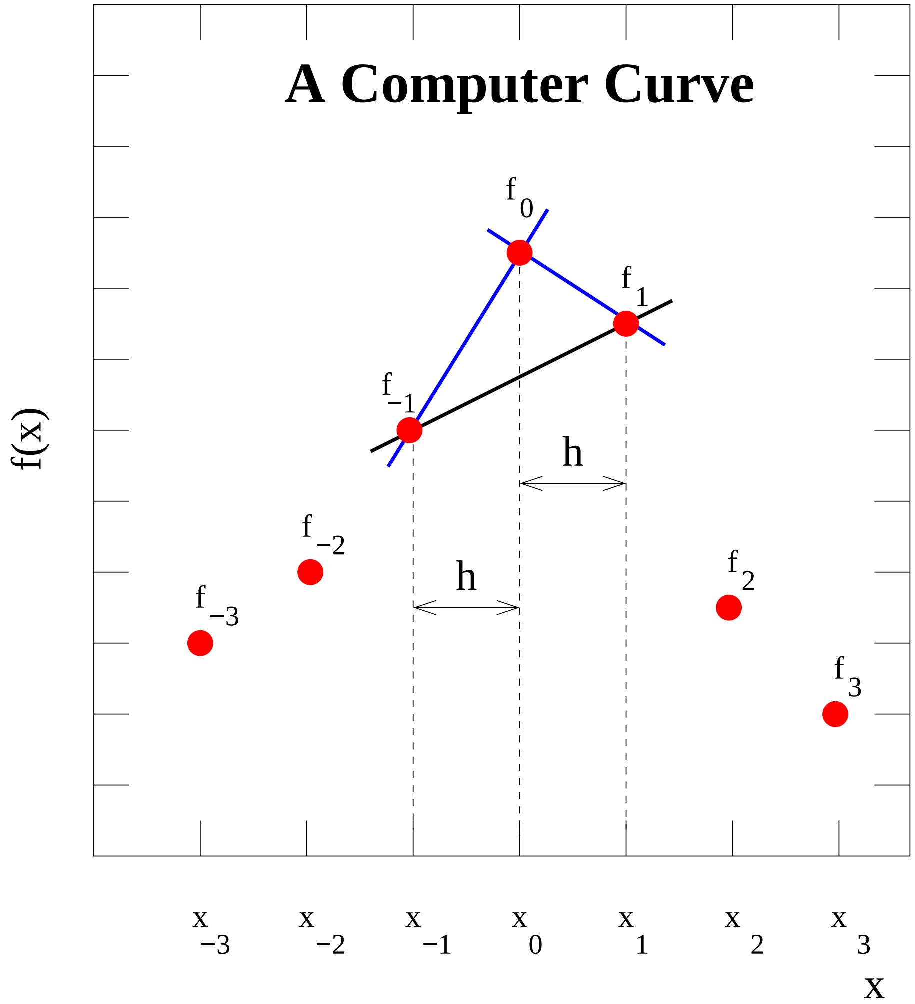

Life is a Differential Equation (DE)

Some definitions first

Ordinary DE	One independent variable; only total derivatives.

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General Solution	Results of integrating the DE; n^{th} order DE require n conditions to fix all the constants.
Particular Solution	General solution plus the n conditions.
Initial Value Problem	Find $v(t)$ and $y(t)$ when $v(t_0) = v_0$ or $y(t_0) = y_0$.

Solving First-Order, Ordinary DEs

1. The differential equation and the initial conditions are given. For example

$$\frac{dy}{dt} = f(t, y) \quad \text{and} \quad y(t_0) = y_0$$

   and $y(t_1)$ is unknown.

2. Divide the range $[t_0, t_1]$ into pieces.
3. Generate a recursion relationship between adjacent points.
4. Perform a step-by-step integration.

Nuclear Decay

The rate of radioactive decay of atomic nuclei is proportional to the number of nuclei N in the sample so

$$\frac{dN}{dt} = -\lambda N$$

where λ is a constant of proportionality (related to the half-life) and the negative sign means the number of nuclei is decreasing. The initial condition is $N(t=0) = N_0 = 1000$.

1. Write down the analytical solution.
2. Generate an algorithm to solve this differential equation and apply it for the first three values of $N(t)$ 'by hand' for $h = \Delta t = 0.1\ s$ and $\lambda = 0.2\ s^{-1}$.
3. Find the solution for the first sixty seconds.

Nuclear Decay Results

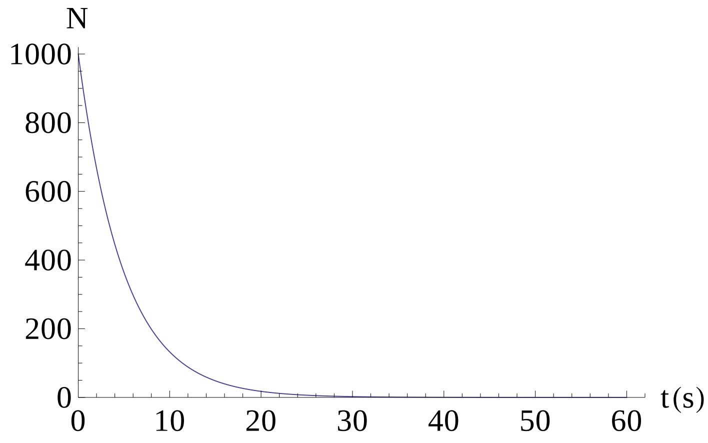
t(s)	Calculation	Result	Analytic Result
0	$N_0 = N_0$	1000	1000
0.1	$N_1 = N_0 (1 - \lambda h)$	980.	980.199
0.2	$N_2 = N_1 (1 - \lambda h)$	960.4	960.789
0.3	$N_3 = N_2 (1 - \lambda h)$	941.192	941.765
0.4	$N_4 = N_3 (1 - \lambda h)$	922.368	923.116
0.5	$N_5 = N_4 (1 - \lambda h)$	903.921	904.837
0.6	$N_6 = N_5 (1 - \lambda h)$	885.842	886.92
0.7	$N_7 = N_6 (1 - \lambda h)$	868.126	869.358
0.8	$N_8 = N_7 (1 - \lambda h)$	850.763	852.144
0.9	$N_9 = N_8 (1 - \lambda h)$	833.748	835.27
1.	$N_{10} = N_9 (1 - \lambda h)$	817.073	818.731

Solution of $\frac{dN}{dt} = -\lambda N$ using an Euler algorithm

```
(* initial parameter values *)
N0 = 1000.0;
lambda = 0.2;
t0 = 0.0;
t1 = 60.0;
h = 0.1;

(* starting point of recursion relationship. *)
Nn = N0;

(* make the table *)
table1 = Table[
   {t,
    Nplus = Nn*(1 - lambda*h);
    Nn = Nplus
    },
   {t, t0 + h, t1, h}];

(* stick the starting point at the front of the table. *)
table1 = Prepend[table1, {t0, N0}];
```

Results

t (s)	N
0.0	1000.0
0.1	980.0
0.2	960.4
0.3	941.2



The Limits of Accuracy

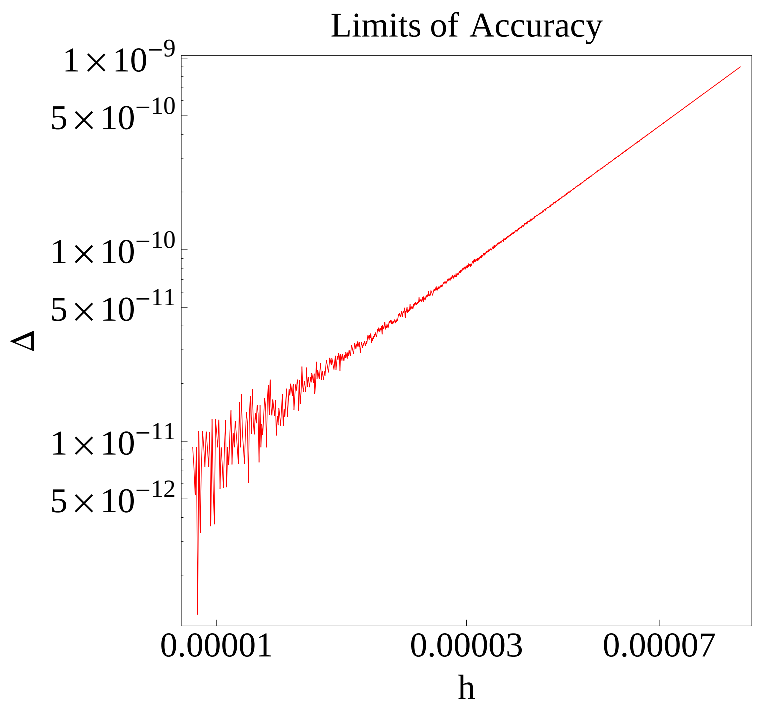
1. Consider the following code fragment. Why does $a \neq b$?

```
a = 1.0*10^17  + 1.0 - 1.0*10^17;
b = 1*10^17 + 1 - 1*10^17;
Print["a=", a, " b=", b]
a=0. b=1
```

2. Consider the following function.

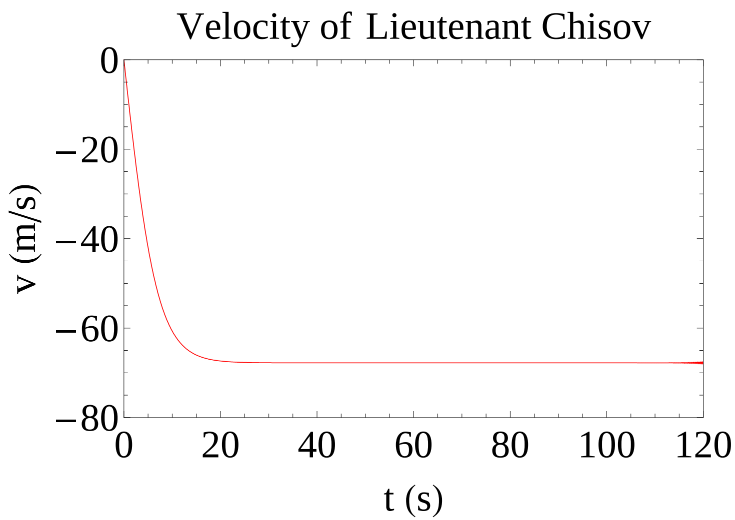
$$\Delta = \cos\theta - \frac{\sin(\theta+h)-\sin(\theta-h)}{2h}$$

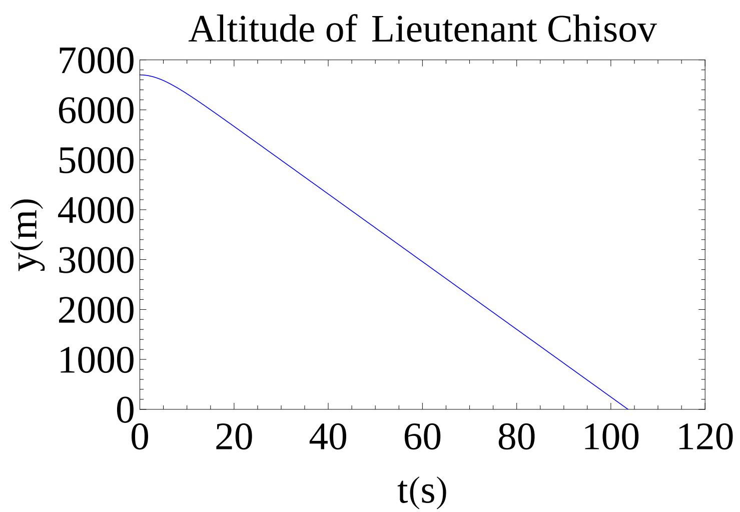
The plot shows the dependence of the function on the stepsize h.



PROBLEMS!!!!!!

Original initial conditions.

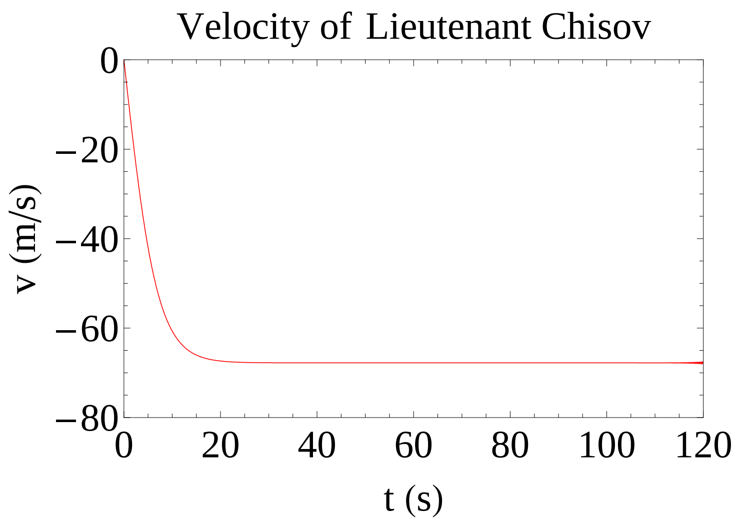


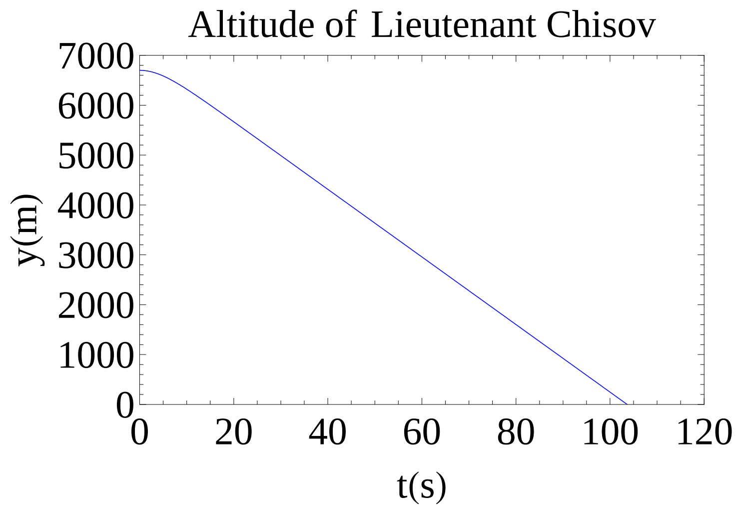


Increase the initial altitude.

PROBLEMS!!!!!!

Original initial conditions.





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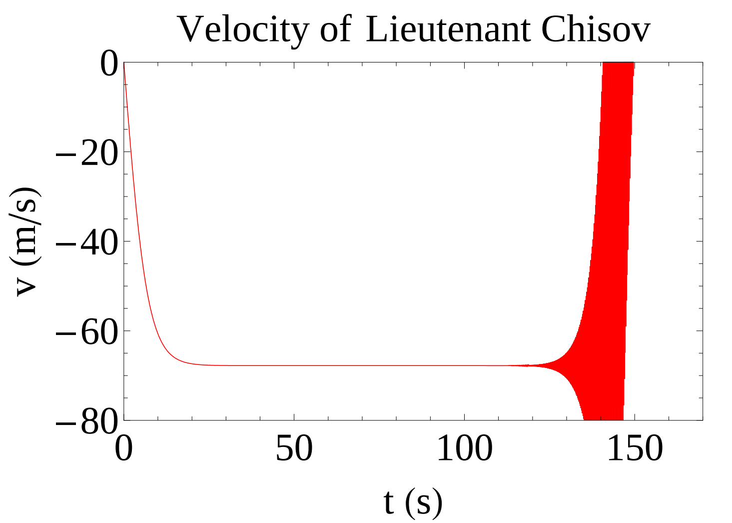
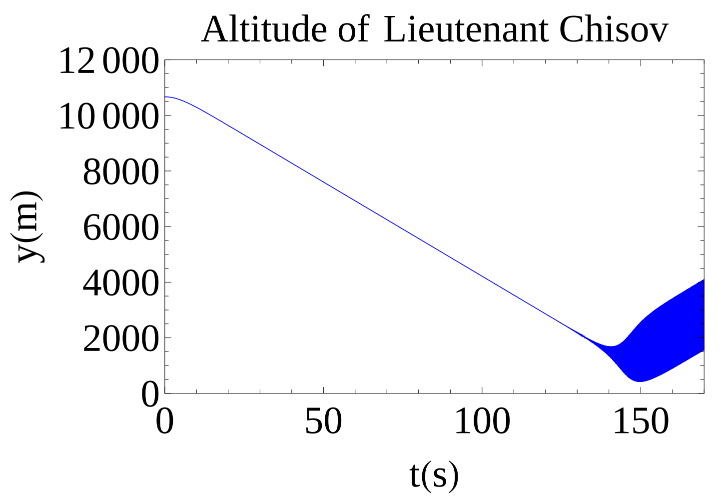
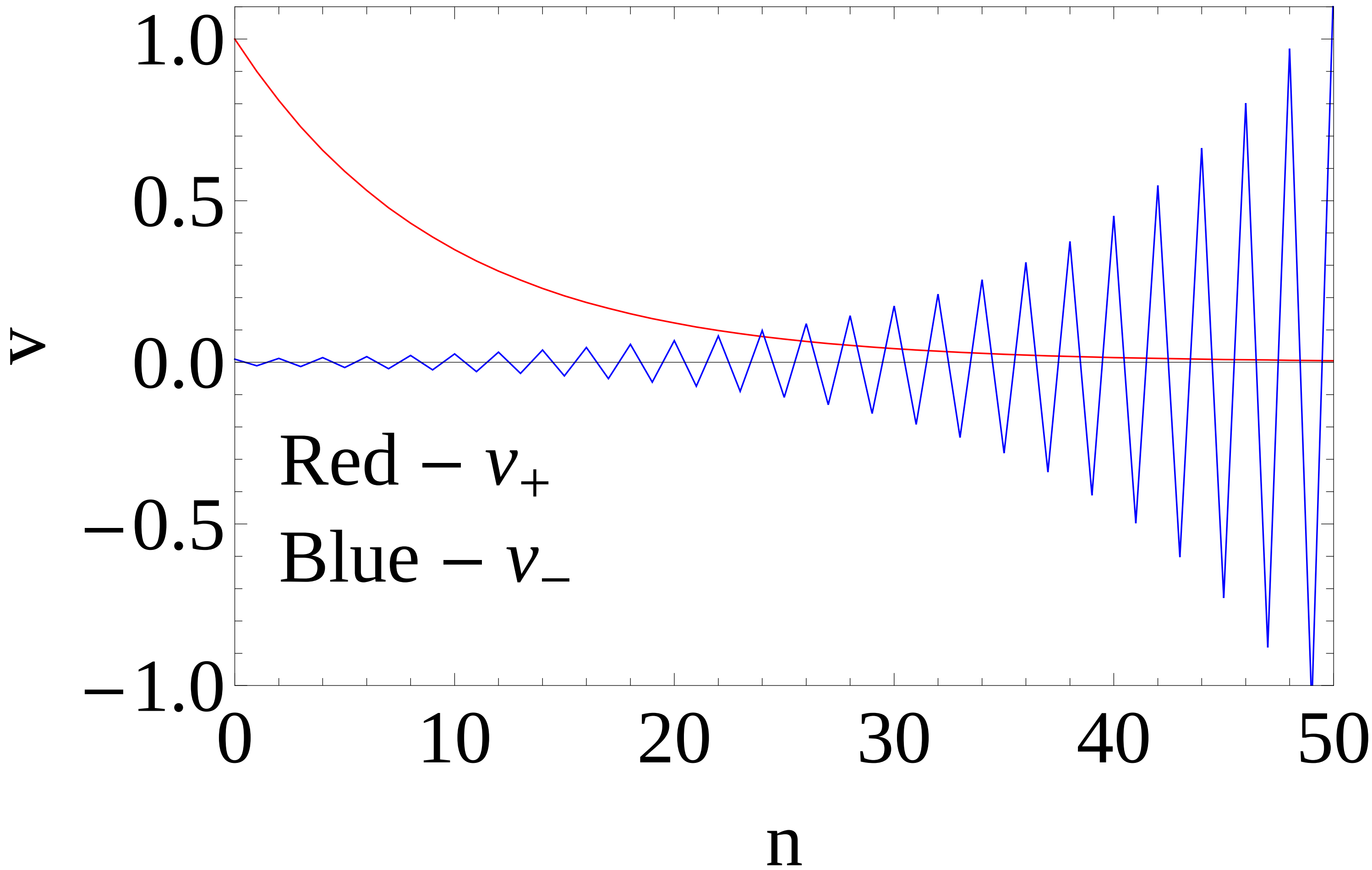


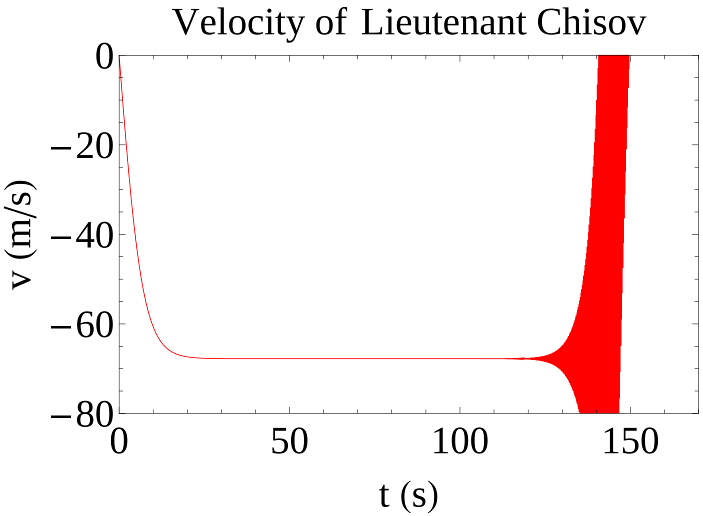


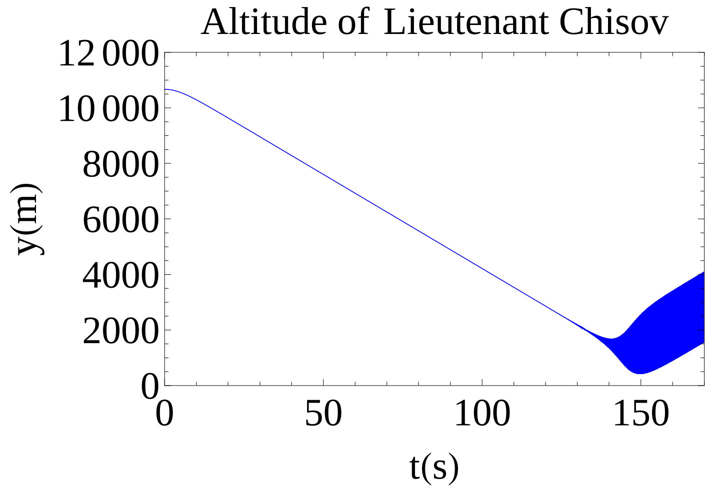
The Stability Problem



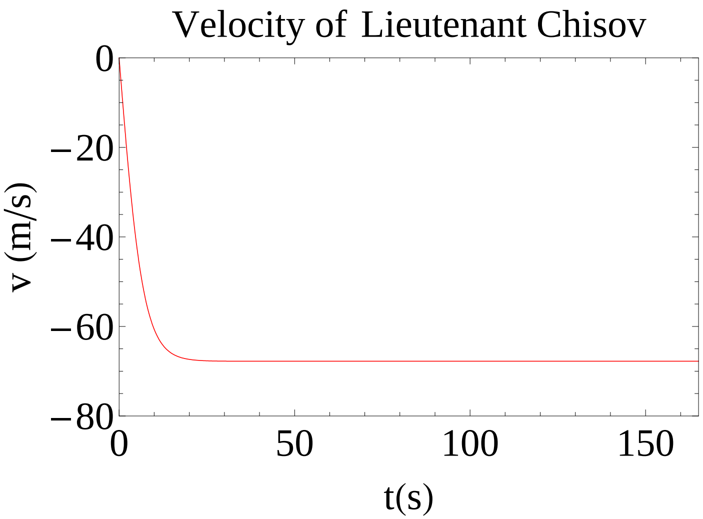
SOLUTIONS!!!!!!

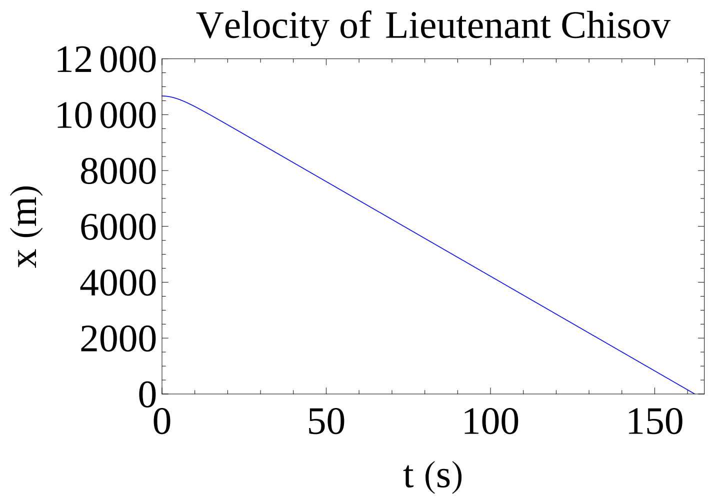
Increase the initial altitude.





The fix is in.





The Harmonic Oscillator - Stating the Problem

Hooke's Law states that

$$F_s = -kx$$

where F_s is the force exerted by a spring (the restoring force) and x is the displacement from equilibrium where there is no net force acting on the mass.

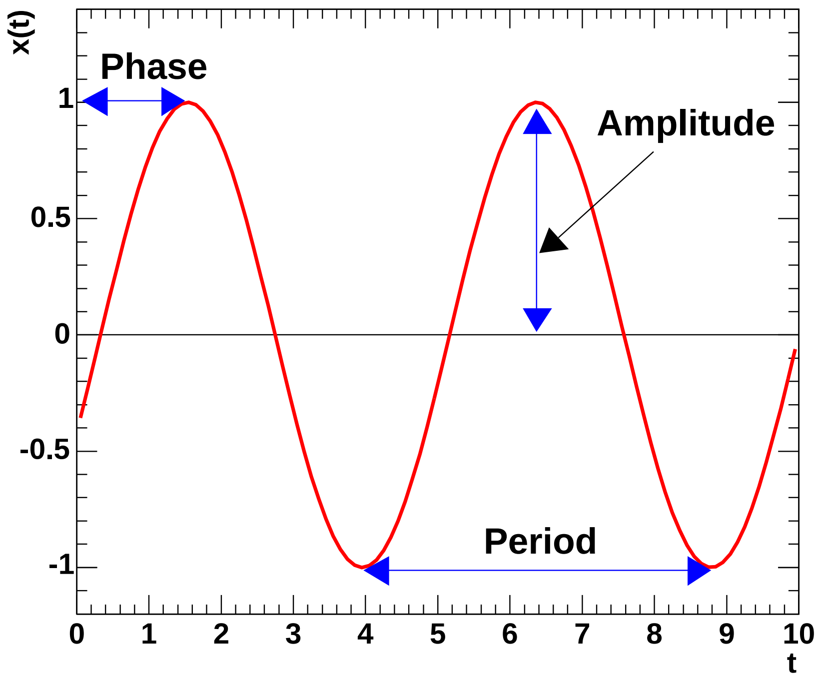
1. What differential equation does x satisfy?
2. What is the solution?
3. How would you test the solution?
4. What is the physical meaning of the constants in the solution?

The Harmonic Oscillator - The Solution

The solution for Hooke's Law is

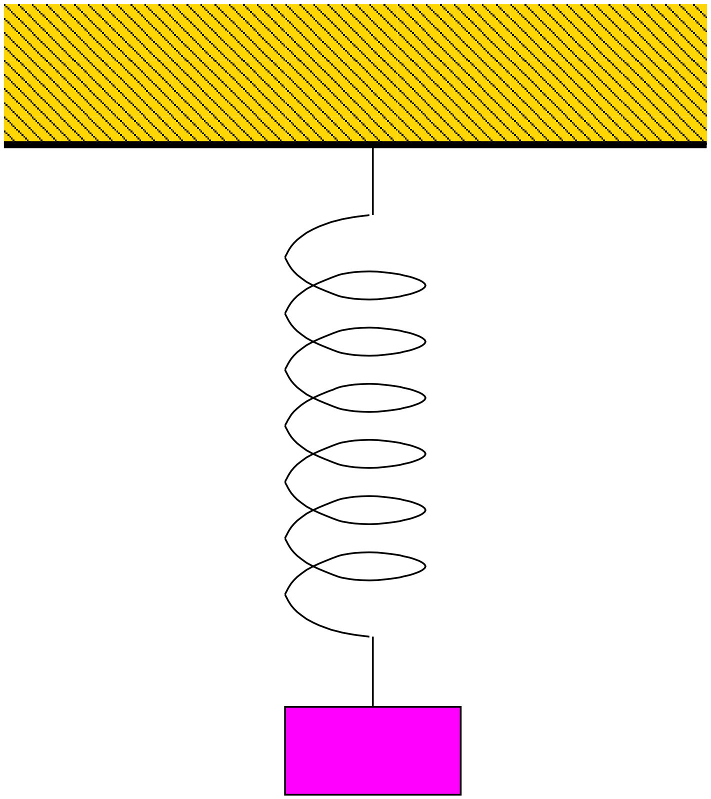
$$x(t) = A\cos(\omega t + \phi)$$

where $x(t)$ is the displacement from equilibrium.



The Simple Harmonic Oscillator - An Example

A harmonic oscillator consists of a block of mass $m = 0.33\ kg$ attached to a spring with spring constant $k = 400\ N/m$. See the figure below. At time $t = 0.0\ s$ the block's displacement from equilibrium and its velocity are $y = 0.100\ m$ and $v = -13.6\ m/s$. (1) Find the particular solution for this oscillator. (2) Use a centered derivative formula to generate an algorithm for solving the equation of motion.



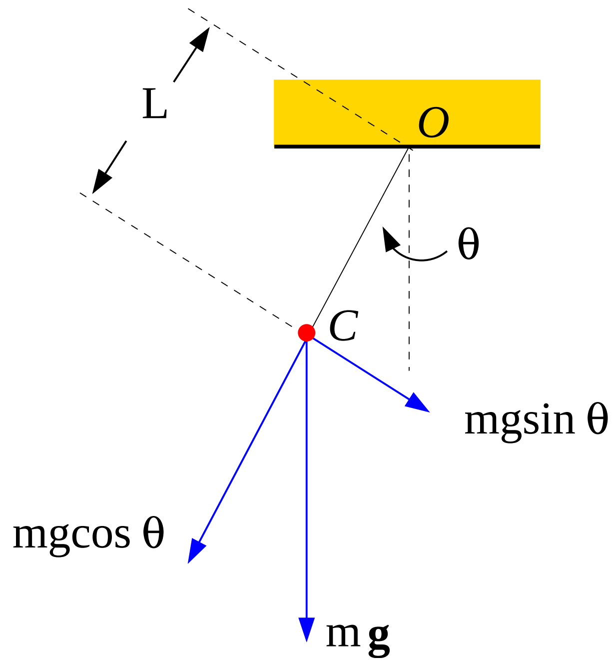
The Pendulum - Stating the Problem

Hooke's Law states that

$$F_s = -kx$$

where F_s is the force exerted by a spring (the restoring force) and x is the displacement from equilibrium where there is no net force acting on the mass. One can show the similarity between the simple pendulum and the harmonic oscillator.

1. What differential equation does θ satisfy for small angles?
2. What is the solution?
3. How would you test the solution?



The Simple Pendulum with Friction

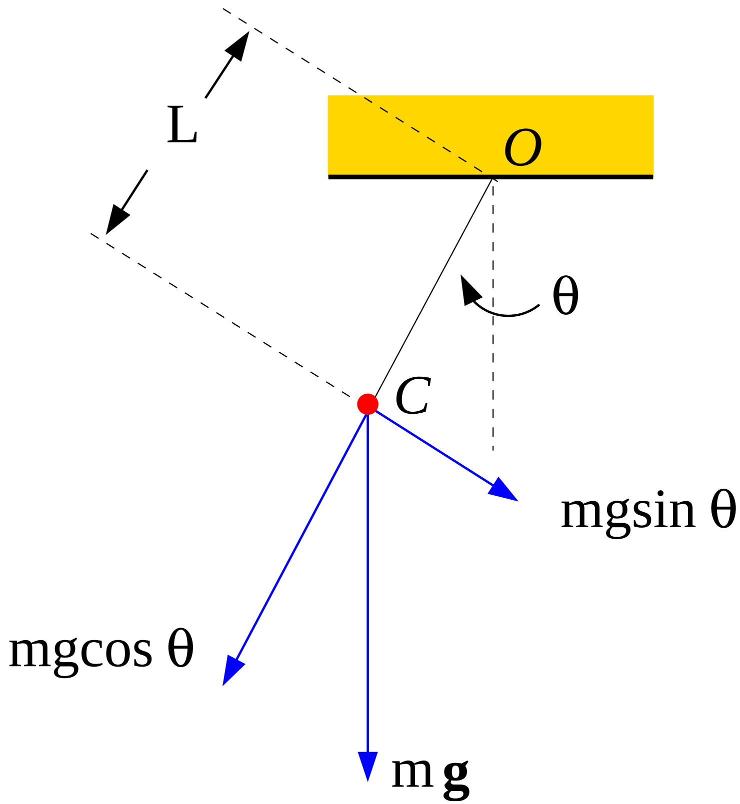
Consider the simple pendulum shown here. What is the differential equation describing the motion when the following forces are included in addition to gravity? For friction use

$$F_{friction} = -\frac{q}{L}v$$

where q is a constant specific to a particular body. For the driving force use

$$F_{driving} = F_D \sin(\Omega t)$$

where F_D is the magnitude of the driving force and Ω is its angular frequency.



The Physical Pendulum

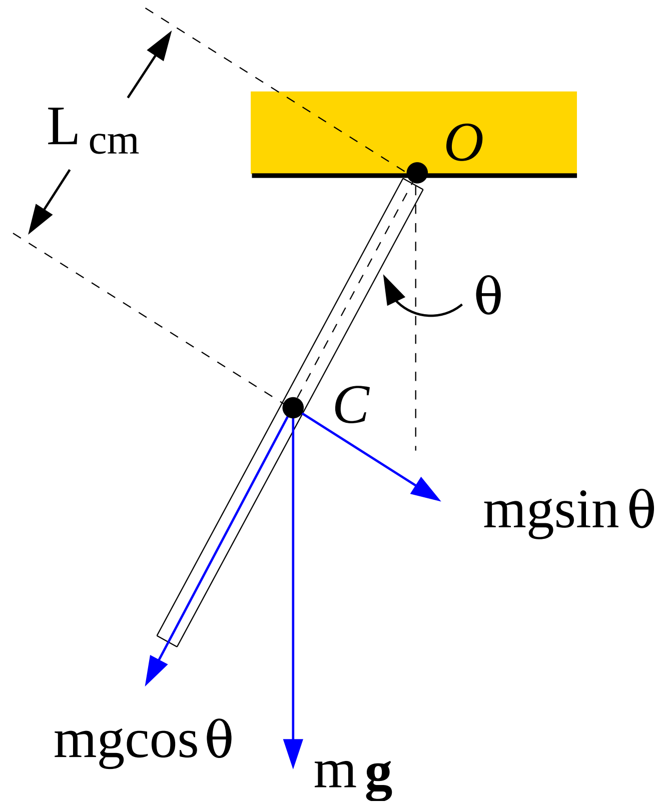
Consider the rod rotating about an end point in the figure. Starting from the definition of the torque $\vec{\tau} = \vec{r} \times \vec{F}$,

(1) derive the differential equation the angular position θ must satisfy.

(2) Derive a new differential equation if the pendulum is damped by a friction force $\vec{F}_f = -b\vec{v}$ where b is some constant describing the the pendulum.

(3) Derive a final differential equation if the pendulum is now also driven by a force $\vec{F}_{drive} = F_D \sin(\Omega t)\hat{\theta}$.

(4) What does the phase space look like for each set of conditions if the initial conditions are $\theta_0 = 25°$ and $\omega_0 = 0\ rad/s$?



Harmonic Oscillator With Coupled Equations - 1

```
(* Solving the mass on a spring problem.
   Initial conditions and parameters *)
x0 = 0.0; (*   initial position in meters *)
v0 = 2.0; (* initial velocity in m/s *)
t0 = 0.0; (* initial time in seconds *)

(* set up the first two points.
   step size *)
step = 0.1;
t1 = t0 + step;
x1 = x0 + v0*step;
v1 = v0 - ( step*kspring*x0/mass);

xminus = x0;   (* initial value of x *)
vminus = v0;   (* initial value of v *)
xmid   = x1;
vmid   = v1;
mass = 0.33; (* the mass in kg *)
kspring = 0.5; (* spring constant in N/m *)
```

Harmonic Oscillator With Coupled Equations - 2

```
(* limits of the iterations. since we already have y(t=0) and we
    have calculated y(t=step), then the first value in the table
    will be for t=2*step. *)
tmin = 2*step;
tmax = 25.0;

(* create a table of ordered (t,x). for each component the next value is
   calculated and then variables are incremented for the next interation.
tpos = Table[
    {t,
      vplus = vminus - (2*step*kspring/mass)*xmid;
      xplus =  xminus + (2*step*vmid);
      vminus = vmid;
      vmid  = vplus;
      xminus = xmid;
      xmid = xplus
     },
    {t, tmin, tmax, step}
   ];
```