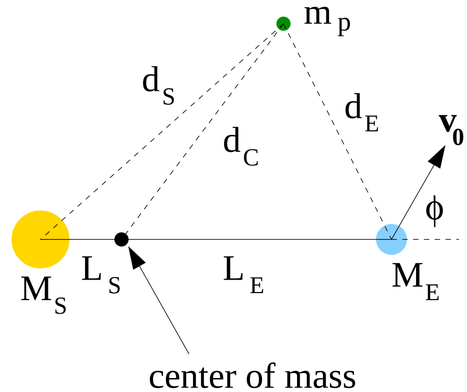
Shooting the Sun

Consider a two-body solar system with just the Sun and and the Earth (see the figure below). Assume the Earth (mass M_E) and the Sun (mass M_S) follow circular orbits around the center of mass of the Earth-Sun system. The Earth-Sun distance is $R_E = L_S + L_E$. The origin is placed at the center of mass (CM) and at $t = 0$ the Earth and Sun are in the configuration shown in the figure. A projectile (mass m_p) is launched from the Earth at $t = 0$ with a mass much, much smaller than the mass of the Earth or the Sun.

1. What is the angular speed ω of the Earth and Sun about the CM?
2. The Earth and Sun are a distance $R_E = 1.5 \times 10^{11}\ m$ apart. What is the distance from the center of mass (CM) to the Earth L_E and from the CM to the Sun L_S in terms of R_E and the masses?
3. What is the expression for the position of the Sun $\vec{r}_E(t)$ and the Earth $\vec{r}_E(t)$ in terms of L_S, L_E, ω, and t?



4. What is the force $\vec{F}_S$ (in full vector form) on the projectile due to the Sun in terms of the coordinates of the projectile (x_p, y_p) and the parameters for the position of the Sun L_S, ω, and t?
5. What is the force $\vec{F}_E$ (in full vector form) between the projectile and the Earth in terms of the coordinates of the projectile (x_p, y_p) and the parameters for the position of the Earth L_E, ω, and t?
6. What is the distance d_S (see the figure) in terms of the coordinates of the projectile (x_p, y_p) and the parameters for the position of the Sun L_S, ω, and t?
7. What is the distance d_E (see the figure) in terms of the coordinates of the projectile (x_p, y_p) and the parameters for the position of the Sun L_E, ω, and t?
8. What is the vector force $\vec{F}_p$ in terms of d_S, d_E, ω, L_S, L_E, and the position coordinates (x_p, y_p) of the projectile?
9. Show that the position coordinates (x_p, y_p) of the projectile obey the following.

$$\ddot{x} = -\frac{GM_S}{d_S^3}(x_p + L_S \cos\omega t) - \frac{GM_E}{d_E^3}(x_p - L_E \cos\omega t)$$

$$\ddot{y} = -\frac{GM_S}{d_S^3}(y_p + L_S \sin\omega t) - \frac{GM_E}{d_E^3}(y_p - L_E \sin\omega t)$$

10. Using the result from Part 9, generate the algorithm to integrate each component x and y of the acceleration on the projectile. Use a three-point, centered formula for the second derivative. At this point just get the recursion relationship.

11. The projectile will be launched from the Earth's surface at an angle ϕ as shown in the figure. What are the position and velocity vectors at $t = 0$? Express the initial velocity of the projectile in terms of the Earth's velocity $\vec{v}_E$ at $t = 0$ and the projectile velocity $\vec{v}_0$ relative to the Earth (see figure). It will be helpful later to express the initial speed v_0 as a multiple s_e of the escape velocity. Calculate the escape velocity for a stationary Earth v_e. Get the initial velocity of the projectile in terms of $\vec{v}_E$, v_e, s_e, and ϕ.

12. In order to use the three-point, centered formula for the recursion relationship we need the first two points on the trajectory of the projectile. We are given the initial position and velocity. How would you obtain the second point on the trajectory?