The Diffusion Equation

1. Consider the one-dimensional diffusion equation corresponding to particles of density $n(x,t)$ in a long pipe of length L with no sources (*i.e.*, $C = 0$).

$$\frac{\partial n(x,t)}{\partial t} = D\frac{\partial^2 n}{\partial x^2}$$

For each of the following descriptions, write down equations for the boundary conditions, calculate the dispersion relationship, and find the particular solution of the separated form $n(x,t) = X(x)T(t)$ which is bounded and nonzero.

   (a) Both ends of the pipe at $x = 0$ and $x = L$ are open and the density goes to zero outside the pipe.

   (b) The end of the pipe at $x = 0$ is open and the outside density is zero. At $x = L$ the pipe is closed so the particle flux J_x vanishes there.

   (c) Both ends of the pipe are closed so the particle flux J_x vanishes there.

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2. Consider the one-dimensional diffusion equation corresponding to particle in a long pipe of length L.

$$\frac{\partial n(x,t)}{\partial t} = D\frac{\partial^2 n}{\partial x^2} + Cn$$

where $n(x,t)$ is the particle density, D is the self-diffusion coefficient, and C is the creation rate. In this problem there IS a source of particles ($C \neq 0$).

   (a) What is the dispersion relationship?

   (b) Suppose the particle density is fixed at the value n_0 at the ends of the pipe so $n(x = 0, t) = n(x = L, t) = n_0$. What is the particular solution?