Physics 215

First Order Differential Equations

1. Generate a finite difference equation for the following equations using the right deriva-
   tive method and the centered derivative method. State the order of the error for each
   result.

$$a. \quad \frac{dy}{dx} \quad = \quad y \, + \, e^{-y} \, + \, 2x$$

$$b. \quad \frac{dy}{dx} \quad = \quad xy^3 \, - \, y$$

$$c. \quad \frac{dy}{dx} \quad = \quad 1 \, - \, x \, + \, y^2$$

2. When an object falls through a resistive medium (like air) its acceleration is retarded
   by a force related to its speed,

$$F_{retarding} = \frac{1}{2}\rho C A v^2 \quad .$$

   Consider an object falling through the air accelerated by the force of gravity and
   retarded by air friction.

   (a) What is the total force on the object?

   (b) Express your result from part (a) as a first-order, linear, ordinary differential
       equation where the speed v is a function of the time t.

   (c) Generate an algorithm to solve the equation from part (b) using the two-point
       formula. What is the order of the error term?

   (d) Generate an algorithm to solve the equation from part (b) using the three-point
       formula. What is the order of the error term?

3. Consider now a baseball struck by Manny Ramirez and subject to a friction force like
   the falling object above.

   (a) What are the components of the total vector force on the object?

   (b) Express your result from part a. as a set of first-order, linear, ordinary differential
       equations where the velocity is a function of the time t.

   (c) Generate an algorithm to solve the equations from part (b) using the Euler method
       (*i.e.*, the two-point formula). What is the order of the error term for each equa-
       tion?

   (d) Generate an algorithm to solve the equations from part (b) using the Taylor series
       method (*i.e.*, the three-point formula). What is the order of the error term for
       each equation?