Physics 215

Differentiation

1. Show that
$$f''(x) = \frac{f_1 - 2f_0 + f_{-1}}{h^2} + O(h^2)$$
by using a Taylor series expansion.

2. Show that
$$f'(x) = \frac{f_{-2} - 8f_{-1} + 8f_1 - f_2}{12h^2} + O(h^4)$$
by using a Taylor series expansion.

3. Write a code that calculates the derivative of the sine of 25° (1) analytically, (2) using the forward 2-point formula, and (3) using the 3-point formula. Make a figure showing the value for each method as a function of the stepsize, h. The stepsize should range from $h \approx 0.001$ to $h \approx 2$.

4. Consider the Taylor series expansion of the exponential function e^x about the origin
$$e^x = \sum_{n=0}^{\infty} \frac{1}{n!} x^n = \lim_{N\to\infty} S(x, N)$$
where $S(x, N)$ is the partial sum with $N + 1$ terms.

   (a) Show the expression above is correct.

   (b) Calculate the fractional error of the sum
$$\left| \frac{S(x, N) - e^x}{e^x} \right|$$
for $N = 5$ and $N = 10$ and for $x = 2,\ 10,\ -2,\ -10$. Discuss your results.