## Physics 215

## **Taylor Series**

1. Write out the first four terms of the Taylor series expansion of

$$\tan \theta \quad \text{about } \theta = \frac{\pi}{4} \quad \text{and} \quad e^x \quad \text{about } x = 0$$

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2. Provide an estimate to third order of the Fresnel integral

$$\int_0^1 \sin(x^2) dx$$

and compare your value with a tabulated one.

3. A body with rest mass  $m_0$  and moving at a speed near that of light, as in the case of an electron close to the nucleus of a heavy atom, has a kinetic energy given by the relativistic formula

$$K = m_0 c^2 \left( \frac{1}{\sqrt{1 - (v^2/c^2)}} - 1 \right)$$

Express K as a series in v and show that for  $v \ll c$  it reduces to the usual nonrelativistic expression.

4. Suppose a constant force F acts on a body, initially at rest, for a time t. Using the relativistic definition of momentum

$$p = \frac{m_0 v}{\sqrt{1 - (v^2/c^2)}}$$

and the definition of force as the time derivative of the linear momentum show that

$$v = c \left( 1 + \left(\frac{m_0 c}{Ft}\right)^2 \right)^{-1/2}$$

(Hint: use force equals the time derivative of the momentum, separate variables and integrate). Then show that if  $Ft \ll m_0 c$  one has the familiar expression

$$Ft = m_0 v$$

or

impulse = momentum.