Physics 303 Test 1

I pledge that I have neither given nor received unauthorized assistance during the completion of this work.

Name ______________________________ Signature ______________________________

Questions (6 pts. apiece) Answer questions 1-3 in complete, well-written sentences WITHIN the spaces provided.

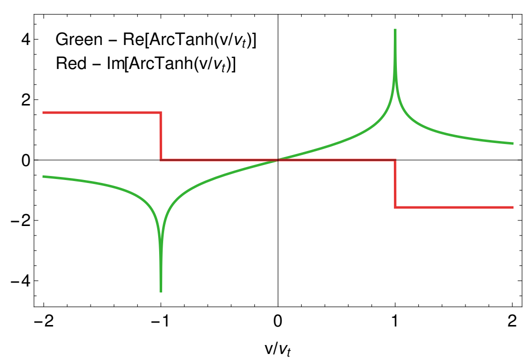
1. What advantage is there, if any, to using the center-of-mass frame? Explain.

2. Consider a 'plucked' harmonic oscillator with initial conditions $x(0) = x_0$ and $\dot{x}(0) = 0$. What is the kinetic energy at $t = 0$. Explain.

3. Recall Lt. Chisov's fall to Earth. From direct integration we obtained

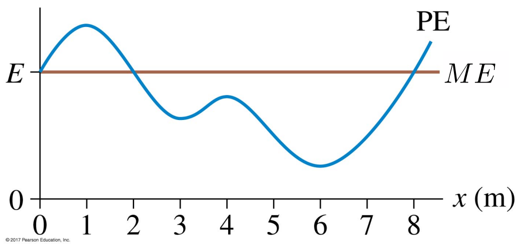
$$\text{arctanh}\left(\frac{v}{v_t}\right) = -\frac{g}{v_t}t + C_1$$

which relates his velocity v to the time t. The figure shows the real and imaginary parts of the left-hand-side of the equation above. What properties of the solution $v(t)$ can you determine from this plot?



DO NOT WRITE BELOW THIS LINE.

4. Consider the potential energy curve shown below with a particle moving to the right with total mechanical energy ME. Where is the particle's speed a maximum or a minimum? Explain. Draw a new ME line where the particle will come to a complete stop and stay there somewhere on its trajectory. Explain.



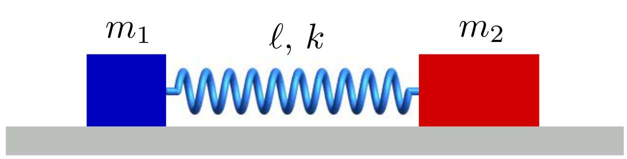
5. When we studied the NaCl potential energy function defined as

$$V(r) = -\frac{A}{r} + \frac{B}{r^2}$$

where A = 24 eV − Å and B = 28 eV − Å^2. To study small oscillations we expanded this potential in a Taylor series about the equilibrium point at r_e. We found the first order term vanished in the Taylor series. Why?

Problems. Clearly show all reasoning for full credit. Use a separate sheet to show your work.

1. 20 pts. Use Langrangian methods to find the equations of motion of the system shown in the figure. Two masses m_1 and m_2 are connected a by a spring of rest length ℓ and spring constant k. The system slides without friction on a horizontal surface in the direction of the spring's length.



2. 23 pts. A physics professor holds a bowling ball suspended as a pendulum at a distance $y_0 = 1.9\ m$ above the floor, The pendulum string is $\ell = 7\ m$ in length, the ceiling height is $h = 7.5\ m$ and the bowling ball has a mass $m = 15\ kg$. The professor gently releases the bowling ball just in front of their nose. In the first cycle (swing out and back) the bowling ball loses an energy $W_{loss} = 2.0\ J$. What is the height of the ball after one swing out and back? What is the initial angle of the ball with the vertical? What is the angle of the ball with the vertical after one cycle?

3. 27 pts. A world class shotputter can put an $m = 7.26 - kg$ shot with an initial velocity at release of $v_0 = 15\ m/s$ at an angle $\theta = 45°$ to the horizontal. The shot is accelerated by the athlete over a distance of $d = 1.5\ m$ at the angle θ. How much weight can this person lift with one hand?

Equations, Conversions, and Constants

$$\vec{F} = m\vec{a} = \dot{\vec{p}} = -\frac{dV}{dx} \quad \vec{F}_G = -\frac{Gm_1m_2}{r^2}\hat{r} \quad \vec{F}_C = \frac{kq_1q_2}{r^2}\hat{r} \quad \vec{F}_g = -mg\hat{y} \quad \vec{F}_s = -kr\hat{r}$$

$$\vec{F}_f = -bv\hat{v} \quad \vec{F}_f = -cv^2\hat{v} \quad x = \frac{a}{2}t^2 + v_0t + x_0 \quad v = at + v_0 \quad \int \frac{df}{dx}dx = \int df \quad f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$\ddot{y} + A\dot{y} + By = 0 \Rightarrow y = Ce^{\lambda t} \quad \ddot{y} + \omega_0^2 y = 0 \Rightarrow y = A\sin(\omega_0 t + \phi) = \alpha_1 e^{i\omega_0 t} + \alpha_2 e^{-i\omega_0 t}$$

$$\ddot{y} + \omega_0^2 y = \omega_0^2 l \Rightarrow y = C + A\sin(\omega_0 t + \phi) \quad \omega_0^2 = \frac{k}{m} \quad \omega_0^2 = \frac{g}{l} \quad \mu = \frac{m_1m_2}{m_1 + m_2}$$

$$V = -\int_{x_s}^{x} \vec{F}(\vec{r}\,')\cdot d\vec{r}\,' \quad E = K + V \quad V_s = \frac{kx^2}{2} \quad V_g = mgy \quad V_G = -\frac{Gm_1m_2}{r} \quad V_C = -\frac{kq_1q_2}{r}$$

$$F = -\frac{dV}{dx} \quad K = \frac{1}{2}mv^2 \quad L = K - V \quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0$$

Speed of light (c)	$3.0 \times 10^8\ m/s$	g	$9.8\ m/s^2$
Gravitation constant (G)	$6.67 \times 10^{-11}\ N - m^2/kg^2$	Earth's radius	$6.37 \times 10^6\ m$
Coulomb constant (k_e)	$8.99 \times 10^9 \frac{N-m^2}{C^2}$	Earth's mass	$5.97 \times 10^{24}\ kg$
Elementary charge (e)	$1.60 \times 10^{-19}\ C$	Proton/Neutron mass	$1.67 \times 10^{-27}\ kg$
Planck's constant (h)	$6.626 \times 10^{-34}\ J - s$	Proton/Neutron mass	$932 \times 10^6\ eV/c^2$
Permittivity constant (ϵ_0)	$8.85 \times 10^{-12} \frac{kg^2}{N-m^2}$	Electron mass	$9.11 \times 10^{-31}\ kg$
Permeability constant (μ_0)	$4\pi \times 10^{-7} N/A^2$	Electron mass	$0.55 \times 10^6\ MeV/c^2$
1 MeV	$10^6\ eV$	1.0 eV	$1.6 \times 10^{-19}\ J$
1 kg	$931.5\ MeV/c^2$	1 u	$1.67 \times 10^{-27}\ kg$

- Hyperbolic sine:

$$\sinh x = \frac{e^x - e^{-x}}{2} = \frac{e^{2x} - 1}{2e^x} = \frac{1 - e^{-2x}}{2e^{-x}}.$$

- Hyperbolic cosine:

$$\cosh x = \frac{e^x + e^{-x}}{2} = \frac{e^{2x} + 1}{2e^x} = \frac{1 + e^{-2x}}{2e^{-x}}.$$

- Hyperbolic tangent:

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{e^{2x} - 1}{e^{2x} + 1} = \frac{1 - e^{-2x}}{1 + e^{-2x}}.$$

- Hyperbolic cotangent: $x \neq 0$

$$\coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}} = \frac{e^{2x} + 1}{e^{2x} - 1} = \frac{1 + e^{-2x}}{1 - e^{-2x}}$$

- Hyperbolic secant:

$$\operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}} = \frac{2e^x}{e^{2x} + 1} = \frac{2e^{-x}}{1 + e^{-2x}}$$

- Hyperbolic cosecant: $x \neq 0$

$$\operatorname{csch} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}} = \frac{2e^x}{e^{2x} - 1} = \frac{2e^{-x}}{1 - e^{-2x}}$$

$$\operatorname{arsinh}(x) = \ln\left(x + \sqrt{x^2 + 1}\right)$$

$$\operatorname{arcosh}(x) = \ln\left(x + \sqrt{x^2 - 1}\right); x \geq 1$$

$$\operatorname{artanh}(x) = \frac{1}{2} \ln\left(\frac{1 + x}{1 - x}\right); |x| < 1$$

$$\operatorname{arcoth}(x) = \frac{1}{2} \ln\left(\frac{x + 1}{x - 1}\right); |x| > 1$$

Odd and even functions:

$$\sinh(-x) = -\sinh x$$
$$\cosh(-x) = \cosh x$$

Hence:

$$\tanh(-x) = -\tanh x$$
$$\coth(-x) = -\coth x$$
$$\operatorname{sech}(-x) = \operatorname{sech} x$$
$$\operatorname{csch}(-x) = -\operatorname{csch} x$$

$$\int \frac{dx}{x^2 - a^2} = -\frac{1}{a} \tanh^{-1}\left(\frac{x}{a}\right) \qquad \int \tanh x dx = \ln\left[\cosh x\right] \qquad \int \coth x dx = \ln[\sinh x]$$

$$\frac{d}{dx} \tan x = \frac{1}{\cos^2 x} \qquad \int \frac{1}{\sqrt{x^2 + a^2}} dx = \sinh^{-1} \frac{x}{a} \qquad \frac{d}{dx} \csc x = -\csc x \cot x$$

$$\frac{d}{dx} \sec x = \sec x \tan x \qquad \frac{d}{dx} \ln ax = \frac{1}{x} \qquad \frac{d}{dx} \cot x = -\frac{1}{\sin^2 x}$$

$$\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1 - x^2}} \qquad \frac{d}{dx} \cos^{-1} x = \frac{-1}{\sqrt{1 - x^2}} \qquad \frac{d}{dx} \tan^{-1} x = \frac{1}{1 + x^2}$$

$$\int \sqrt{a^2 - x^2} dx = \frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2} \sin^{-1}\left(\frac{x}{a}\right) \qquad \int \sqrt{x^2 - a^2} dx = \frac{x\sqrt{x^2 - a^2}}{2} + \frac{a^2}{2} \ln\left[x + \sqrt{x^2 - a^2}\right]$$

$$\int \tanh^2(x) dx = x - \tanh x \qquad \int \tanh^3(x) dx = \ln\left[\cosh x\right] + \frac{\operatorname{sech}^2(x)}{2}$$

$$\int \sqrt{\tanh x} dx = -\tan^{-1}\left[\sqrt{\tanh x}\right] - \frac{1}{2}\ln\left[1 - \sqrt{\tanh x}\right] + \frac{1}{2}\ln\left[1 + \sqrt{\tanh x}\right]$$

$$\frac{d}{dx}\tanh x = \operatorname{sech}^2 x \qquad \frac{d}{dx}\coth x = -\operatorname{csch}^2 x \qquad \frac{d}{dx}\sinh x = \cosh x$$

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + ... + \frac{f^n(a)}{n!}(x-a)^n + ...$$