Physics 303 Test 1

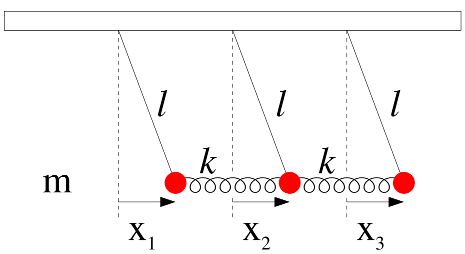
I pledge that I have neither given nor received unauthorized assistance during the completion of this work.

Signature ________________________________

Questions (5 pts. apiece) Answer questions 1-3 in complete, well-written sentences WITHIN the spaces provided.

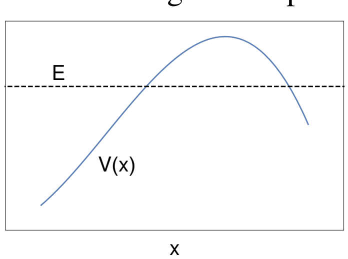
1. Why can't a falling object exceed terminal velocity?

2. Consider the three, identical, coupled pendula each of length l and mass m in the figure below. The displacement of each pendulum from equilibrium is represented by the x_i's and is small. The springs both have spring constant k. What is the potential energy due to the two springs ONLY? Explain your reasoning.

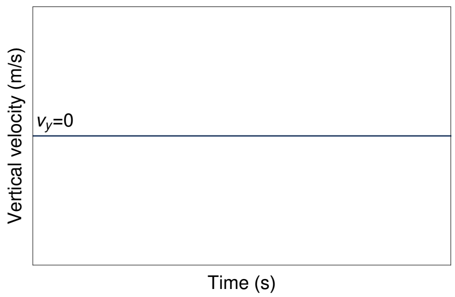


3. You're a program manager at DARPA and the proposal you're evaluating claims to produce changes in the natural frequency $\Delta\omega$ of a cantilever-based biosensor of $200\ kHz$ when exposed to anthrax spores. We can detect frequency changes of $\approx 170\ kHz$. Will this work? Explain.

4. Consider the plot of the potential energy $V(x)$ and the total energy E in the figure. For a particle launched from the left-hand side, describe qualitatively the motion of the particle. What are the significant points on the particle's trajectory? Explain your reasoning.

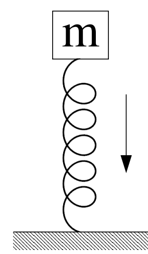


5. Consider a projectile shot from a cannon at an elevation of $45°$ to the horizontal and at a high speed. In the graph below draw a solid curve representing the vertical velocity of the projectile during its flight assuming there is no drag force. The horizontal line in the middle of the plot represents zero velocity. Treat the upwards direction as positive vertical velocity. Draw a second, dashed curve representing the vertical velocity in the presence of a drag force. Explain your reasoning.



Problems. Clearly show all reasoning for full credit. Use a separate sheet to show your work.

1. 20 pts. A massless spring of rest length l and spring constant k has a mass m attached to one end. The system is set on a table with the mass vertically above the spring as shown. What is the new equilibrium height of the mass above the table in terms of l, m, k, and any other constants? What is the differential equation describing the system, *i.e.* the equation of motion?



2. 25 pts. The vertical velocity of a parachutist is

$$v(t) = -v_t \tanh^2\left(\frac{gt}{v_t}\right)$$

where v_t is the terminal velocity and $\tanh x$ is the hyperbolic tangent of x. Get the general solution for $y(t)$ and then use the initial conditions that at $t = 0$, $y = y_0$ to find the particular solution.

3. 30 pts. The draw force $F(x)$ of a Turkish bow versus the bowstring displacement x (for x negative) is approximately represented by a quadrant of the ellipse

$$\left(\frac{F(x)}{F_{max}}\right)^2 + \left(\frac{x+d}{d}\right)^2 = 1$$

where F_{max} is the maximum force exerted by the bow when it drawn back, x is the displacement from equilibrium, and d is the maximum draw. What is the work done by the bow in terms of F_{max} and d and any other constants?

Equations, Conversions, and Constants

$$\vec{F} = m\vec{a} = \dot{\vec{p}} = -\frac{dV}{dx} \quad \vec{F}_G = -\frac{Gm_1m_2}{r^2}\hat{r} \quad \vec{F}_C = \frac{kq_1q_2}{r^2}\hat{r} \quad \vec{F}_g = -mg\hat{y} \quad \vec{F}_s = -kr\hat{r}$$

$$\vec{F}_f = -bv\hat{v} \quad \vec{F}_f = -cv^2\hat{v} \quad \int \frac{df}{dx}dx = \int df \quad \ddot{y} + A\dot{y} + By = 0 \Rightarrow y = Ce^{\lambda t}$$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n \quad \ddot{y} + \omega_0^2 y = 0 \Rightarrow y = A\sin(\omega_0 t + \phi) = \alpha_1 e^{i\omega_0 t} + \alpha_2 e^{-i\omega_0 t}$$

$$\ddot{y} + \omega_0^2 y = \omega_0^2 l \Rightarrow y = C + A\sin(\omega_0 t + \phi) \quad \omega_0^2 = \frac{k}{m} \quad \omega_0^2 = \frac{g}{l} \quad \mu = \frac{m_1m_2}{m_1+m_2}$$

$$V = -\int_{x_s}^{x} \vec{F}(\vec{r}\,')\cdot d\vec{r}\,' \quad E = K+V \quad V_s = \frac{kx^2}{2} \quad V_g = mgy \quad V_G = -\frac{Gm_1m_2}{r} \quad V_C = -\frac{kq_1q_2}{r}$$

$$F = -\frac{dV}{dx} \quad K = \frac{1}{2}mv^2 \quad L = K - V \quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0$$

Speed of light (c)	$3.0 \times 10^8\ m/s$	g	$9.8\ m/s^2$
Gravitation constant (G)	$6.67 \times 10^{-11}\ N-m^2/kg^2$	Earth's radius	$6.37 \times 10^6\ m$
Coulomb constant (k_e)	$8.99 \times 10^9 \frac{N-m^2}{C^2}$	Earth's mass	$5.97 \times 10^{24}\ kg$
Elementary charge (e)	$1.60 \times 10^{-19}\ C$	Proton/Neutron mass	$1.67 \times 10^{-27}\ kg$
Planck's constant (h)	$6.626 \times 10^{-34}\ J-s$	Proton/Neutron mass	$932 \times 10^6\ eV/c^2$
Permittivity constant (ϵ_0)	$8.85 \times 10^{-12} \frac{kg^2}{N-m^2}$	Electron mass	$9.11 \times 10^{-31}\ kg$
Permeability constant (μ_0)	$4\pi \times 10^{-7} N/A^2$	Electron mass	$0.55 \times 10^6\ MeV/c^2$
1 MeV	$10^6\ eV$	1.0 eV	$1.6 \times 10^{-19}\ J$
1 kg	$931.5\ MeV/c^2$	1 u	$1.67 \times 10^{-27}\ kg$

$$\int \frac{dx}{x^2-a^2} = -\frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right) \quad \int \tanh x dx = \ln[\cosh x] \quad \int \coth x dx = \ln[\sinh x]$$

$$\frac{d}{dx}\tan x = \frac{1}{\cos^2 x} \quad \int \frac{1}{\sqrt{x^2+a^2}}dx = \sinh^{-1}\frac{x}{a} \quad \frac{d}{dx}\csc x = -\csc x \cot x$$

$$\frac{d}{dx}\sec x = \sec x \tan x \quad \frac{d}{dx}\ln ax = \frac{1}{x} \quad \frac{d}{dx}\cot x = -\frac{1}{\sin^2 x}$$

$$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}} \quad \frac{d}{dx}\cos^{-1} x = \frac{-1}{\sqrt{1-x^2}} \quad \frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$$

$$\int \sqrt{a^2-x^2}dx = \frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) \qquad \int \sqrt{x^2-a^2}dx = \frac{x\sqrt{x^2-a^2}}{2} + \frac{a^2}{2}\ln\left[x+\sqrt{x^2-a^2}\right]$$

$$\int \tanh^2(x)dx = x - \tanh x \qquad \int \tanh^3(x)dx = \ln\left[\cosh x\right] + \frac{\operatorname{sech}^2(x)}{2}$$

$$\int \sqrt{\tanh x}dx = -\tan^{-1}\left[\sqrt{\tanh x}\right] - \frac{1}{2}\ln\left[1-\sqrt{\tanh x}\right] + \frac{1}{2}\ln\left[1+\sqrt{\tanh x}\right]$$

$$\frac{d}{dx}\tanh x = \operatorname{sech}^2 x \qquad \frac{d}{dx}\coth x = -\operatorname{csch}^2 x \qquad \frac{d}{dx}\sinh x = \cosh x$$

- Hyperbolic sine:

$$\sinh x = \frac{e^x - e^{-x}}{2} = \frac{e^{2x}-1}{2e^x} = \frac{1-e^{-2x}}{2e^{-x}}.$$

- Hyperbolic cosine:

$$\cosh x = \frac{e^x + e^{-x}}{2} = \frac{e^{2x}+1}{2e^x} = \frac{1+e^{-2x}}{2e^{-x}}.$$

- Hyperbolic tangent:

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \\ = \frac{e^{2x}-1}{e^{2x}+1} = \frac{1-e^{-2x}}{1+e^{-2x}}.$$

- Hyperbolic cotangent: $x \neq 0$

$$\coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}} = \\ = \frac{e^{2x}+1}{e^{2x}-1} = \frac{1+e^{-2x}}{1-e^{-2x}}$$

- Hyperbolic secant:

$$\operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}} = \\ = \frac{2e^x}{e^{2x}+1} = \frac{2e^{-x}}{1+e^{-2x}}$$

- Hyperbolic cosecant: $x \neq 0$

$$\operatorname{csch} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}} = \\ = \frac{2e^x}{e^{2x}-1} = \frac{2e^{-x}}{1-e^{-2x}}$$

$$\operatorname{arsinh}(x) = \ln\left(x+\sqrt{x^2+1}\right)$$
$$\operatorname{arcosh}(x) = \ln\left(x+\sqrt{x^2-1}\right); x \geq 1$$
$$\operatorname{artanh}(x) = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right); |x| < 1$$
$$\operatorname{arcoth}(x) = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right); |x| > 1$$

Odd and even functions:

$$\sinh(-x) = -\sinh x$$
$$\cosh(-x) = \cosh x$$

Hence:

$$\tanh(-x) = -\tanh x$$
$$\coth(-x) = -\coth x$$
$$\operatorname{sech}(-x) = \operatorname{sech} x$$
$$\operatorname{csch}(-x) = -\operatorname{csch} x$$