Physics 303 Test 1

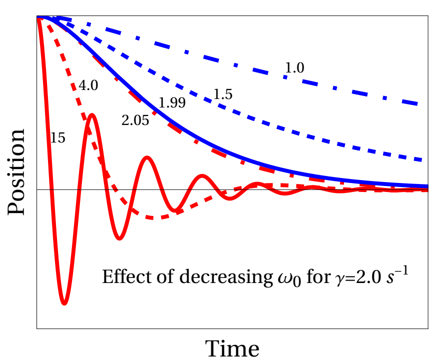
I pledge that I have neither given nor received unauthorized assistance during the completion of this work.

Name ______________________________ Signature ______________________________

Questions (6 pts. apiece) Answer questions 1-5 in complete, well-written sentences WITHIN the spaces provided.

1. What is terminal velocity for a falling object? What conclusions can you draw, if any, about the forces in action?

2. Consider a cantilever in our biosensor. The figure below shows a set of possible curves for the response of the cantilever for different angular frequencies ω_0. Each curve is labeled with the angular frequency. Which curve would be the best choice for the biosensor? Explain.



3. What is the classical mechanics program?

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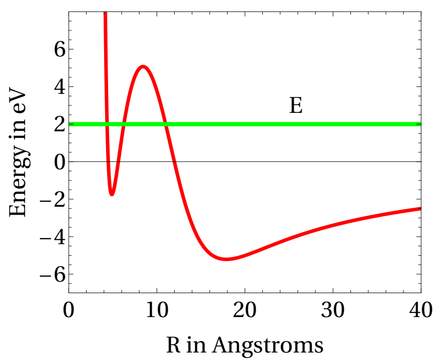
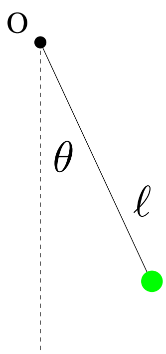
4. Consider the potential energy curve shown below. The horizontal green line represents the total mechanical energy E in the system. Where are the turning points of the system? Explain how you chose your points. Clearly label the points on the graph.



5. The figure shows the configuration of a simple pendulum. Starting from the expression for the kinetic energy in Cartesian coordinates show how you would convert from (x, y) coordinates to spherical coordinates (r, θ) and obtain a new expression for the kinetic energy in terms of spherical coordinates and their derivatives.



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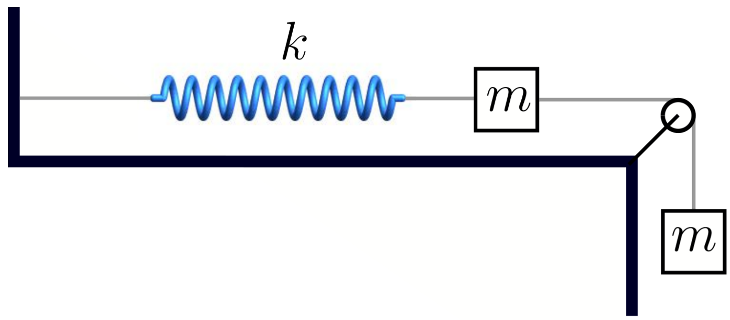
Problems. Work out Problems 1-3 on a separate sheet to show your work. Clearly show all reasoning for full credit.

1. 20 pts. Two equal masses are constrained by the spring-and-pulley system shown in the figure below. Assume a massless pulley and a frictionless surface. Let x be the extension of the spring from its relaxed length.

   (a) What is the Lagrangian for the motion?

   (b) What are the equations of motion? Use the Lagrangian to obtain them.



2. 25 pts. An archer using a bow with a spring constant $k = 186\ kg/s$ and draw $d = 0.72\ m$ aims horizontally at a target a distance $x_1 = 40\ m$ away. How far below the aiming point will an arrow with mass $m = 0.02\ kg$ strike? Neglect air resistance.

3. 25 pts. For the damped oscillator with $\gamma^2 < \omega_o^2$ the general solution is

$$y(t) = c_1 e^{(-\gamma + i\Omega')t} + c_2 e^{-(\gamma + i\Omega')t} \qquad (1)$$

   where $\Omega' = \sqrt{\omega_0^2 - \gamma^2}$. This definition makes the imaginary component of the solution explicit. Apply the following boundary conditions

$$\text{for } t = 0 \Longrightarrow \ y = 0 \text{ and } \dot{y} = v_0 \qquad (2)$$

   and obtain the constants c_1 and c_2 in terms of γ, Ω', v_0, and any other constants. Is the solution oscillatory? In other words, can it be written in terms of periodic functions like sine or cosine?

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Equations, Conversions, and Constants

$$\vec{F} = m\vec{a} = \dot{\vec{p}} = -\frac{dV}{dx} \quad \vec{F}_G = -\frac{Gm_1m_2}{r^2}\hat{r} \quad \vec{F}_C = \frac{kq_1q_2}{r^2}\hat{r} \quad \vec{F}_g = -mg\hat{y} \quad \vec{F}_s = -kr\hat{r}$$

$$\vec{F}_f = -bv\hat{v} \quad \vec{F}_f = -cv^2\hat{v} \quad x = \frac{a}{2}t^2 + v_0t + x_0 \quad v = at + v_0 \quad \int \frac{df}{dx}dx = \int df \quad f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$\ddot{y} + A\dot{y} + By = 0 \Rightarrow y = Ce^{\lambda t} \quad \ddot{y} + \omega_0^2 y = 0 \Rightarrow y = A\sin(\omega_0 t + \phi) = \alpha_1 e^{i\omega_0 t} + \alpha_2 e^{-i\omega_0 t}$$

$$\ddot{y} + \omega_0^2 y = \omega_0^2 l \Rightarrow y = C + A\sin(\omega_0 t + \phi) \quad \omega_0^2 = \frac{k}{m} \quad \omega_0^2 = \frac{g}{l} \quad \mu = \frac{m_1m_2}{m_1 + m_2}$$

$$V = -\int_{\vec{r}_s}^{\vec{r}} \vec{F}(\vec{r}\,')\cdot d\vec{r}\,' \quad E = K + V \quad V_s = \frac{kx^2}{2} \quad V_g = mgy \quad V_G = -\frac{Gm_1m_2}{r} \quad V_C = -\frac{kq_1q_2}{r}$$

$$F = -\frac{dV}{dx} \quad K = \frac{1}{2}mv^2 \quad L = K - V \quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0$$

$$e^{\pm ix} = \cos x \pm i\sin x \quad \cos x = \frac{e^{ix} + e^{-ix}}{2} \quad \sin x = \frac{e^{ix} - e^{-ix}}{2i}$$

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) \quad \cos A - \cos B = 2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{B-A}{2}\right)$$

Speed of light (c)	$3.0 \times 10^8\ m/s$	g	$9.8\ m/s^2$
Gravitation constant (G)	$6.67 \times 10^{-11}\ N - m^2/kg^2$	Earth's radius	$6.37 \times 10^6\ m$
Coulomb constant (k_e)	$8.99 \times 10^9 \frac{N-m^2}{C^2}$	Earth's mass	$5.97 \times 10^{24}\ kg$
Elementary charge (e)	$1.60 \times 10^{-19}\ C$	Proton/Neutron mass	$1.67 \times 10^{-27}\ kg$
Planck's constant (h)	$6.626 \times 10^{-34}\ J - s$	Proton/Neutron mass	$932 \times 10^6\ eV/c^2$
Permittivity constant (ϵ_0)	$8.85 \times 10^{-12} \frac{kg^2}{N-m^2}$	Electron mass	$9.11 \times 10^{-31}\ kg$
Permeability constant (μ_0)	$4\pi \times 10^{-7} N/A^2$	Electron mass	$0.55 \times 10^6\ MeV/c^2$
1 MeV	$10^6\ eV$	1.0 eV	$1.6 \times 10^{-19}\ J$
1 kg	$931.5\ MeV/c^2$	1 u	$1.67 \times 10^{-27}\ kg$

$$\int \frac{dx}{x^2 - a^2} = -\frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right) \qquad \int \tanh x dx = \ln\left[\cosh x\right] \qquad \int \coth x dx = \ln[\sinh x]$$

$$\frac{d}{dx}\tan x = \frac{1}{\cos^2 x} \qquad \int \frac{1}{\sqrt{x^2 + a^2}}\, dx = \sinh^{-1}\frac{x}{a} \qquad \frac{d}{dx}\csc x = -\csc x \cot x$$

$$\frac{d}{dx}\sec x = \sec x \tan x \qquad \frac{d}{dx}\ln ax = \frac{1}{x} \qquad \frac{d}{dx}\cot x = -\frac{1}{\sin^2 x}$$

$$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1 - x^2}} \qquad \frac{d}{dx}\cos^{-1} x = \frac{-1}{\sqrt{1 - x^2}} \qquad \frac{d}{dx}\tan^{-1} x = \frac{1}{1 + x^2}$$

$$\int \sqrt{a^2 - x^2} dx = \frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) \qquad \int \sqrt{x^2 - a^2} dx = \frac{x\sqrt{x^2 - a^2}}{2} + \frac{a^2}{2}\ln\left[x + \sqrt{x^2 - a^2}\right]$$

$$\int \tanh^2(x) dx = x - \tanh x \qquad \int \tanh^3(x) dx = \ln\left[\cosh x\right] + \frac{\operatorname{sech}^2(x)}{2}$$

$$\int \sqrt{\tanh x} dx = -\tan^{-1}\left[\sqrt{\tanh x}\right] - \frac{1}{2}\ln\left[1 - \sqrt{\tanh x}\right] + \frac{1}{2}\ln\left[1 + \sqrt{\tanh x}\right]$$

$$\frac{d}{dx}\tanh x = \operatorname{sech}^2 x \qquad \frac{d}{dx}\coth x = -\operatorname{csch}^2 x \qquad \frac{d}{dx}\sinh x = \cosh x$$

$$f(x) = f(a) + \frac{f'(a)}{1!}(x - a) + \frac{f''(a)}{2!}(x - a)^2 + \frac{f'''(a)}{3!}(x - a)^3 + ... + \frac{f^n(a)}{n!}(x - a)^n + ...$$

- Hyperbolic sine:

$$\sinh x = \frac{e^x - e^{-x}}{2} = \frac{e^{2x} - 1}{2e^x} = \frac{1 - e^{-2x}}{2e^{-x}}.$$

- Hyperbolic cosine:

$$\cosh x = \frac{e^x + e^{-x}}{2} = \frac{e^{2x} + 1}{2e^x} = \frac{1 + e^{-2x}}{2e^{-x}}.$$

- Hyperbolic tangent:

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \\ = \frac{e^{2x} - 1}{e^{2x} + 1} = \frac{1 - e^{-2x}}{1 + e^{-2x}}.$$

- Hyperbolic cotangent: $x \neq 0$

$$\coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}} = \\ = \frac{e^{2x} + 1}{e^{2x} - 1} = \frac{1 + e^{-2x}}{1 - e^{-2x}}$$

- Hyperbolic secant:

$$\operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}} = \\ = \frac{2e^x}{e^{2x} + 1} = \frac{2e^{-x}}{1 + e^{-2x}}$$

- Hyperbolic cosecant: $x \neq 0$

$$\operatorname{csch} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}} = \\ = \frac{2e^x}{e^{2x} - 1} = \frac{2e^{-x}}{1 - e^{-2x}}$$

$$\operatorname{arsinh}(x) = \ln\left(x + \sqrt{x^2 + 1}\right)$$
$$\operatorname{arcosh}(x) = \ln\left(x + \sqrt{x^2 - 1}\right); x \geq 1$$
$$\operatorname{artanh}(x) = \frac{1}{2} \ln\left(\frac{1 + x}{1 - x}\right); |x| < 1$$
$$\operatorname{arcoth}(x) = \frac{1}{2} \ln\left(\frac{x + 1}{x - 1}\right); |x| > 1$$

Odd and even functions:

$$\sinh(-x) = -\sinh x$$
$$\cosh(-x) = \cosh x$$

Hence:

$$\tanh(-x) = -\tanh x$$
$$\coth(-x) = -\coth x$$
$$\operatorname{sech}(-x) = \operatorname{sech} x$$
$$\operatorname{csch}(-x) = -\operatorname{csch} x$$