Physics 303 Final

I pledge that I have neither given nor received unauthorized assistance during the completion of this work.

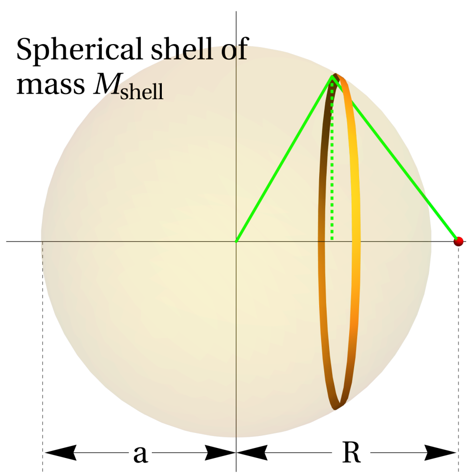
Signature ________________________________

Questions (5 pts. apiece) Answer questions 1-8 in complete, well-written sentences WITHIN the spaces provided.

1. What is the evidence for the existence of an exoplanet orbiting 51 Pegasus? Explain.

2. Consider a 'plucked' harmonic oscillator with initial conditions $x(0) = x_0$ and $\dot{x}(0) = 0$. What is the kinetic energy at $t = 0$. Explain.

3. In proving Newton's Theorem about the gravitational force from a uniform spherical mass we divided the mass into spherical shells and the shells into circular rings. We used the surface mass density σ of the shell to calculate the mass dM_{ring} of each ring. What is the surface area of each ring and the mass dM_{ring}? Use the appropriate geometric properties shown in the figure. Clearly define any quantities you introduce in your answer.



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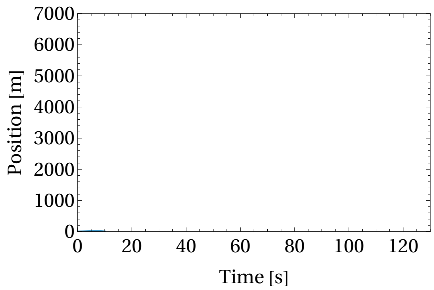
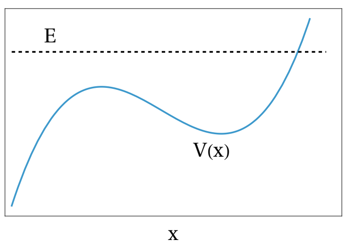
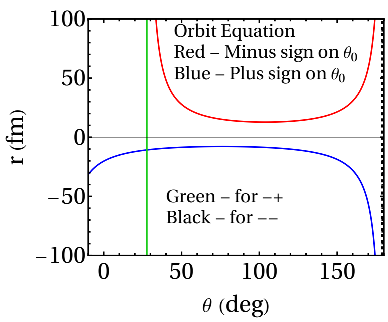
4. Consider two identical balls that start falling with the same initial conditions. One is subject to air drag and the other is not. On the figure below sketch the position of each ball as a function of time. Clearly label your sketch. Explain any differences in their trajectories.



5. Consider the plot of the potential energy $V(x)$ and the total energy E of a particle in the figure. If launched from the left-hand side, describe qualitatively the motion of the particle. What are the significant points on the particle's trajectory? Explain.

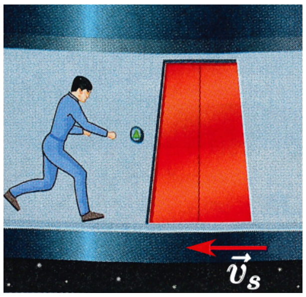


6. Recall our calculation of the trajectory of a ^{6}Li projectile with a ^{208}Pb target. The plot shows the separation r between the nuclei versus the angle θ of the ^{6}Li. There are two solutions of the orbit equation (red and blue curves) for two sign choices of the parameter θ_0. What quantities are represented by the vertical green and dashed lines? Which curve is the right one? Explain.



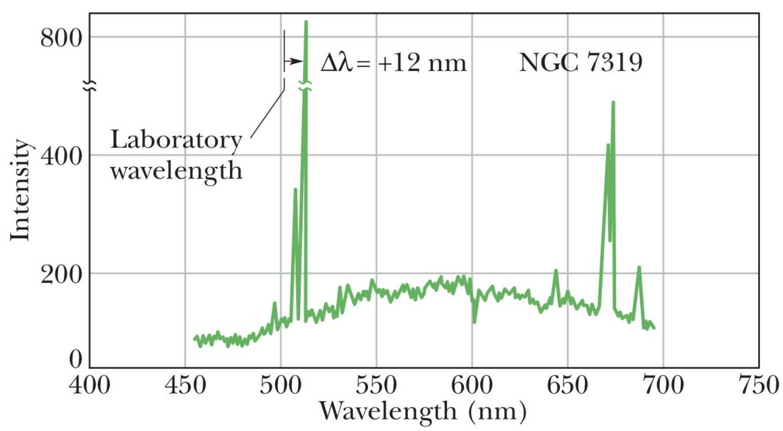
7. Suppose an asteroid struck a glancing, inelastic blow on the Earth and went off into space. Would this situation be better or worse than a perfectly inelastic collision like the dinosaur killer? Explain.

8. The figure below shows a section of a circular space station that rotates about its center to give apparent weight W_a to each crew member. One of the crew is shown at the outer wall of the station which has a velocity $\vec{v}_s$ as shown. If the astronaut runs along the outer wall in the direction opposite $\vec{v}_s$ with a speed less than the magnitude of $\vec{v}_s$ does their apparent weight W_a increase, decrease, or stay the same? Explain.



Problems (1-6). Clearly show all reasoning for full credit. Use a separate sheet to show your work.

1. 8 pts. The figure below is a graph of intensity versus wavelength for light from galaxy NGC 7319, which is about $3 \times 10^8 \ell y$ away. The most intense light is emitted by the oxygen in NGC 7319. In the lab that emission is at $\lambda = 513\ nm$, but the light from NGC 7319 it has been shifted to $525\ nm$ due to the Doppler effect (all emissions from NGC 7319 have been shifted). (a) What is the radial speed of NGC 7319 relative to Earth? (b) Is the relative motion toward or away from our planet? Explain.



2. 8 pts. The density of a spherical planet of radius R with a molten core of radius $R/2$ is given by ρ for $R/2 < r < R$ and 5ρ for $r < R/2$, where ρ is a constant. What is the enclosed mass $M(r)$ in terms of ρ and R for $R/2 < r < R$?

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3. 10 pts. For the damped oscillator we showed the general solution to the differential equation

$$\ddot{y} + 2\gamma\dot{y} + \omega_0^2 y = 0 \tag{1}$$

is

$$y(t) = c_1 e^{(-\gamma + i\Omega')t} + c_2 e^{-(\gamma + i\Omega')t} \tag{2}$$

where

$$\Omega' = \sqrt{\omega_0^2 - \gamma^2}$$

and Ω' is real for $\omega_0^2 > \gamma^2$. Apply the following boundary conditions and determine c_1 only.

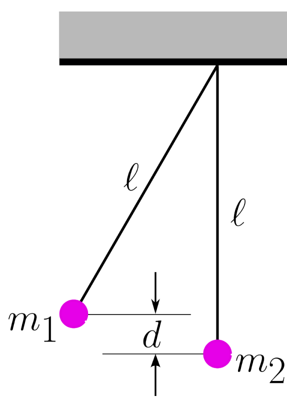
$$\text{for } t = 0 \implies y = y_0 \text{ and } \dot{y} = 0 \tag{3}$$

4. 10 pts. A boat is slowed by a drag force $F(v)$. Its velocity decreases according to the formula

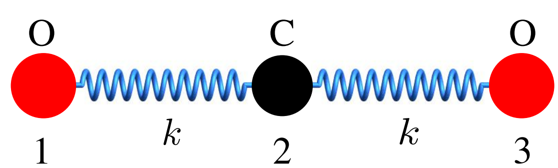
$$v = c^2(t - t_1)^2$$

where c is a constant and t_1 is the time at which it stops. What is the force $F(v)$ as a function of v?

5. 12 pts. Two pendula, both of length ℓ are initially situated as shown in the figure. The left-hand pendulum is released and strikes the other. Assume the collision is perfectly inelastic and neglect the masses of the string and any frictional effects. How high does the center-of-mass of the pendulum system rise after the collision?



6. 12 pts. Our classical model of the CO_2 molecule is shown below. What is the Lagrangian for this system? What are the equations of motion?



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Physics 303 Final Equation Sheet

$$\vec{F} = m\vec{a} = \dot{\vec{p}} = -\frac{dV}{dr}\hat{r} = -\nabla V \quad \vec{F}_G = -\frac{Gm_1m_2}{r^2}\hat{r} \quad \vec{F}_C = \frac{kq_1q_2}{r^2}\hat{r} \quad \vec{F}_g = -mg\hat{y} \quad \vec{F}_s = -kr\hat{r}$$

$$|\vec{F}_{cent}| = m\frac{v^2}{r} \quad \vec{F}_f = -bv\hat{v} \quad \vec{F}_f = -cv^2\hat{v} \quad x = \frac{a}{2}t^2 + v_0t + x_0 \quad v = at + v_0 \quad \frac{df(y)}{dx} = \frac{df(y)}{dy}\frac{dy}{dx}$$

$$\int \frac{df}{dx} = \int df \quad \ddot{y} + A\dot{y} + By = 0 \Rightarrow y = Ce^{\lambda t} \quad \ddot{y} + \omega_0^2 y = 0 \Rightarrow y = A\sin(\omega_0 t + \phi) = \alpha_1 e^{i\omega_0 t} + \alpha_2 e^{-i\omega_0 t}$$

$$\omega_0^2 = \frac{k}{m} \quad \omega_0^2 = \frac{g}{l} \quad k = \left.\frac{d^2V(x)}{dx^2}\right|_{x=x_e} \text{(small oscillations)} \quad E = K + V \quad f(x) = \sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$V = -\int_{x_s}^{x} \vec{F}(\vec{r}\,')\cdot d\vec{r}\,' \quad V_s = \frac{kx^2}{2} \quad V_g = mgy \quad V_G = -\frac{Gm_1m_2}{r} \quad V_C = \frac{kq_1q_2}{r} = \frac{Z_1Z_2e^2}{r} \quad \Phi = \frac{V}{m_2 \text{ or } e_2}$$

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m} = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) \quad \mathscr{L} = K - V \quad \frac{d}{dt}\left(\frac{\partial \mathscr{L}}{\partial \dot{q}}\right) - \frac{\partial \mathscr{L}}{\partial q} = 0 \quad p_q = \frac{\partial \mathscr{L}}{\partial \dot{q}}$$

$$\underset{\sim}{\mathbf{A}}\vec{C} = -\omega^2 \underset{\sim}{\mathbf{I}}\vec{C} \quad \vec{L} = \vec{r}\times\vec{p} \quad \vec{N} = \frac{d\vec{L}}{dt} \quad L = \mu r^2\dot{\theta} \quad V_{cent} = \frac{L^2}{2\mu r^2} \quad \vec{p} = m\vec{v} \quad \vec{p}_i = \vec{p}_f \quad K_i = K_f$$

$$\frac{1}{r} = \frac{\mu\alpha}{L^2}\left(1 + \epsilon\cos(\theta - \theta_0)\right) \quad \theta_0 = -\arccos\left(\frac{1}{\epsilon}\right) \quad \epsilon = \sqrt{1 + \frac{2EL^2}{\mu\alpha^2}} \quad \sin\left(\frac{\theta_s}{2}\right) = \frac{1}{\epsilon} \quad \mu = \frac{m_1m_2}{m_1 + m_2}$$

$$\alpha = -Gm_1m_2 \text{ or } e^2Z_1Z_2 \quad E_{cm} = \frac{m_{tgt}}{m_{tgt} + m_{beam}}E_{lab} \quad V_{eff}(r) = \frac{L^2}{2\mu r^2} + V(r) \quad c = \frac{\lambda}{T} = \lambda f$$

$$d\sigma = 2\pi b db \quad d\Omega = \sin\theta d\theta d\phi \quad \frac{d\sigma}{d\Omega} = \left(\frac{\alpha}{4E_{cm}}\right)^2 \frac{1}{\sin^4\left(\frac{\theta_s}{2}\right)} \quad r_{min} = \frac{\ell^2}{\mu\alpha}\frac{1}{1-\epsilon} = -\frac{\alpha}{2E_{cm}}(1+\epsilon)$$

$$\vec{R}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2} = \vec{0} \quad v_T = r\dot{\theta} = \frac{2\pi r_{sp}}{T} \approx \sqrt{\frac{GM_s}{r_{sp}}} = v_p \quad r_{sp} = \left[\frac{GT^2M_s}{4\pi^2}\right]^{1/3} \quad r_s = \frac{m_p}{m_p + m_s}r_{sp}$$

$$\mu\ddot{r}_{sp} - \mu r_{sp}\dot{\theta}^2 + \frac{\alpha}{r_{sp}} = 0 \quad \lambda_\pm = \sqrt{\frac{1 \mp v_s/c}{1 \pm v_s/c}}\,\lambda_0 \begin{array}{l} +\ blue \\ -\ red \end{array} \quad dM_{ring} = 2\pi a^2\sin\theta d\theta\sigma \quad M(r) = \int_0^r \text{mass in r } dr'$$

$$\sigma = \frac{M_{shell}}{4\pi a^2} \quad d\Phi_{ring} = -\frac{GM_{shell}dr}{2aR} \quad \Phi_{out} = -\frac{GM_{shell}}{R}\ (R > a) \quad \Phi_{in} = -\frac{GM_{shell}}{a}\ (R < a)$$

More Equations, Conversions, and Constants

Speed of light (c)	$3.0 \times 10^8\ m/s$	g	$9.8\ m/s^2$
Gravitation constant (G)	$6.67 \times 10^{-11}\ N - m^2/kg^2$	Earth's radius	$6.37 \times 10^6\ m$
Coulomb constant (k_e)	$8.99 \times 10^9 \frac{N-m^2}{C^2}$	Earth's mass	$5.97 \times 10^{24}\ kg$
Elementary charge (e)	$1.60 \times 10^{-19}\ C$	Proton/Neutron mass	$1.67 \times 10^{-27}\ kg$
Planck's constant (h)	$6.626 \times 10^{-34}\ J - s$	Proton/Neutron mass	$932 \times 10^6\ eV/c^2$
Permittivity constant (ϵ_0)	$8.85 \times 10^{-12} \frac{kg^2}{N-m^2}$	Electron mass	$9.11 \times 10^{-31}\ kg$
Permeability constant (μ_0)	$4\pi \times 10^{-7} N/A^2$	Electron mass	$0.55 \times 10^6\ MeV/c^2$
1 MeV	$10^6\ eV$	1.0 eV	$1.6 \times 10^{-19}\ J$
1 kg	$931.5\ MeV/c^2$	1 u	$1.67 \times 10^{-27}\ kg$
$\hbar c$	$197\ MeV - fm$	e^2	$\frac{hbarc}{137}$

$$\int \frac{dx}{x^2 - a^2} = -\frac{1}{a} \tanh^{-1}\left(\frac{x}{a}\right) \qquad \int \tanh x dx = \ln\left[\cosh x\right] \qquad \int \coth x dx = \ln[\sinh x]$$

$$\frac{d}{dx}\tan x = \frac{1}{\cos^2 x} \qquad \int \frac{1}{\sqrt{x^2+a^2}} dx = \sinh^{-1}\frac{x}{a} \qquad \frac{d}{dx}\csc x = -\csc x \cot x$$

$$\frac{d}{dx}\sec x = \sec x \tan x \qquad \frac{d}{dx}\ln ax = \frac{1}{x} \qquad \frac{d}{dx}\cot x = -\frac{1}{\sin^2 x}$$

$$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\cos^{-1} x = \frac{-1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$$

$$\int \sqrt{a^2 - x^2} dx = \frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) \qquad \int \sqrt{x^2 - a^2} dx = \frac{x\sqrt{x^2-a^2}}{2} + \frac{a^2}{2}\ln\left[x + \sqrt{x^2-a^2}\right]$$

$$\int \tanh^2(x) dx = x - \tanh x \qquad \int \tanh^3(x) dx = \ln\left[\cosh x\right] + \frac{\operatorname{sech}^2(x)}{2}$$

$$\int \sqrt{\tanh x} dx = -\tan^{-1}\left[\sqrt{\tanh x}\right] - \frac{1}{2}\ln\left[1 - \sqrt{\tanh x}\right] + \frac{1}{2}\ln\left[1 + \sqrt{\tanh x}\right]$$

$$\frac{d}{dx}\tanh x = \operatorname{sech}^2 x \qquad \frac{d}{dx}\coth x = -\operatorname{csch}^2 x \qquad \frac{d}{dx}\sinh x = \cosh x$$

$$\int \frac{1}{\sqrt{ar^4 + br^3 - r^2}} dr = \frac{r\sqrt{-1+br+ar^2}\arctan\left(\frac{-2+br}{2\sqrt{-1+br+ar^2}}\right)}{\sqrt{r^2(-1+br+ar^2)}}$$

$$\int \frac{1}{\sqrt{ax^2+bx+c}} dx = -\frac{1}{\sqrt{-a}}\arcsin\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)$$

$$\int \frac{1}{ax^2+bx+c} dx = \frac{2}{\sqrt{4ac-b^2}}\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k} = \hat{r}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta} + \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}$$

$$e^{\pm ix} = \cos x \pm i \sin x \quad \cos x = \frac{e^{ix} + e^{-ix}}{2} \quad \sin x = \frac{e^{ix} - e^{-ix}}{2i}$$

$$\sin A = \cos(A - \pi/2) \quad \cos A = -\sin(A - \pi/2) \quad \arccos(-x) = \pi - \arccos(x)$$

$$\cos(\arcsin(x)) = \sin(\arccos x) = \sqrt{1 - x^2}$$

$$\cos(\arctan(x)) = \frac{1}{\sqrt{1 - x^2}} \quad \tan(\arccos x) = \frac{\sqrt{1 - x^2}}{x} \quad \tan(\arcsin x) = \sin(\arctan x) = \frac{x}{\sqrt{1 - x^2}}$$

$$\cos A + \cos B = 2\cos\left(\frac{A + B}{2}\right)\cos\left(\frac{A - B}{2}\right) \quad \cos A - \cos B = 2\sin\left(\frac{A + B}{2}\right)\sin\left(\frac{B - A}{2}\right)$$

- Hyperbolic sine:

$$\sinh x = \frac{e^x - e^{-x}}{2} = \frac{e^{2x} - 1}{2e^x} = \frac{1 - e^{-2x}}{2e^{-x}}.$$

- Hyperbolic cosine:

$$\cosh x = \frac{e^x + e^{-x}}{2} = \frac{e^{2x} + 1}{2e^x} = \frac{1 + e^{-2x}}{2e^{-x}}.$$

- Hyperbolic tangent:

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}} = = \frac{e^{2x} - 1}{e^{2x} + 1} = \frac{1 - e^{-2x}}{1 + e^{-2x}}.$$

- Hyperbolic cotangent: $x \neq 0$

$$\coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}} = = \frac{e^{2x} + 1}{e^{2x} - 1} = \frac{1 + e^{-2x}}{1 - e^{-2x}}$$

- Hyperbolic secant:

$$\operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}} = = \frac{2e^x}{e^{2x} + 1} = \frac{2e^{-x}}{1 + e^{-2x}}$$

- Hyperbolic cosecant: $x \neq 0$

$$\operatorname{csch} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}} = = \frac{2e^x}{e^{2x} - 1} = \frac{2e^{-x}}{1 - e^{-2x}}$$

$$\operatorname{arsinh}(x) = \ln\left(x + \sqrt{x^2 + 1}\right)$$

$$\operatorname{arcosh}(x) = \ln\left(x + \sqrt{x^2 - 1}\right); x \geq 1$$

$$\operatorname{artanh}(x) = \frac{1}{2}\ln\left(\frac{1 + x}{1 - x}\right); |x| < 1$$

$$\operatorname{arcoth}(x) = \frac{1}{2}\ln\left(\frac{x + 1}{x - 1}\right); |x| > 1$$

Odd and even functions:

$$\sinh(-x) = -\sinh x$$
$$\cosh(-x) = \cosh x$$

Hence:

$$\tanh(-x) = -\tanh x$$
$$\coth(-x) = -\coth x$$
$$\operatorname{sech}(-x) = \operatorname{sech} x$$
$$\operatorname{csch}(-x) = -\operatorname{csch} x$$