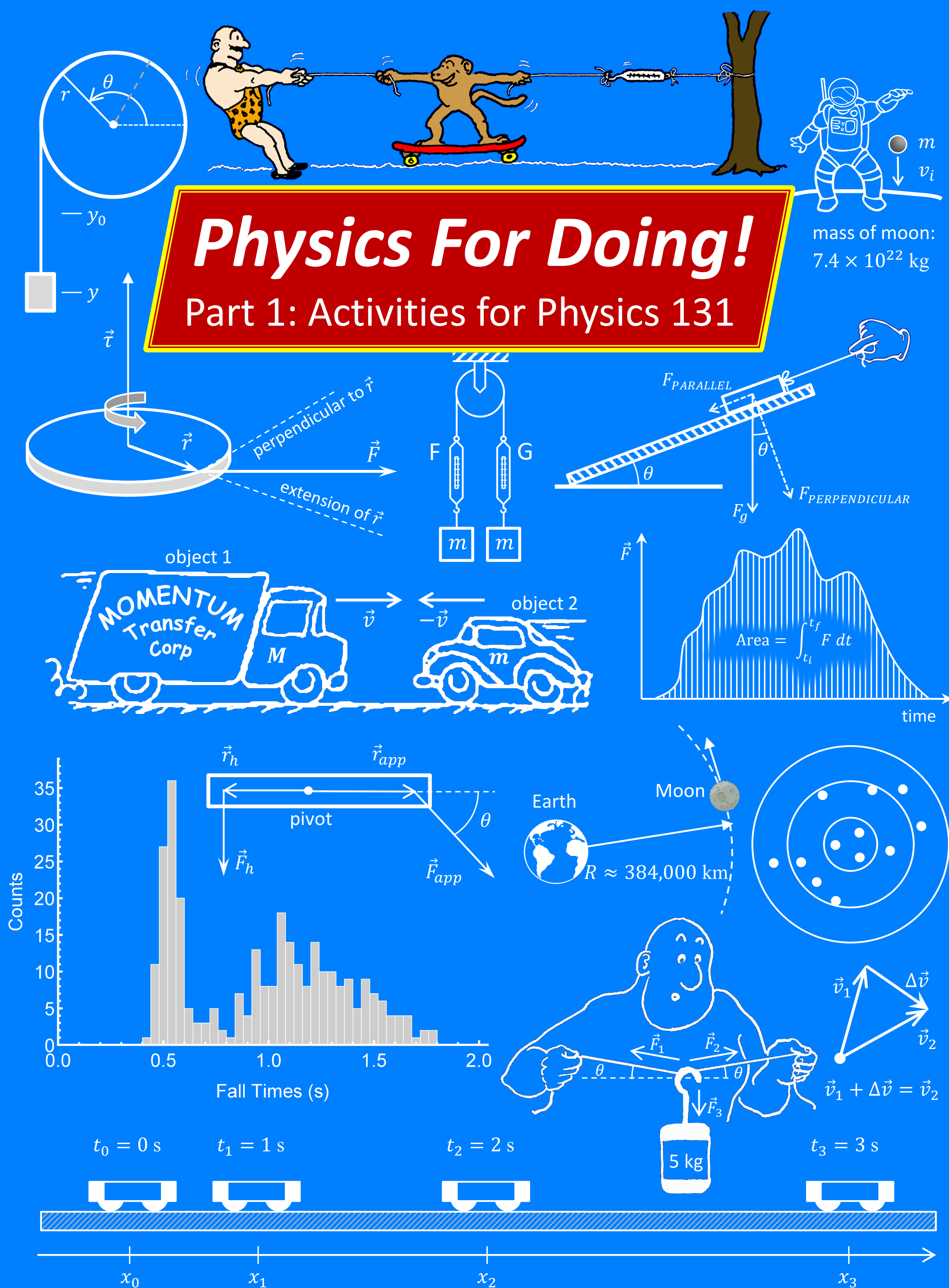
Physics For Doing!

Part 1: Activities for Physics 131



Cover art: Various graphics and diagrams from the activities in this manual. You'll be doing lots of stuff.

Physics For Doing!
Part 1: Activities for Physics 131
Fall 2024

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Welcome to Physics 131!

The exercises in this manual have been developed to support an investigative physics course that emphasizes active learning. Your written work will consist primarily of documenting your class activities by filling in the entries in the spaces provided in the units. The entries consist of observations, derivations, calculations, and answers to questions. Although you may use the same data and graphs as your partner(s) and discuss concepts with your classmates, all entries should reflect your own understanding of the concepts and the meaning of the data and graphs you are presenting. Thus, each entry should be written in your own words. It is very important to your success in this course that your entries reflect a sound understanding of the phenomena you are observing and analyzing.

Some of these exercises have been taken from the Workshop Physics project at Dickinson College and the Tools for Scientific Thinking project at Tufts University and modified for use at the University of Richmond. Others have been developed locally. We wish to acknowledge the support we have received for this project from the University of Richmond and the Instrumentation and Laboratory Improvement program of the National Science Foundation.

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Lab 1 Position *vs.* Time Graphs[1]

Name: ______________________ Lab Partner(s): ______________________

Objectives

- To learn about two of the ways that physicists can describe motion in one dimension: words and graphs.
- To learn how to relate graphs of position *vs.* time to the motions they represent.

Apparatus

- *Capstone* software (*P_Graphs.cap* experiment file)
- Masking tape
- Wireless motion sensor
- Wooden board

Introduction

The focus of this unit on kinematics is to be able to describe your position as a function of time using words and graphs. You will use a wireless motion sensor connected to a computer to learn to describe one-dimensional motion.

The motion sensor sends out a series of sound pulses that are of too high a frequency to hear (ultrasonic). These pulses reflect from objects in the vicinity of the motion sensor and some of the sound energy returns to the sensor. The computer is able to record the time it takes for reflected sound waves to return to the sensor and then, by knowing the speed of sound in air, figure out how far away the reflecting object is. There are several things to watch out for when using a motion sensor. (1) Do not get closer than 0.15 meters from the sensor because it cannot record reflected pulses which come back too soon. (2) The ultrasonic waves come out in a cone of about 15°. It will see the closest object. Be sure there is a clear path between the object whose motion you want to track and the motion sensor. (3) The motion sensor is very sensitive and will detect slight motions. You can try to glide smoothly along the floor, but don't be surprised to see small bumps in velocity graphs. (4) Some objects like bulky sweaters are good sound absorbers and may not be "seen" well by a motion sensor. Hold a wooden board in front of you when doing the activities below so the motion sensor can "see" a smooth surface.

Position *vs.* Time Graphs of Your Motion

The purpose of this unit is to learn how to relate graphs of position as a function of time to the motions they represent. How does a position *vs.* time graph look when you move slowly? Quickly? What happens when you move toward the motion sensor? Away? After completing the next few activities, you should be able to look at a position *vs.* time graph and describe the motion of the object. You should also be able to look at the motion of an object and sketch a graph representing that motion.

Note that the motion sensor measures the distance of an object from the sensor, and that the motion sensor is located at the origin of each graph. It is common to refer to the distance of an object from some origin as the position of the object. Therefore, it is better to refer to these graphs as position *vs.* time graphs than distance *vs.* time graphs.

You will use the *Capstone* software to do the following activities. Launch the *P_Graphs.cap* file by going to the *Phys131* folder on the desktop. Next, turn on the sensor at your station by pressing the power button located on its side (when on, the status LED will blink). In *Capstone*, select **Hardware Setup** in the **Tools** palette. In **Hardware Setup**, the sensors are ordered by proximity to the device. To connect a sensor to the computer via Bluetooth, select the correct address that matches the Device ID Number (XXX-XXX) found on the motion sensor at your station.

To start a data run, click the **Record** button in the **Controls** palette. To stop a data run, click the **Stop** button in the **Controls** palette. After a data run, the graph can be expanded by clicking on the **Scale to Fit** button in

[1]1990-93 Dept. of Physics and Astronomy, Dickinson College. Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Portions of this material may have been modified locally and may not have been classroom tested at Dickinson College.

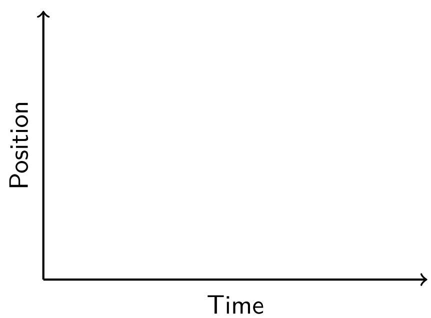
the upper left corner of the Graph window. Multiple data sets can be displayed on the same graph. Data can be removed from the graph by opening the **Delete Last Run** menu on the **Controls** palette and selecting **Delete All Runs** or **Delete Run #X**. When you are finished with the activities, choose **Exit** from the **File** menu and do not save this activity.

Before you begin the activities, you should mark a position scale on the floor. To do this, position one person at approximately 1 meter in front of the motion sensor and take data for 1 second. The computer will display a horizontal line showing the position measured by the sensor. The person standing in front of the sensor should then adjust their position and the procedure repeated until the 1 meter position is established. Mark the 1 meter position on the floor with a piece of masking tape and then mark the 2, 3 and 4 meter positions using the 2 meter stick.

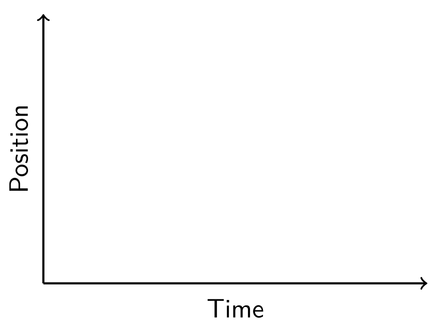
Activity 1: Making Position *vs.* Time Graphs

Make position-time graphs for the following motions and sketch the graph you observe in each case:

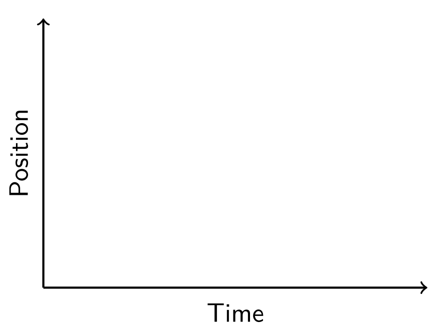
(a) Starting at 0.5 m, walk away from the origin (i.e., the motion sensor) slowly and steadily.



(b) Walk away from the origin medium-fast and steadily.



(c) Walk toward the origin slowly and steadily.

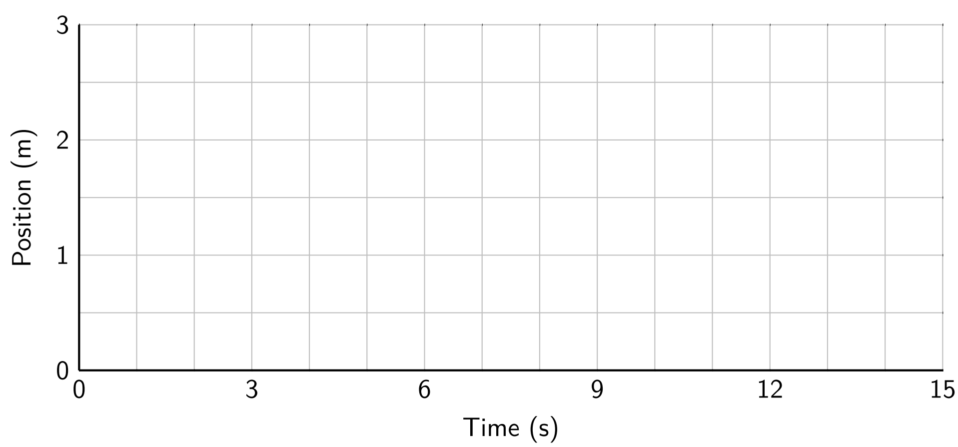


(d) Describe the difference between the graph you made by walking away slowly and the one made by walking away more quickly.

(e) Describe the difference between the graph made by walking toward and the one made walking away from the origin.

Activity 2: Predicting a Position *vs.* Time Graph

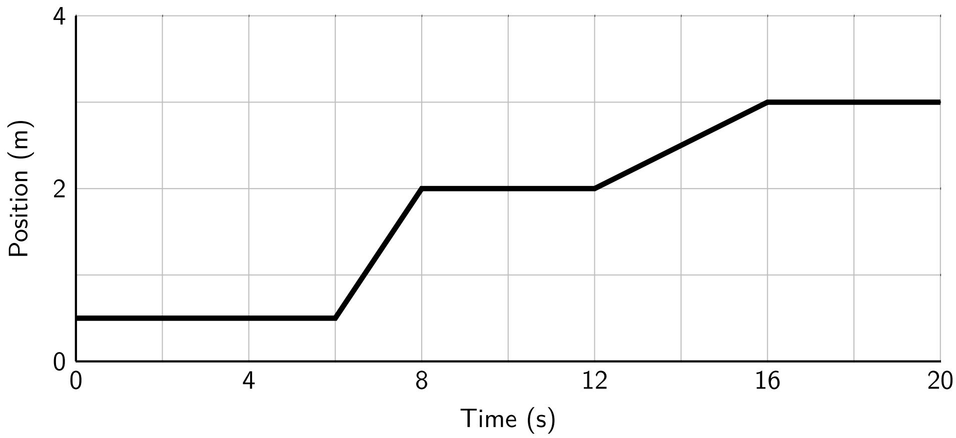
(a) Suppose you were to start 1.0 m in front of the motion sensor and walk away slowly and steadily for 4 seconds, stop for 4 seconds, and then walk toward the sensor quickly. Sketch your prediction on the axes below using a dashed line.



(b) Test your prediction by moving in the way described and making a graph of your motion with the motion sensor. Sketch the trace of your actual motion on the above graph with a solid line.

(c) Is your prediction the same as the final result? If not, describe how you would move to make a graph that looks like your prediction.

Activity 3: Matching Position *vs.* Time Graphs

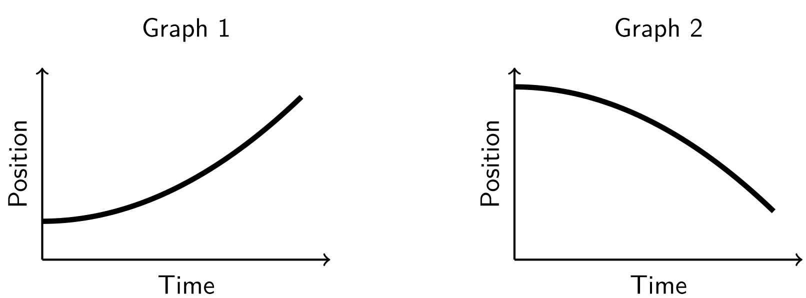


(a) Describe in your own words how you would move in order to match the graph shown above.

(b) Move to match the above graph on the computer screen. You may try a number of times. It helps to work as a team. Get the times right. Get the positions right. Do this for yourself. (Each person in your group should do their own match. You will not learn very much by just watching!)

(c) What was the difference in the way you moved to produce the two differently sloped parts of the graph you just matched?

(d) Make curved position *vs.* time graphs like those shown below.



(e) Describe how you must move to produce a position *vs.* time graph with each of the shapes shown.

Graph 1 answer:

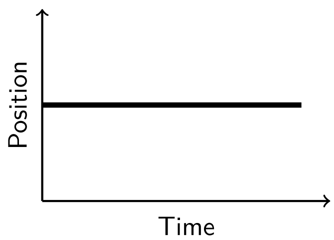
Graph 2 answer:

(f) What is the general difference between motions which result in a straight-line position *vs.* time graph and those that result in a curved-line position vs. time graph?

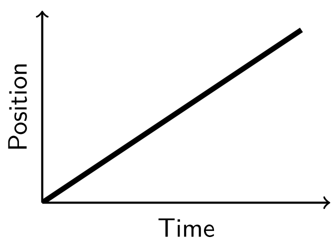
Homework

Answer the following questions in the spaces provided.

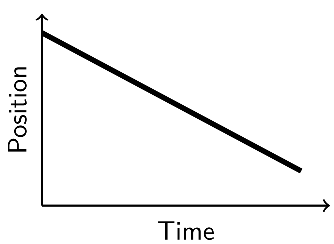
1.What do you do to create a horizontal line on a position-time graph?



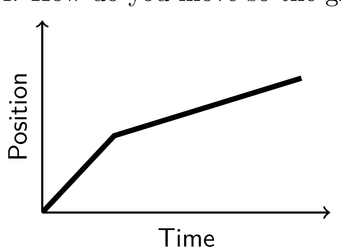
2. How do you walk to create a straight line that slopes up?



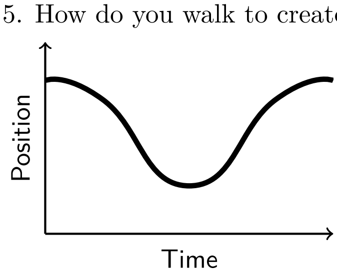
3. How do you walk to create a straight line that slopes down?



4. How do you move so the graph goes up steeply at first, then continues up gradually?



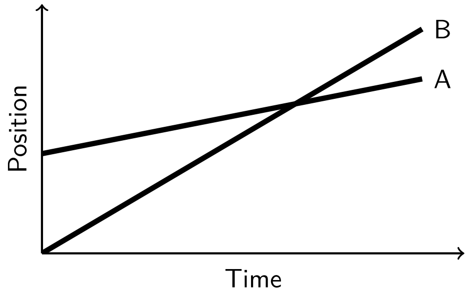
5. How do you walk to create a U-shaped graph?



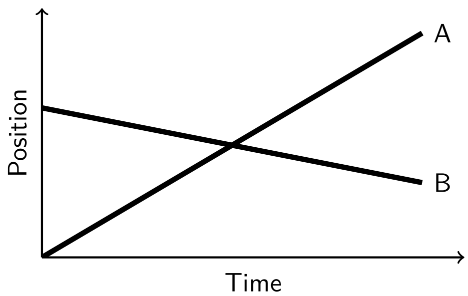
Answer the following about the two objects A and B, whose motion produced the following position-time graphs.

6. (a) Which object is moving faster? (b) Which starts ahead? Define what you mean by "ahead."

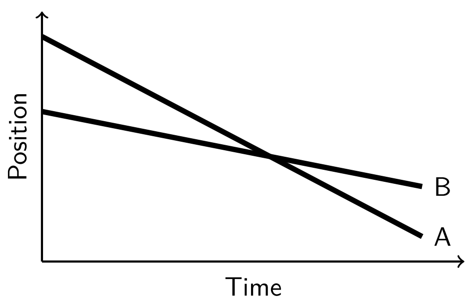
(c) What does the intersection mean?



7. (a) Which object is moving faster? (b) Which object has a negative velocity according to the convention we have established?

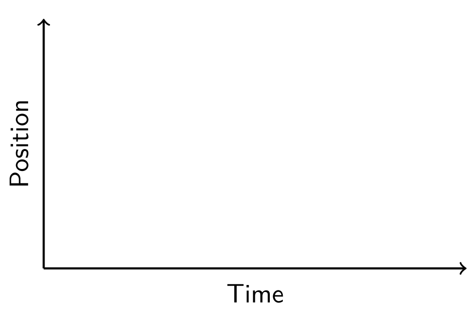


8. (a) Which object is moving faster? (b) Which starts ahead? Define what you mean by "ahead."

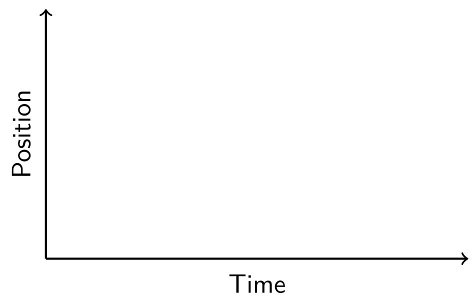


Sketch the position-time graph corresponding to each of the following descriptions of the motion of an object.

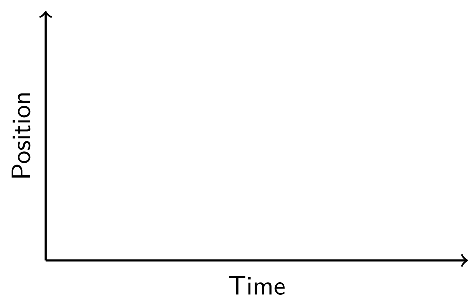
9. The object moves with a steady (constant) velocity away from the origin.



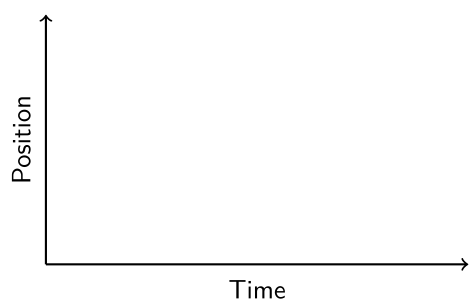
10. The object is standing still.



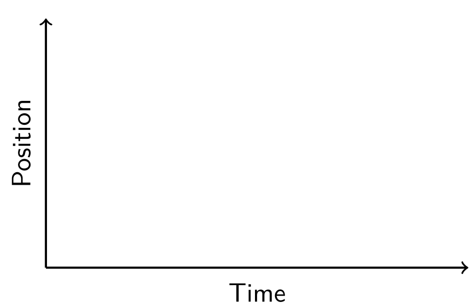
11. The object moves with a steady (constant) velocity toward the origin for 5 seconds and stands still for 5 seconds.



12. The object moves with a steady velocity away from the origin for 5 seconds, then reverses direction and moves at the same speed toward the origin for 5 seconds.



13. The object moves away from the origin, starting slowly and speeding up.



Lab 2 Velocity *vs.* Time Graphs[1]

Name: ____________________ Lab Partner(s): ______________________________

Objectives

- To acquire an intuitive understanding of speed and velocity in one dimension.
- To learn how to relate graphs of velocity *vs.* time to the motions they represent.

Apparatus

- *Capstone* software (*V_Graphs.cap* experiment file)
- Wireless motion sensor
- Wooden board

Introduction

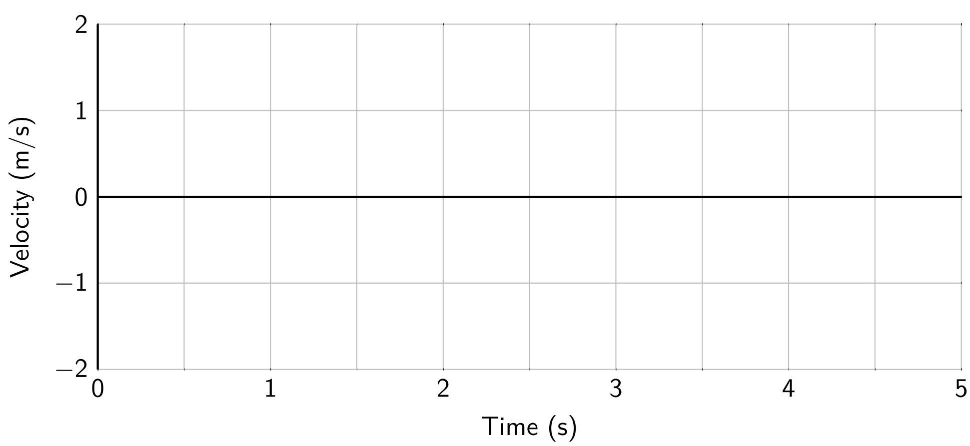
You have already plotted your position as a function of time. Another way to represent your motion during an interval of time is with a graph which describes how fast and in what direction you are moving from moment to moment. How fast you move is known as your speed. Velocity is a quantity which takes into account your speed and the direction you are moving. It is the rate of change of position with respect to time. Thus, when you examine the motion of an object moving along a line, its velocity can be positive or negative depending on whether the object is moving in the positive or negative direction.

Graphs of velocity over time are more challenging to create and interpret than those for position. A good way to learn to interpret them is to create and examine velocity *vs.* time graphs of your own body motions, as you will do in the next few activities.

Activity 1: Making Velocity *vs.* Time Graphs

To make the graphs in the following activities, open the *V_Graphs.cap* application in the *Phys131* folder. Turn on the sensor at your station. To connect it to the computer via Bluetooth, select **Hardware Setup** and select the correct address that matches its Device ID Number (XXX-XXX). To start a data run, click the **Record** button. To stop a data run, click the **Stop** button.

(a) Make a velocity graph by walking away from the sensor slowly and steadily. Try again until you get a graph you're satisfied with and then sketch your result on the graph that follows. (We suggest you draw smooth patterns by ignoring smaller bumps that are mostly due to your steps.)



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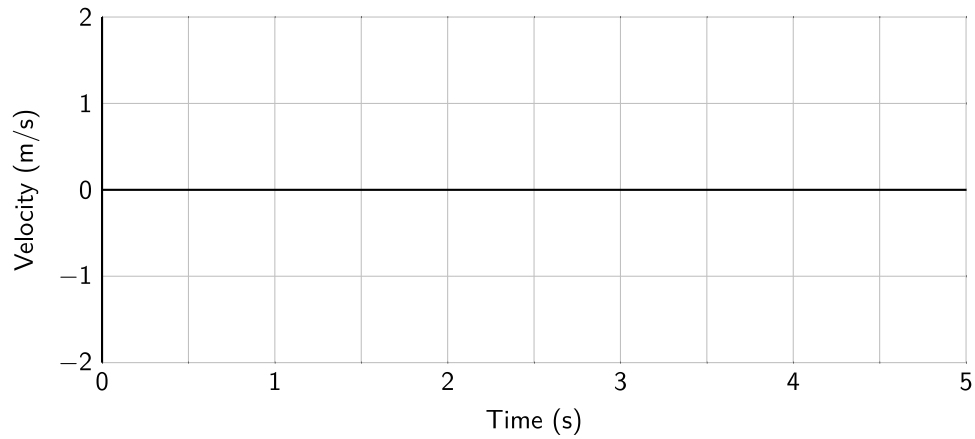
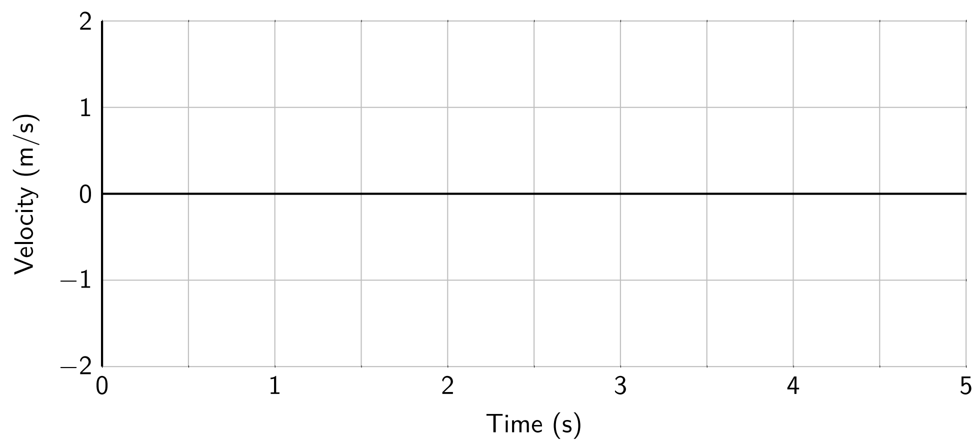
(b) Make a velocity graph, walking away from the sensor steadily at a medium speed. Sketch your graph below.



(c) Make a velocity graph, walking toward the sensor slowly and steadily. Sketch your graph below.



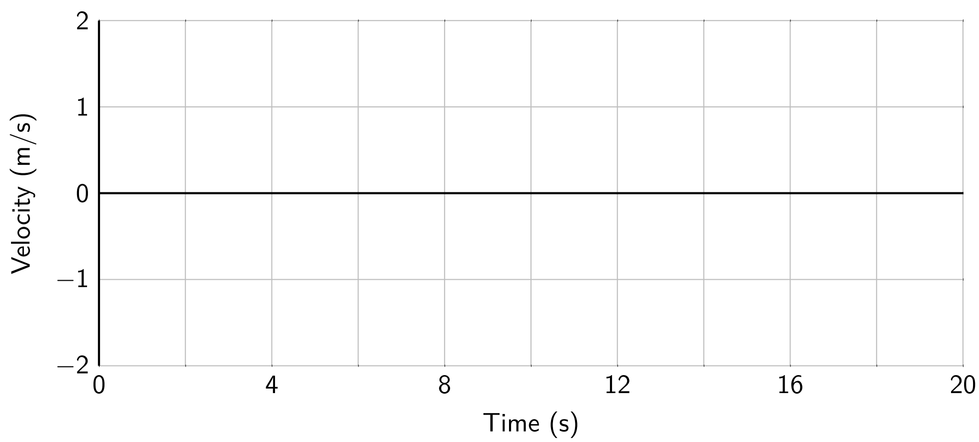
(d) What is the most important difference between the graph made by slowly walking away from the sensor and the one made by walking away more quickly?

(e) How are the velocity *vs.* time graphs different for motion away and motion toward the sensor?

Activity 2: Predicting a Velocity *vs.* Time Graph

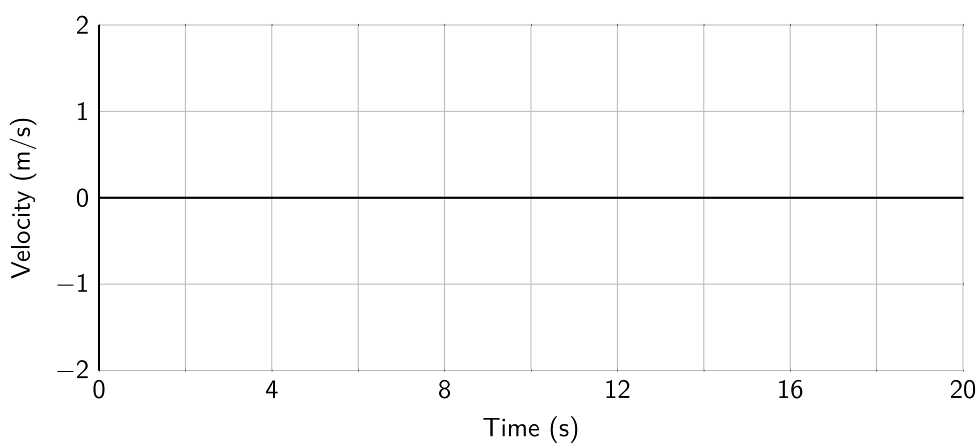
Suppose you were to undergo the following sequence of motions: (1) walk away from the sensor slowly and steadily for 6 seconds, (2) stand still for 6 seconds, (3) walk toward the sensor steadily about twice as fast as before.

(a) Use a dashed line in the graph that follows to record your prediction of the shape of the velocity graph that will result from the motion described above.



(b) Compare predictions with your partner(s) and see if you can all agree. Use a solid line to sketch your group prediction in the graph above.

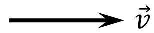
(c) Adjust the sampling time to 15 s and then test your prediction. Repeat your motion until you are confident that it matches the description in words and then draw the actual graph on the axes below. Be sure the 6-second stop shows clearly.



(d) Did your prediction match your real motion? If not, what misunderstanding of what elements of the graph did you have?

Velocity Vectors

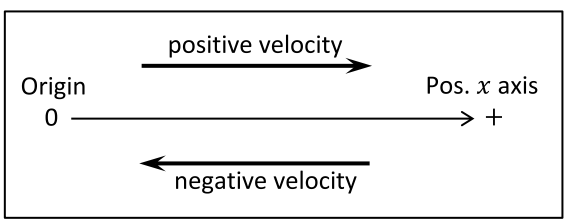
The two ideas of speed and direction can be combined and represented by vectors. A velocity vector is represented by an arrow pointing in the direction of motion. The length of the arrow is drawn proportional to the speed; the longer the arrow, the larger the speed. If you are moving toward the right, your velocity vector can be represented by the arrow shown below.

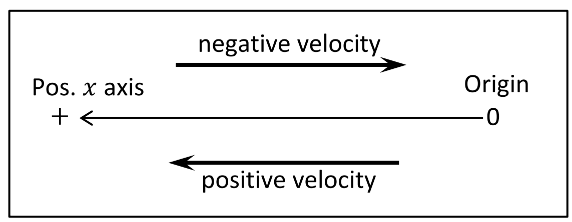


If you were moving twice as fast toward the right, the arrow representing your velocity vector would look like:

while moving twice as fast toward the left would be represented by the following arrow:

What is the relationship between a one-dimensional velocity vector and the sign of velocity? This depends on the way you choose to set the positive x-axis.





In both diagrams the top vectors represent velocity toward the right. In the left diagram, the x-axis has been drawn so that the positive x-direction is toward the right. Thus the top arrow represents positive velocity. However, in the right diagram, the positive x-direction is toward the left. Thus the top arrow represents negative velocity. Likewise, in both diagrams the bottom arrows represent velocity toward the left. In the left diagram this is negative velocity, and in the right diagram it is positive velocity.

Activity 3: Sketching Velocity Vectors

Sketch below velocity vectors representing the three parts of the motion described in the prediction you made in Activity 2.

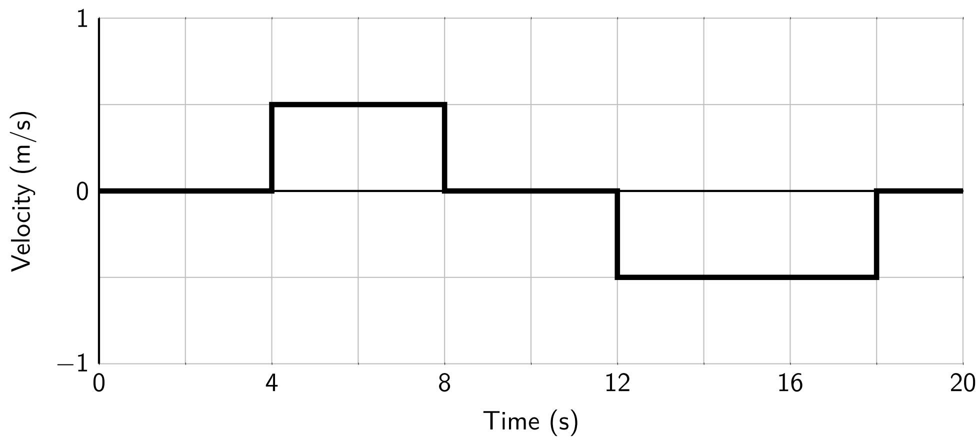
(a) Walking slowly away from the sensor:

(b) Standing still:

(c) Walking rapidly toward the sensor:

Activity 4: Matching a Velocity Graph

(a) Describe how you think you will have to move in order to match the velocity graph shown below.



(b) Move in such a way that you can reproduce the graph shown. You may have to practice a number of times to get the movements right. Work as a team and plan your movements. Get the times right. Get the velocities right. You and each person in your group should take a turn. Then sketch your group's best match on the above graph.

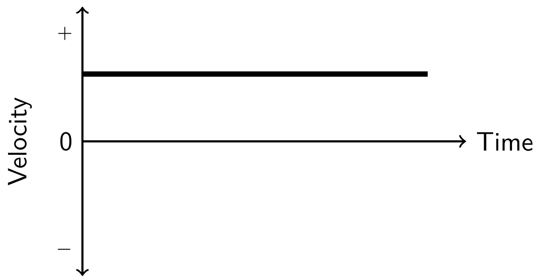
(c) Is it possible for an object to move so that it produces an absolutely vertical line on a velocity-time graph? Explain.

(d) Did you run into the motion sensor on your return trip? If so, why did this happen? How did you solve the problem? Does a velocity graph tell you where to start? Explain.

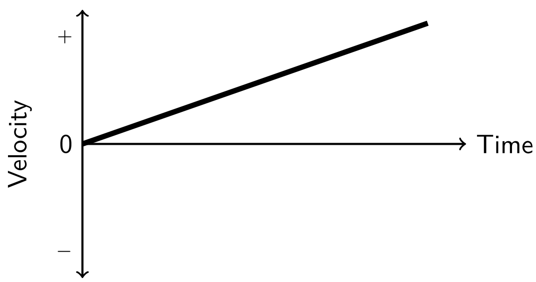
Homework

Answer the following questions in the spaces provided.

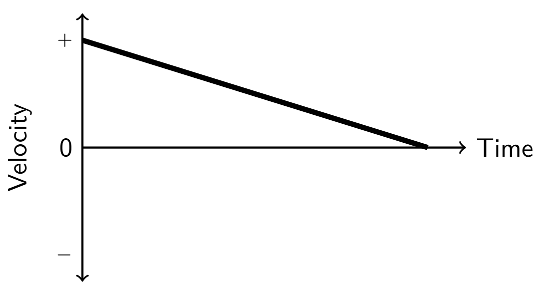
1. How do you move to create a horizontal line in the positive part of a velocity-time graph, as shown below?



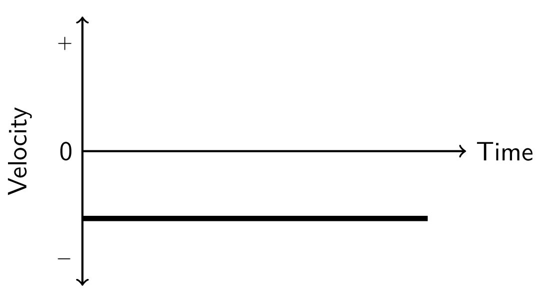
2. How do you move to create a straight-line velocity-time graph that slopes up from zero, as shown below?



3. How do you move to create a straight-line velocity-time graph that slopes down, as shown below?

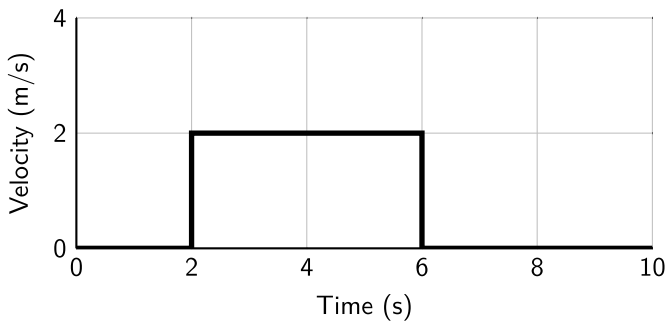


4. How do you move to make a horizontal line in the negative part of a velocity-time graph, as shown below?

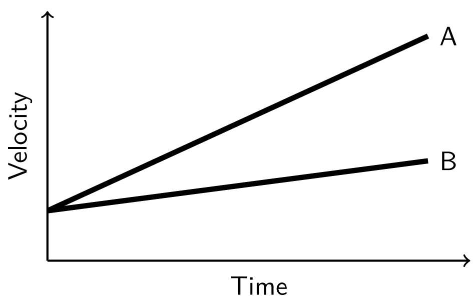


5. The velocity-time graph of an object is shown below. Figure out the total change in position (displacement) of the object. Show your work.

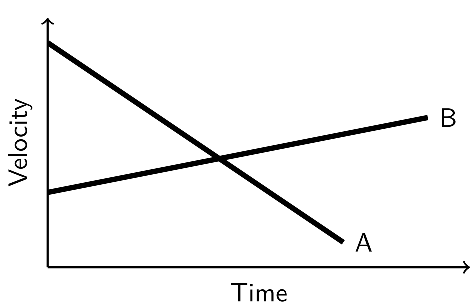
Displacement = ___________________ meters.



6. The velocity graph below shows the motion of two objects, A and B. Answer the following questions separately. Explain your answers when necessary. (a) Is one faster than the other? If so,which one is faster? (A or B) (b) What does the intersection mean? (c) Can one tell which object is “ahead”? (define “ahead”) (d) Does either object A or B reverse direction? Explain.



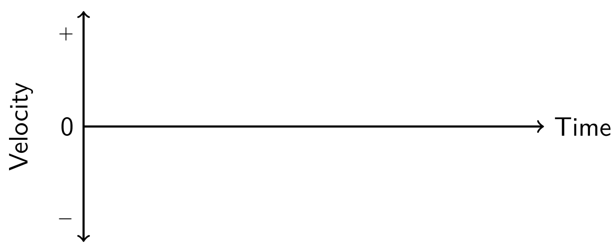
7. The velocity graph below shows the motion of two objects, A and B. Answer the following questions separately. Explain your answers when necessary. (a) Is one faster than the other? If so,which one is faster? (A or B) (b) What does the intersection mean? (c) Can one tell which object is “ahead”? (define “ahead”) (d) Does either object A or B reverse direction? Explain.



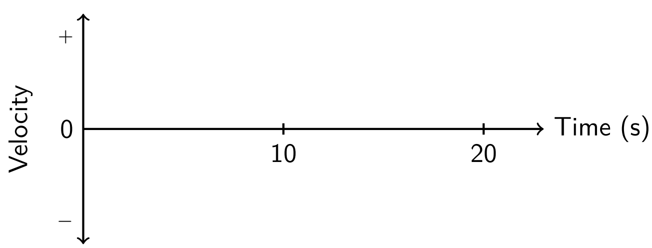
An object moves along a line (the + position axis). Sketch the velocity-time graph corresponding to each of the following descriptions of its motion.

8. The object is moving away from the origin at a constant velocity.

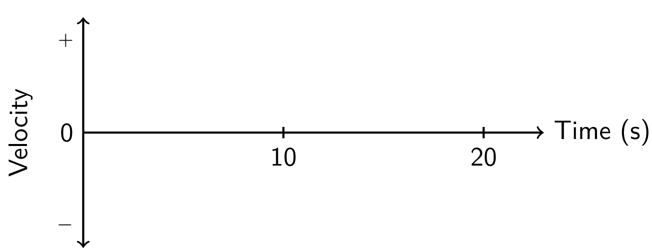
9. The object is standing still.



10. The object moves toward the origin at a steady (constant) velocity for 10 s and then stands still for 10 s.



11. The object moves away from the origin at a steady (constant) velocity for 10 s, reverses direction and moves back toward the origin at the same speed for 10 s.



Lab 3   Relating Position and Velocity Graphs[1]

Name: ______________________   Lab Partner(s): ______________________

Apparatus

- *Capstone* software (*P_V_Graphs.cap* experiment file)
- Wireless motion sensor
- Wooden board

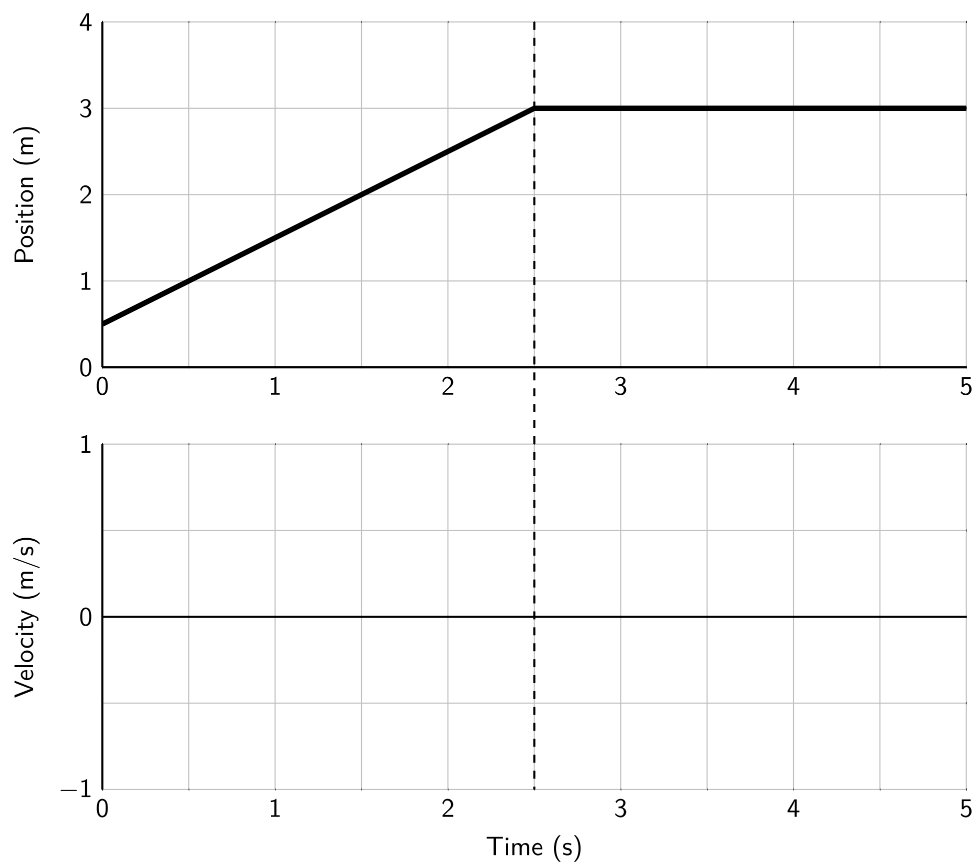
Introduction

You have looked at position and velocity *vs.* time graphs separately. Since position *vs.* time and velocity *vs.* time graphs are different ways to represent the same motion, it ought to be possible to figure out the velocity at which someone is moving by examining their position *vs.* time graph. Conversely, you ought to be able to figure out how far someone has traveled (change in position) from a velocity *vs.* time graph.

Activity 1: Predicting Velocity Graphs from Position Graphs

Open the *P_V_Graphs.cap* application in the *Phys131* folder. Turn on the sensor at your station and connect it to the computer via Bluetooth. For some of the runs it may be necessary to adjust the time axis for one of the graphs so that the time scales are the same for the position and velocity graphs.

(a) Carefully study the position graph shown below and predict the velocity *vs.* time graph that would result from the motion. Using a dashed line, sketch your prediction of the corresponding velocity *vs.* time graph on the velocity axes.



[1]1990-93 Dept. of Physics and Astronomy, Dickinson College. Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Portions of this material may have been modified locally and may not have been classroom tested at Dickinson College.

(b) After each person in your group has sketched a prediction, test your prediction by matching the position *vs.* time graph shown. When you have made a good duplicate of the position graph, sketch your actual graph over the existing position *vs.* time graph. Use a solid line to draw the actual velocity graph on the same graph with your prediction. (Do not erase your prediction).

(c) How would the position graph be different if you moved faster? Slower?

(d) How would the velocity graph be different if you moved faster? Slower?

Activity 2: Average Velocity Calculations

(a) Find your average velocity *during the time you were moving at constant speed* from your velocity graph in the previous activity. To do this, use the **Statistics** function on the graph menu bar to determine the average velocity and the standard deviation on the *left* part of the graph (where the velocity is approximately constant). See **Appendix A: Capstone** for instructions on how to do this.

Average value of the velocity: ______________ m/s

Standard deviation: ______________ m/s (This can be taken as an uncertainty.)

Average velocity with uncertainty: _____________________ m/s

(b) Average velocity during a particular time interval can also be calculated as the change in position divided by the change in time. (The change in position is often called the displacement.) By definition, this is also the slope of the position *vs.* time graph for that time period. Select the position *vs.* time graph and open the **Delta Tool** function on the Graph menu bar. Set the cursor on one point on the position graph, right click on this point and select **Show Delta Tool**. There will then be two cursors. Set one cursor on one point on the position graph and the other on another point on the graph. The changes in position and time will then be shown on the graph. For a more accurate answer, use two points as far apart as possible but still typical of the motion, *and within the time interval over which you took velocity readings in part (a).* Record the values of Δx and Δt here:

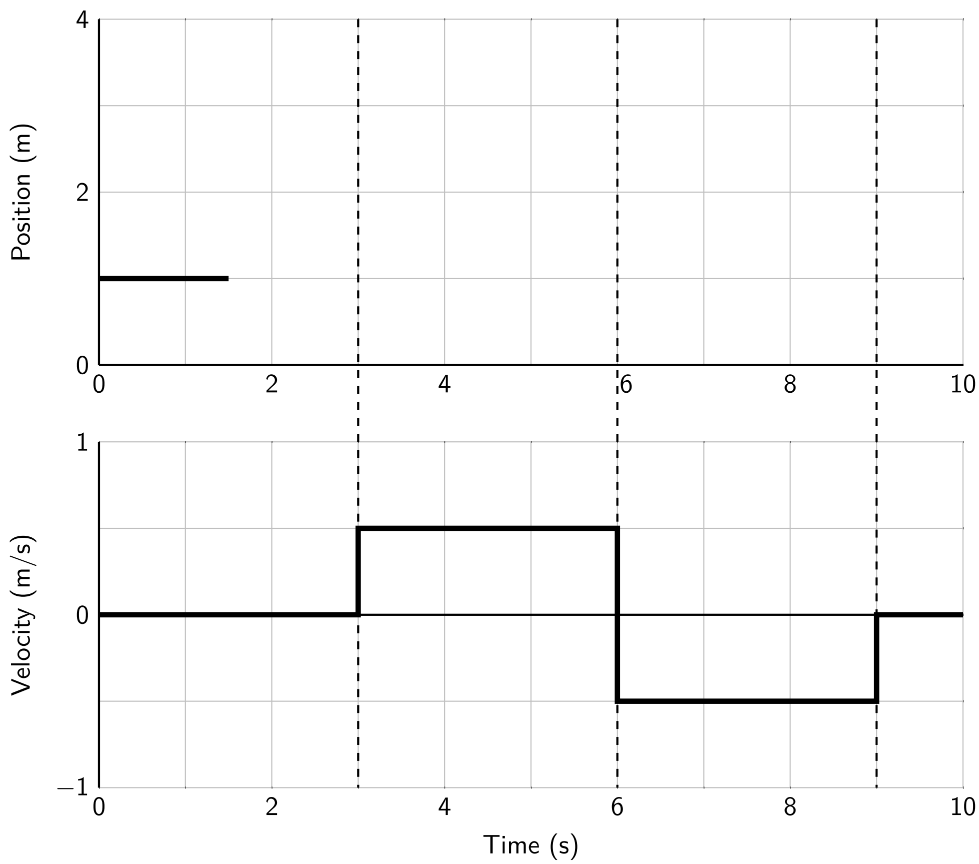
(c) Divide the change in position by the change in time to calculate the average velocity. Show your calculation here:

(d) Is the average velocity positive or negative? Is this what you expected?

(e) Does the average velocity you just calculated from the position graph fall within the range of values determined in part (a) from the velocity graph? Would you expect them to agree? How would you account for any differences?

Activity 3: Finding Position from a Velocity Graph

(a) Carefully study the velocity graph that follows. Using a dashed line, sketch your prediction of the corresponding position graph on the top set of axes. (Assume that you started at the 1-meter mark.)



(b) After each person has sketched a prediction, do your group's best to duplicate the bottom (velocity *vs.* time) graph by walking. When you have made a good duplicate of the velocity *vs.* time graph, draw your actual result over the existing velocity *vs.* time graph. Use a solid line on the top graph to draw the actual position *vs.* time graph on the same axes with your prediction. Do not erase your prediction.

(c) How can you tell from a velocity *vs.* time graph that the moving object has changed direction?

(d) What is the velocity at the moment the direction changes?

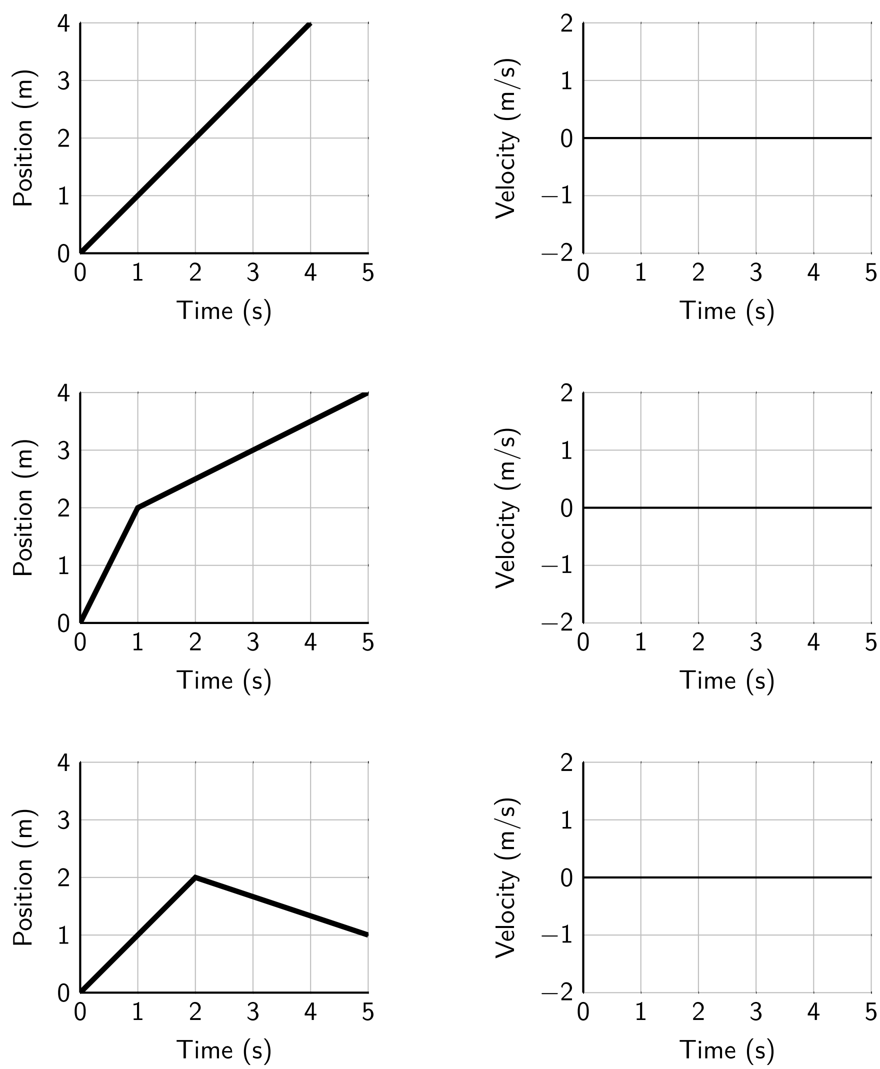
(e) Is it possible to actually move your body (or an object) to make vertical lines on a position *vs.* time graph? Why or why not? What would the velocity be for a vertical section of a position *vs.* time graph?

(f) How can you tell from a position *vs.* time graph that your motion is steady (motion at a constant velocity)?

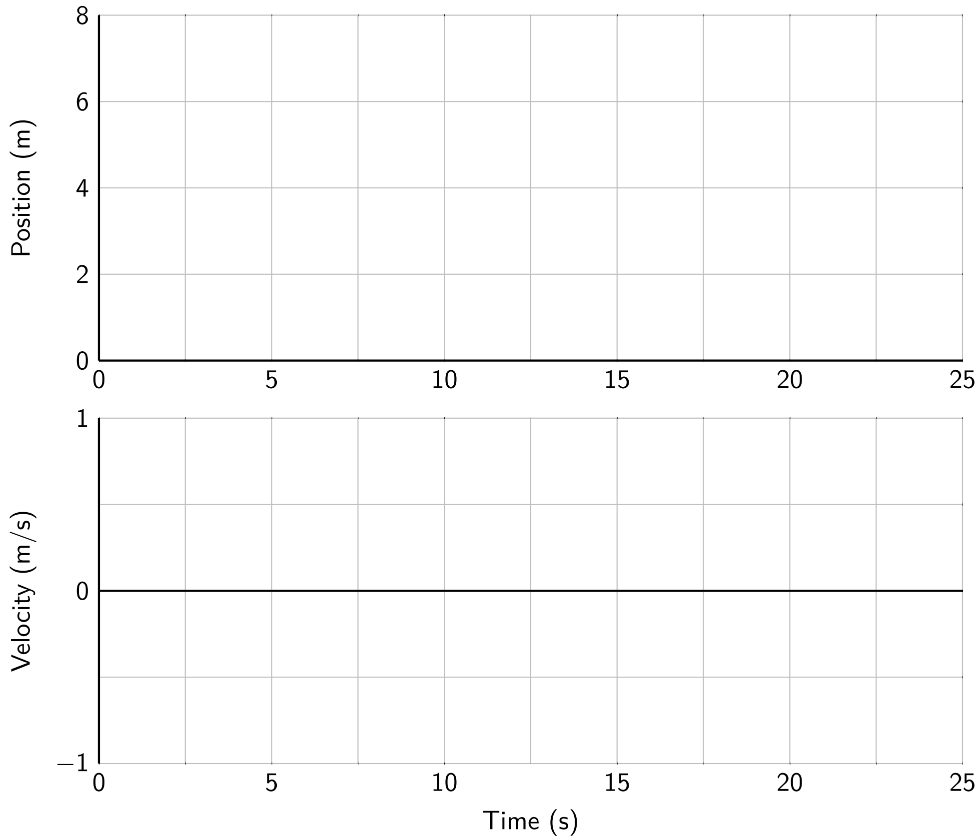
(g) How can you tell from a velocity *vs.* time graph that your motion is steady (constant velocity)?

Homework

1. Draw the velocity graphs for an object whose motion produced the position-time graphs shown below on the left. Note: Unlike most real objects, you can assume these objects can change velocity so quickly that it looks instantaneous with this time scale.



2. Draw careful graphs below of position and velocity for a cart that (a) moves away from the origin at a slow and steady (constant) velocity for the first 5 seconds; (b) moves away at a medium-fast, steady (constant) velocity for the next 5 seconds; (c) stands still for the next 5 seconds; (d) moves toward the origin at a slow and steady (constant) velocity for the next 5 seconds; (e) stands still for the last 5 seconds.



Lab 4 Changing Motion[1]

Name: ____________________ Lab Partner(s): ______________________________

Objectives

- To learn how to relate graphs of acceleration *vs.* time to the motions they represent.
- To understand the relationship between position *vs.* time, velocity *vs.* time, and acceleration *vs.* time graphs.

Introduction: Velocity and Acceleration Graphs

We are interested in having you learn to describe simple motions in which the velocity of an object is changing. In order to learn to describe motion in more detail for some simple situations, you will be asked to observe and describe the motion of a dynamics cart on a track. Although graphs and words are still important representations of these motions, you will also be asked to draw velocity vectors, arrows that indicate both the direction and speed of a moving object. Thus, you will also learn how to represent simple motions with velocity diagrams.

In the last session, you looked at position *vs.* time and velocity *vs.* time graphs of the motion of your body as you moved at a "constant" velocity. The data for the graphs were collected using a motion detector. Your goal in this session is to learn how to describe various kinds of motion in more detail. It is not enough when studying motion in physics to simply say that "the object is moving toward the right" or "it is standing still." You have probably realized that a velocity *vs.* time graph is better than a position vs. time graph when you want to know how fast and in what direction you are moving at each instant in time as you walk. When the velocity of an object is changing, it is also important to know how it is changing. The rate of change of velocity is known as the acceleration.

In order to get a feeling for acceleration, it is helpful to create and learn to interpret velocity *vs.* time and acceleration *vs.* time graphs for some relatively simple motions of a cart on a track. You will be observing the motion of the cart as it moves at a constant velocity and as it changes its velocity at a constant rate.

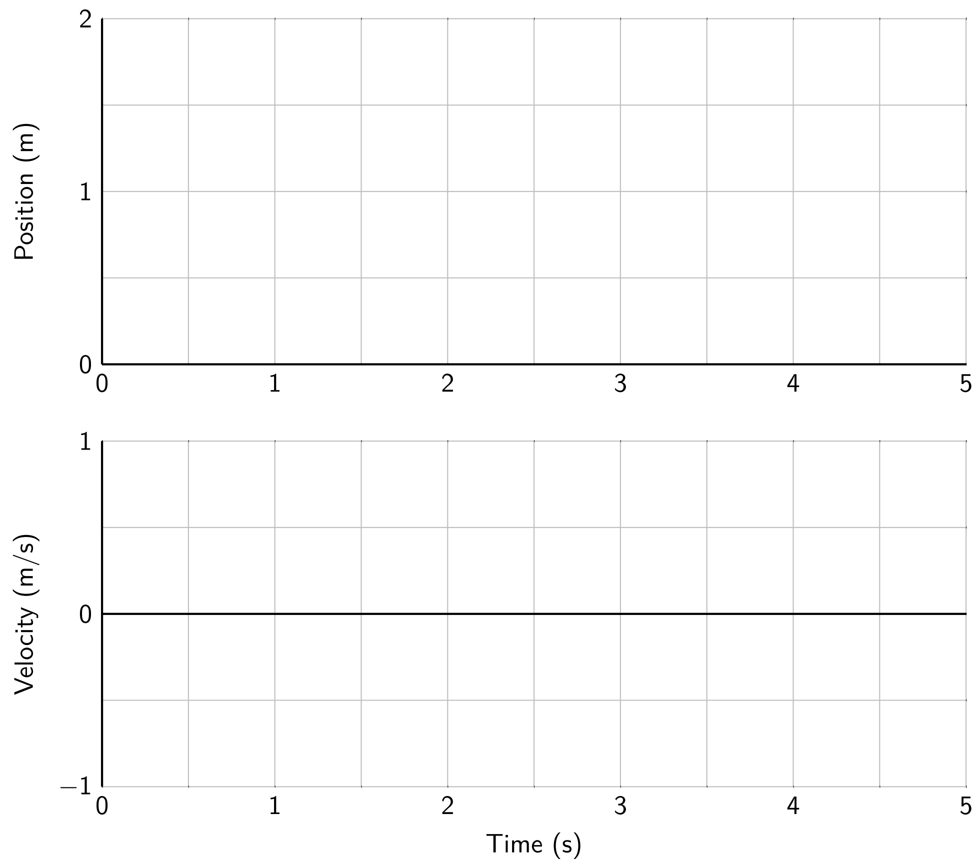
Apparatus

- *Capstone* software (*P_V_A_Graphs.cap* experiment file)
- Dynamics track
- Lab stand
- Wireless smart cart

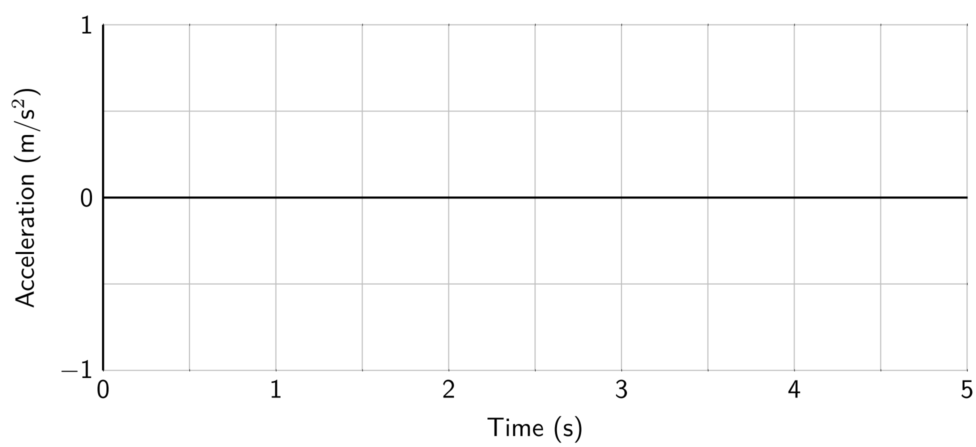
Activity 1: Graphs of Constant Velocity

(a) Based on your observations of the motions of your body in the last session, what would the position and velocity graphs look like if a cart were to move at a constant velocity in the positive direction, starting at the origin? Sketch your predictions with dashed lines on the axes that follow.

[1] 1990-93 Dept. of Physics and Astronomy, Dickinson College. Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Portions of this material may have been modified locally and may not have been classroom tested at Dickinson College.



(b) Acceleration is defined as the time rate of change of velocity. Sketch your prediction of the cart's acceleration on the axes below using a dashed line.



(c) To test your predictions, open the *P_V_A_Graphs.cap* application in the *Phys131* folder. The wireless smart cart includes built-in sensors which measure its position (relative to its starting point), velocity, and acceleration. Turn on the cart at your station and connect it to the computer via Bluetooth. At the beginning of the experiment, the measurement from the built-in acceleration sensor may not be zero when the acceleration is actually zero. To zero (or tare) the sensor, select the desired sensor (in this case, the **Smart Cart Acceleration Sensor**) in the **Controls** palette and then click **Zero Sensor Now** while the cart is at rest.

Give the cart a push in the positive direction (defined by the x-axis printed on the top of the cart) along the level track and graph its motion. Try several times until you get a fairly constant velocity. Sketch your results with solid lines on the axes shown above. The acceleration *vs.* time graphs may exhibit small fluctuations due to irregularities in the motion of the cart. You should ignore these fluctuations and draw smooth patterns.

(d) Did your graphs agree with your predictions? What characterizes constant velocity motion on a position *vs.* time graph?

(e) What characterizes constant velocity motion on a velocity *vs.* time graph?

(f) What characterizes constant velocity motion on an acceleration *vs.* time graph?

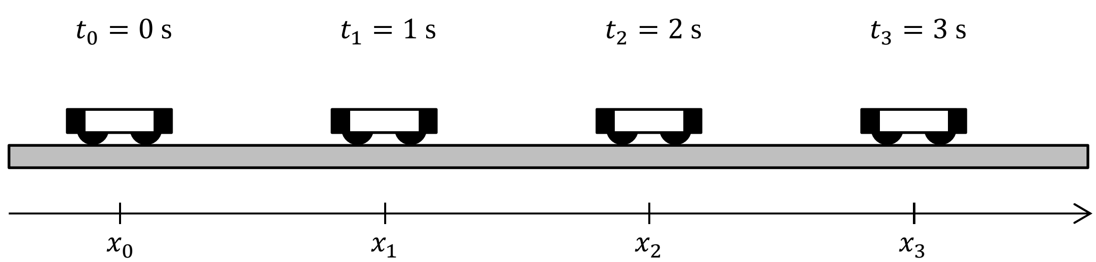
Activity 2: Representing Acceleration

To find the average acceleration of the cart during some time interval (the average time rate of change of its velocity), you must measure its velocity at two different times, calculate the difference between the final value and the initial value and divide by the time interval.

To find the acceleration vector from two velocity vectors, you must first find the vector representing the change in velocity by subtracting the initial velocity vector from the final one. Then you divide this vector by the time interval.

(a) Calculate the average acceleration during some time interval from your velocity graph in Activity 1. Does the result agree with your acceleration graph in Activity 1?

(b) The diagram below shows the positions of the cart at equal time intervals. (This is like taking snapshots of the cart at equal time intervals.) At each indicated time, sketch, and label, a vector above the cart which might represent the velocity of the cart at that time while it is moving at a constant velocity in the positive direction.

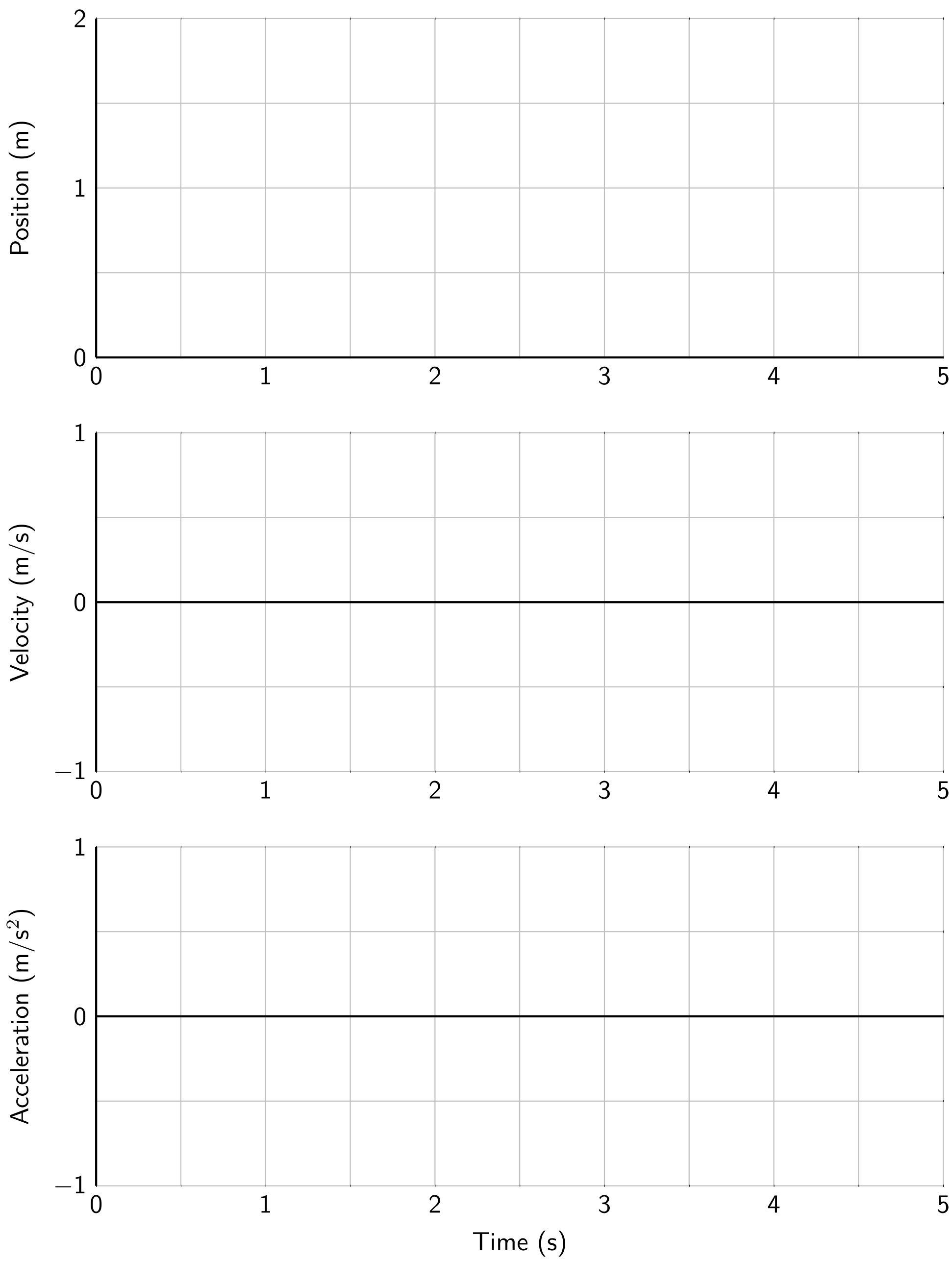


(c) Show how you would find the vector representing the change in velocity between the times $t_1 = 1$ s and $t_2 = 2$ s by creating a vector diagram using the vectors above. From the resultant vector, what value would you calculate for the acceleration? Is this value in agreement with the acceleration graph you obtained in Activity 1?

Activity 3: Graphs Depicting Speeding Up

In this activity you will look at velocity and acceleration graphs of the motion of a cart when its velocity is changing. You will be able to see how these two representations of the motion are related to each other when the cart is speeding up.

(a) Predict the shape of the position, velocity, and acceleration *vs.* time graphs for a cart moving in the positive direction while speeding up at a constant rate. Sketch your predictions on the following axes using dashed lines.



(b) To test your predictions, use the lab stand to raise the track several centimeters at one end. Release the cart from rest near the raised end of the track and create graphs of its motion as it moves in the positive direction (defined by the x-axis printed on the top of the cart) down the incline. Stop the cart before it hits the end of the track. Sketch the graphs neatly on the above axes using solid lines.

(c) How does your position graph differ from the position graphs for steady (constant velocity) motion?

(d) What feature of your velocity graph signifies that the motion was in the positive direction?

(e) What feature of your velocity graph signifies that the cart was speeding up? How would a graph of motion with a constant velocity differ?

(f) During the time that the cart is speeding up, is the acceleration positive or negative? How does speeding up while moving in the positive direction result in this sign of acceleration? Hint: Remember that acceleration is the rate of change of velocity. Look at how the velocity is changing.

(g) How does the velocity vary in time as the cart speeds up? Does it increase at a steady rate or in some other way?

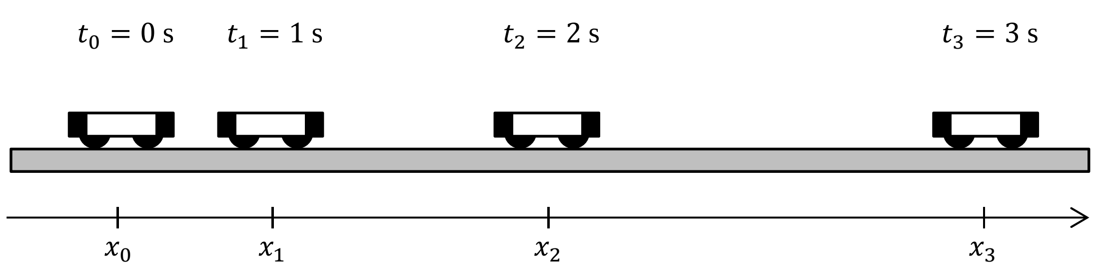
(h) How does the acceleration vary in time as the cart speeds up? Is this what you expect based on the velocity graph? Explain.

(i) Do not delete the graphs from the computer screen. They will be used in Activity 5.

Activity 4: Using Vectors to Describe Acceleration

Let's return to the Vector Diagram representation and use it to describe the acceleration.

(a) The diagram that follows shows the positions of the cart at equal time intervals. At each indicated time, sketch, and label, a vector above the cart which might represent the velocity of the cart at that time while it is moving in the positive direction and speeding up.



(b) Show below how you would find the approximate length and direction of the vector representing the change in velocity between the times $t_1 = 1$ s and $t_2 = 2$ s by creating a vector diagram using the vectors above. No quantitative calculations are needed. Based on the direction of the resultant vector and the direction of the positive x-axis, what is the sign of the acceleration? Does this agree with your answer to Activity 3 (f)?

Activity 5: Measuring and Calculating Accelerations

In this investigation you will analyze the motion of your accelerated cart quantitatively. This analysis will be quantitative in the sense that your results will consist of numbers. You will determine the cart's acceleration from the slope of your velocity *vs.* time graph and compare it to the average acceleration read from the acceleration *vs.* time graph.

(a) Using the **Statistics** function on your acceleration graph from Activity 3, determine the average acceleration and the standard deviation by selecting only values from the portion of the graph after the cart was released and before you stopped it. See **Appendix A: Capstone** for information on how to use the **Statistics** function. Record these values here:

(b) Write the acceleration with its uncertainty (the standard deviation) here:

(c) The average acceleration during a particular time period is defined as the change in velocity divided by the change in time. This is the average rate of change of velocity. By definition, the rate of change of a quantity graphed with respect to time is also the slope of the curve. Thus the (average) slope of an object's velocity *vs.* time graph is the (average) acceleration of the object.

Select the velocity *vs.* time graph from Activity 3 and open the **Delta Tool** function on the graph menu bar. Determine quantities Δv and Δt for two points on the velocity graph using the Delta Tool, in the same way that you did on the position *vs.* time graph in the previous experiment. Record the values of Δv and Δt here:

(d) Divide the change in velocity by the change in time. This is the average acceleration. Show your calculation here:

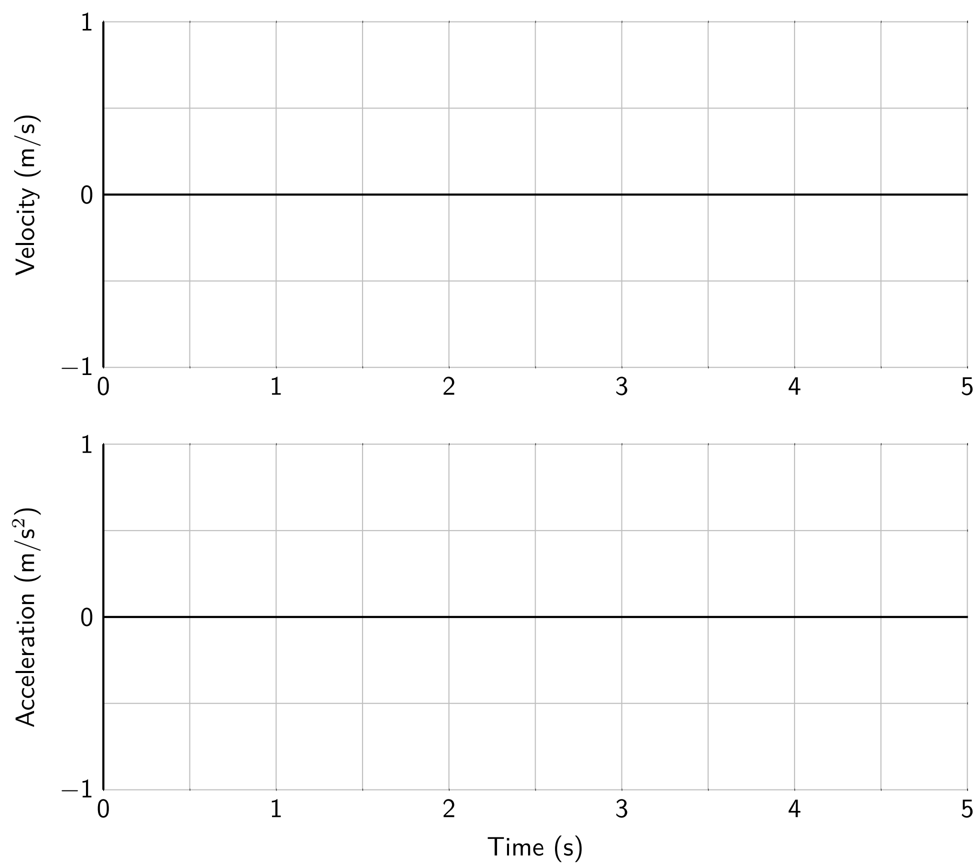
(e) Is the acceleration positive or negative? Is this what you expected?

(f) Does the average acceleration you just calculated agree with the average acceleration you calculated from the acceleration *vs.* time graph? Do you expect them to agree? How would you account for any differences?

Activity 6: Speeding Up at a Faster Rate

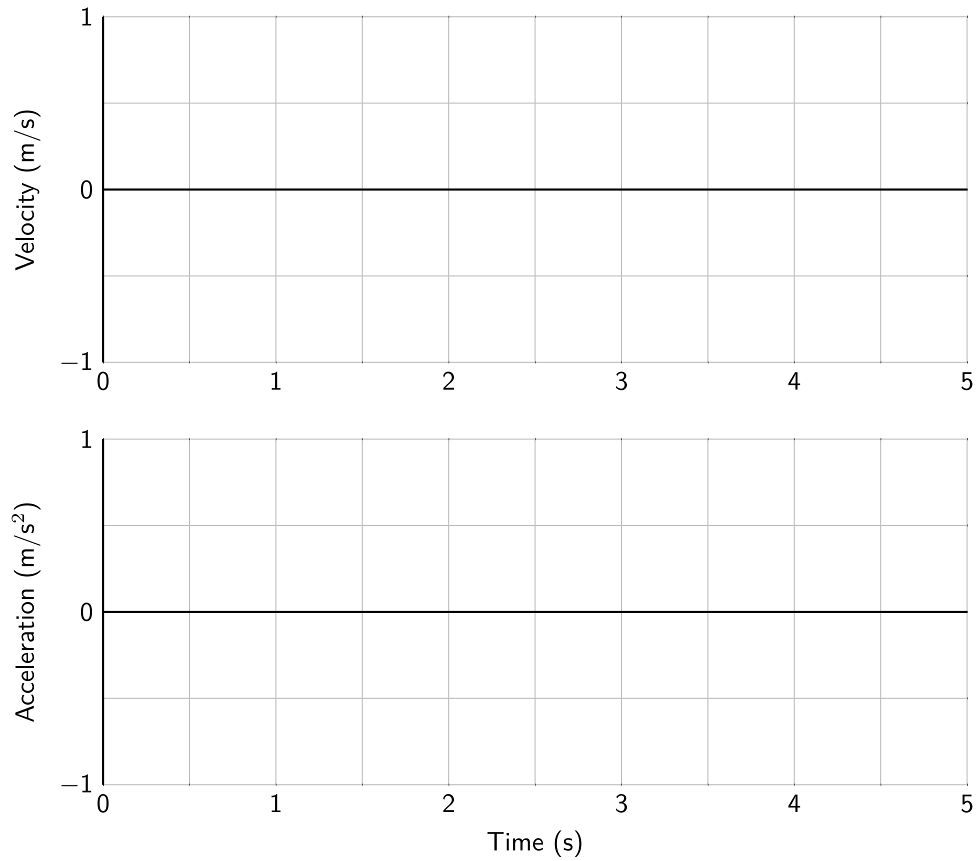
(a) Suppose that you accelerate your cart at a faster rate by raising the end of the track several more centimeters. How would your velocity and acceleration graphs change?

(b) Resketch the velocity and acceleration graphs you found in Activity 3 using the axes that follow.



(c) In the previous set of axes, use a dashed line or another color to sketch your predictions for the general graphs that depict a cart speeding up at a faster rate. Exact predictions are not expected. We just want to know how you think the general shapes of the graphs will change.

(d) Test your predictions by accelerating the cart with the end of the track raised several centimeters more than in Activity 3. Repeat if necessary to get nice graphs and then sketch the results using the axes that follow.

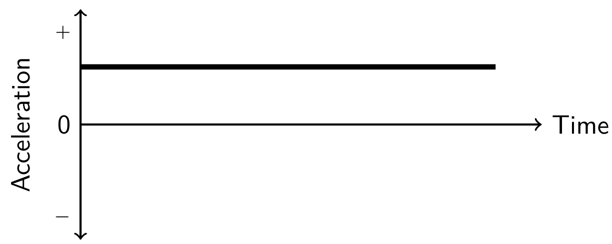


(e) Did the general shapes of your velocity and acceleration graphs agree with your predictions? How is the greater magnitude (size) of acceleration represented on a velocity *vs.* time graph?

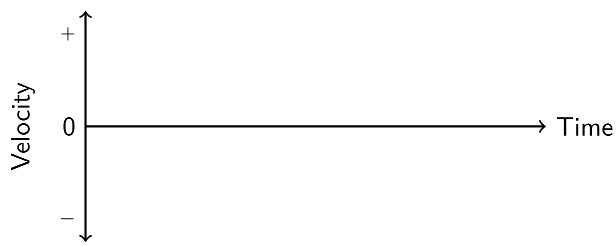
(f) How is the greater magnitude (size) of acceleration represented on an acceleration *vs.* time graph?

Homework

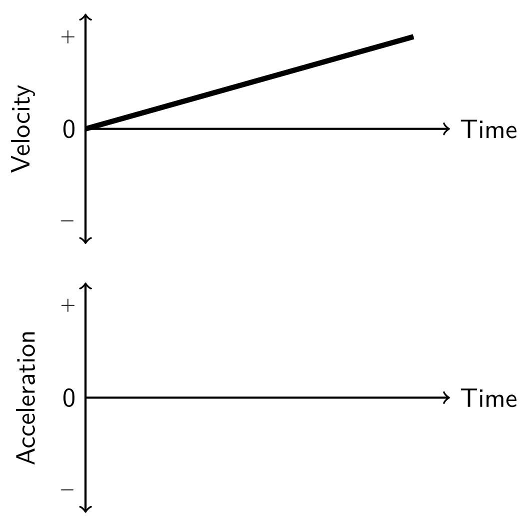
1. An object moving along a line (the + position axis) has the acceleration-time graph shown below. Describe how the object might move to create this graph if it is moving away from the origin.

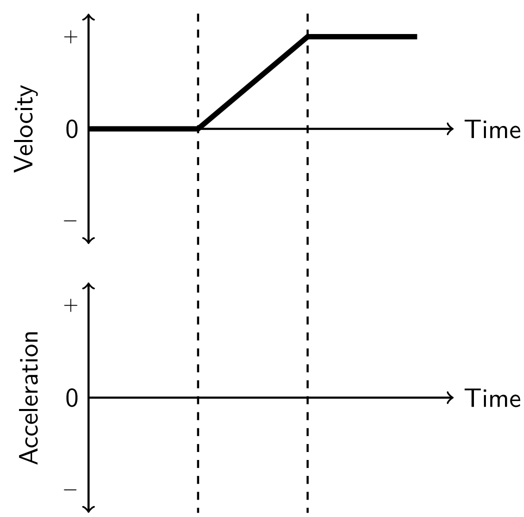


2. Sketch on the axes below a velocity-time graph that goes with the above acceleration-time graph.

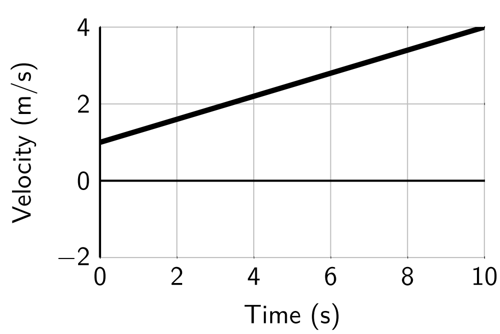


3. For each of the velocity-time graphs below, sketch the shape of the acceleration-time graph that goes with it.

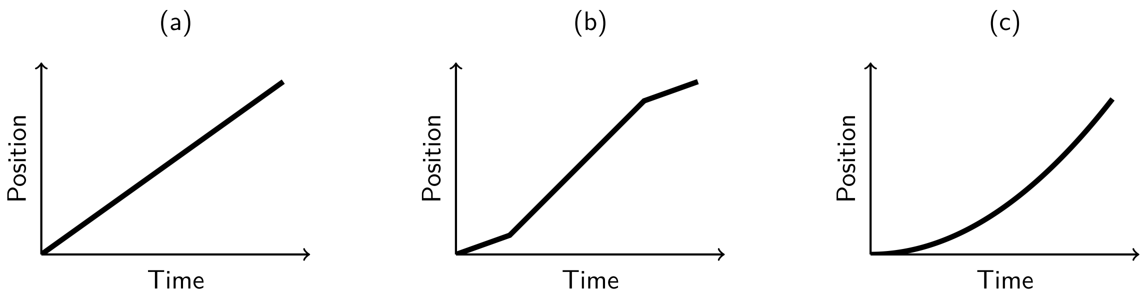




4. The following is a velocity-time graph for a car. What is the car's average acceleration? Show your work below.

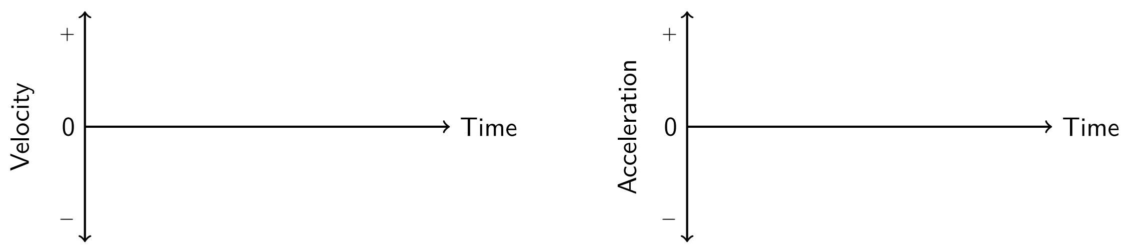


5. Which position-time graph below could be that for a cart that is steadily accelerating away from the origin?

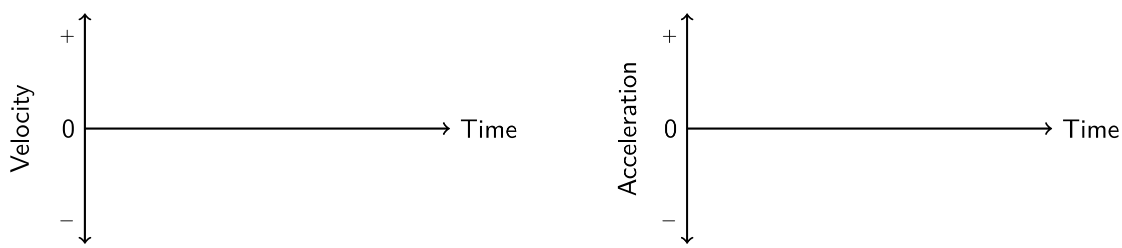


A car can move along a line (the + position axis). Sketch velocity-time and acceleration-time graphs which correspond to each of the following descriptions of the car's motion.

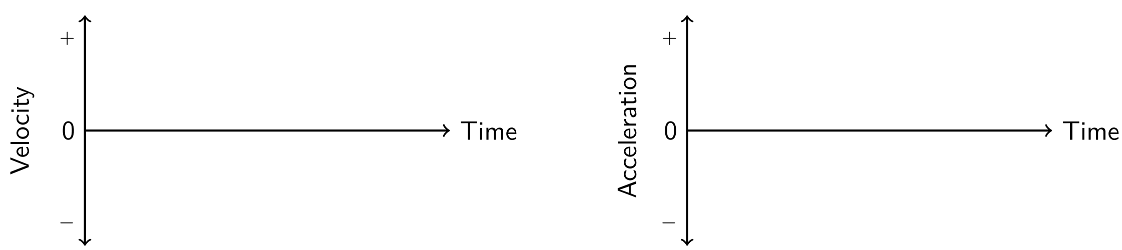
6. The car starts from rest and moves away from the origin increasing its speed at a steady rate.



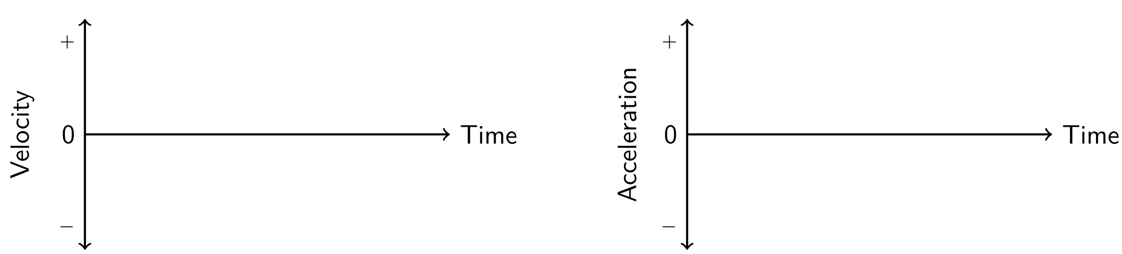
7. The car is moving away from the origin at a constant velocity.



8. The car starts from rest and moves away from the origin increasing its speed at a steady rate twice as large as in (6) above.



9. The car is moving away from the origin at a constant velocity twice as large as in (7) above.



Lab 5 Slowing Down, Speeding Up, and Turning[1]

Name: ______________________ Lab Partner(s): ______________________

Objectives

- To learn how to relate graphs of acceleration *vs.* time to the motions they represent.
- To understand the relationship between velocity *vs.* time and acceleration vs. time graphs.

About Slowing Down, Speeding Up and Turning

In the previous session, you explored the characteristics of position *vs.* time, velocity *vs.* time and acceleration *vs.* time graphs. In the cases examined, the object was always moving in the positive direction, either at a constant velocity or speeding up with a constant acceleration. Under these conditions, the velocity and acceleration are both positive. You also learned how to find the magnitude of the acceleration from velocity *vs.* time and acceleration *vs.* time graphs, and how to represent the velocity and acceleration using vectors.

In the motions you studied in the last session, the velocity and acceleration vectors representing the motion of the object both pointed in the same direction. In order to get a better feeling for acceleration, it will be helpful to examine velocity *vs.* time and acceleration *vs.* time graphs for some slightly more complicated motions. As before, you will examine the motion of a cart as its velocity changes at a constant rate. Only this time the motion may be in the positive or negative direction, and the cart may be speeding up or slowing down.

Apparatus

- *Capstone* software (*P_V_A_Graphs.cap* experiment file)
- Dynamics track
- Lab stand
- Tennis ball
- Wireless smart cart

Slowing Down and Speeding Up

In this activity you will look at a cart moving in the positive direction and slowing down. Later you will examine the motion of a cart moving in the negative direction and speeding up. In both cases, we are interested in the shapes of the velocity *vs.* time and acceleration *vs.* time graphs, as well as the vectors representing velocity and acceleration.

Activity 1: Graphs Depicting Slowing Down

(a) Predict the shape of the velocity and acceleration *vs.* time graphs for a cart moving in the positive direction while slowing down at a constant rate. Sketch your predictions on the following axes using dashed lines.

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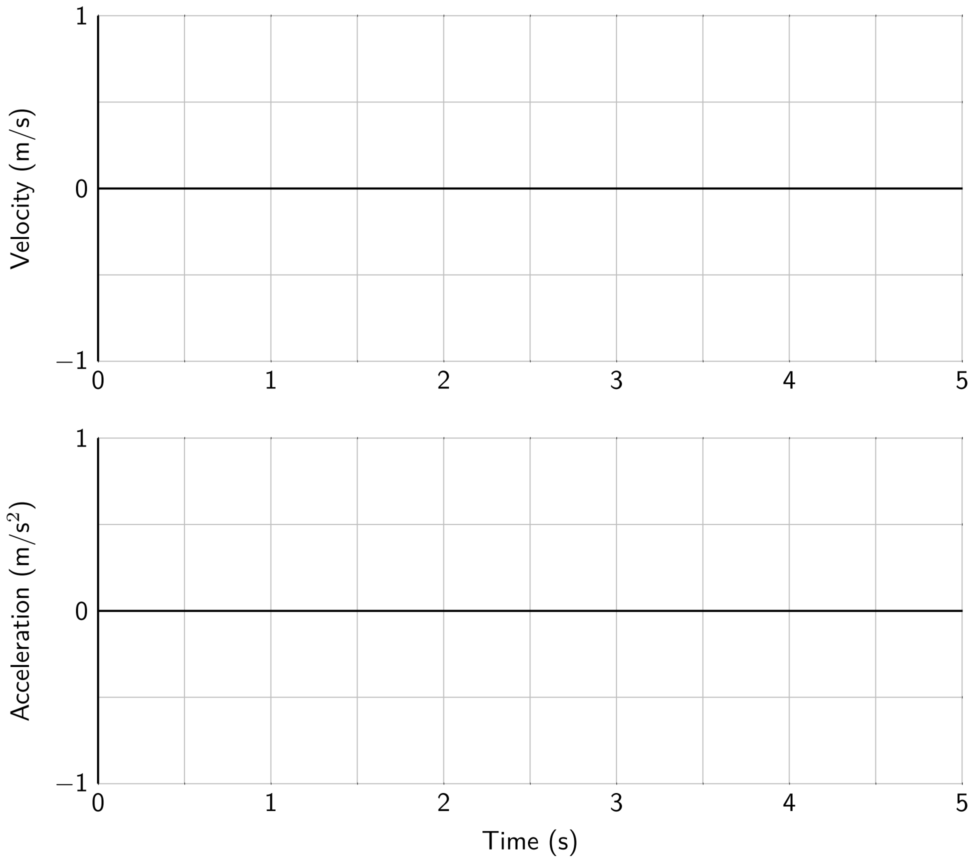


(b) To test your predictions, open the *P_V_A_Graphs.cap* application in the *Phys131* folder. Turn on the cart at your station and connect it to the computer via Bluetooth. You may need to zero (or tare) the built-in acceleration sensor. Use the lab stand to raise the track several centimeters at one end. Give the cart a gentle push near the lowered end of the track and create graphs of its motion as it moves in the positive direction (defined by the x-axis printed on the top of the cart) up the incline. Stop the cart before it turns around or hits the end of the track. Sketch the graphs neatly on the above axes using solid lines. The acceleration *vs.* time graphs may exhibit small fluctuations due to irregularities in the motion of the cart. You should ignore these fluctuations and draw smooth patterns.

(c) Did the shapes of your velocity and acceleration graphs agree with your predictions? How is the sign of the acceleration represented on a velocity *vs.* time graph?

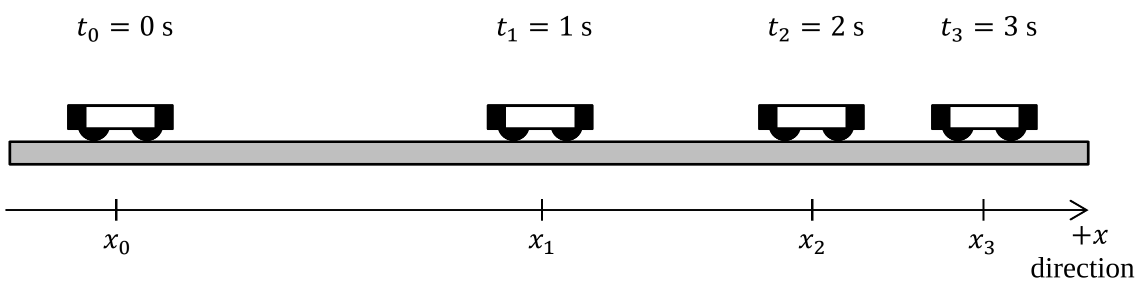
(d) How is the sign of the acceleration represented on an acceleration *vs.* time graph?

(e) Is the sign of the acceleration what you predicted? How does slowing down while moving in the positive direction result in this sign of acceleration? Hint: Remember that acceleration is the rate of change of velocity. Look at how the velocity is changing.

Activity 2: Constructing Acceleration Vectors for Slowing Down

Let's consider a diagrammatic representation of a cart which is slowing down and use vector techniques to figure out the direction of the acceleration.

(a) The diagram that follows shows the positions of the cart at equal time intervals. (This is like taking snapshots of the cart at equal time intervals.) At each indicated time, sketch, and label, a vector above the cart which might represent the velocity of the cart at that time while it is moving in the positive direction and slowing down.



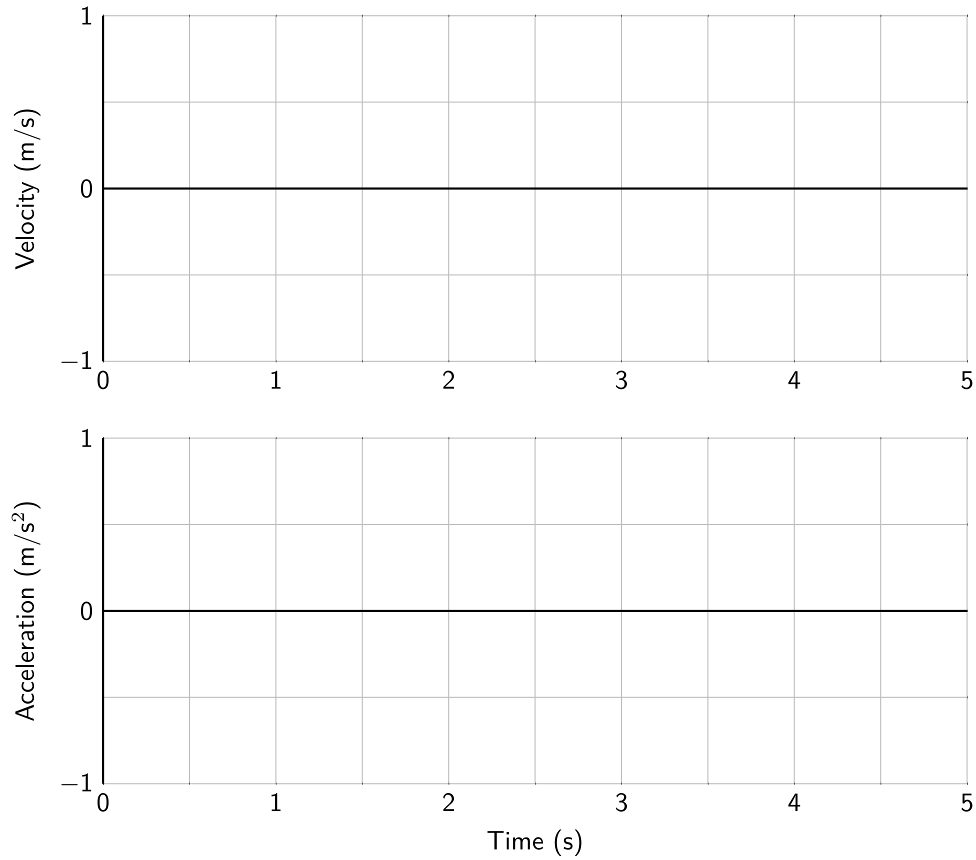
(b) Show below how you would find the vector representing the change in velocity between the times $t_1 = 1$ s and $t_2 = 2$ s by creating a vector diagram using the vectors above. Based on the direction of the resultant vector and the direction of the positive x-axis, what is the sign of the acceleration? Does this agree with your answer to Question (e) in Activity 1?

(c) By filling in the following table, state the general rules to predict the sign of the acceleration if you know the sign of the velocity (i.e., the direction of motion) and whether the object is speeding up or slowing down. Positive velocities have been investigated in this experiment and the previous one. Negative velocities we investigate below, so these are essentially predictions.

Velocity		Acceleration
positive	speeding up	
positive	slowing down	
negative	speeding up	
negative	slowing down	

Activity 3: Speeding Up While Moving in the Negative Direction

(a) Predict the shape of the velocity and acceleration *vs.* time graphs for a cart moving in the negative direction while speeding up at a constant rate. Sketch your predictions on the following axes using dashed lines.



(b) To test your predictions, release the cart from rest near the raised end of the track. The cart should be oriented so that the positive direction (the x-axis printed on the top of the cart) points up the incline, so that the cart's motion is in the negative direction. Stop the cart before it hits the end of the track. Sketch the graphs neatly on the above axes using solid lines.

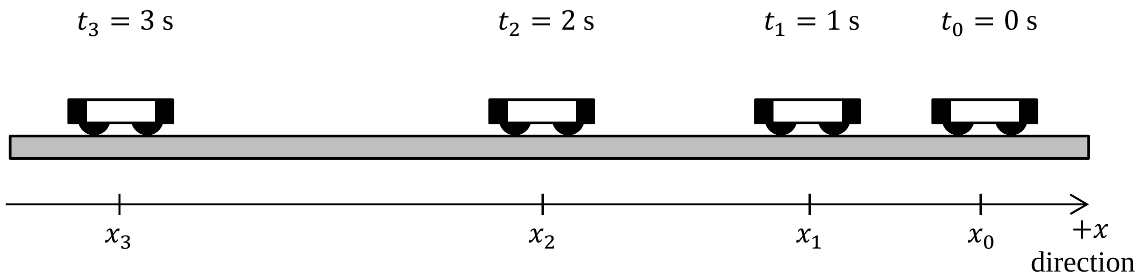
(c) How does your velocity graph show that the cart was moving in the negative direction?

(d) During the time that the cart was speeding up, is the acceleration positive or negative? Does this agree with your prediction? Explain how speeding up while moving in the negative direction results in this sign of acceleration. Hint: Think about how the velocity is changing.

Activity 4: Constructing Acceleration Vectors for Speeding Up

Let's consider a diagrammatic representation of a cart which is speeding up and use vector techniques to figure out the direction of the acceleration.

(a) The diagram that follows shows the positions of the cart at equal time intervals. (This is like taking snapshots of the cart at equal time intervals.) At each indicated time, sketch, and label, a vector above the cart which might represent the velocity of the cart at that time while it is moving in the negative direction and speeding up.



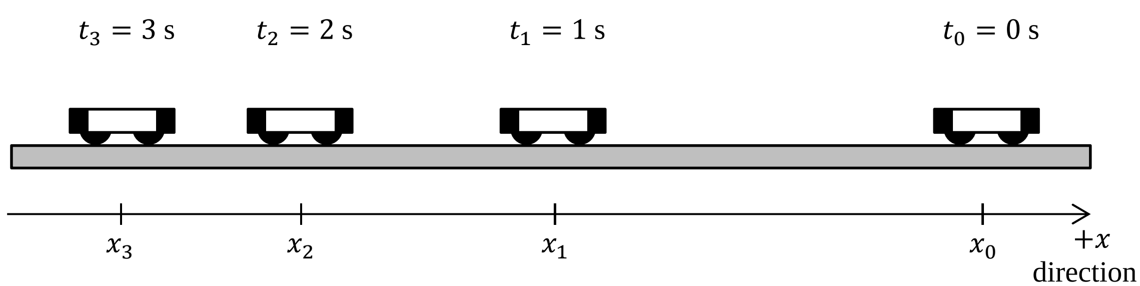
(b) Show below how you would find the vector representing the change in velocity between the times $t_1 = 1$ s and $t_2 = 2$ s as you did in Activity 2(b). Based on the direction of the resultant vector and the direction of the positive x-axis, what is the sign of the acceleration? Does this agree with your answer to Question (d) in Activity 3?

Activity 5: Slowing Down While Moving in the Negative Direction

There is one more possible combination of velocity and acceleration for the cart, that of moving in the negative direction while slowing down.

(a) Use your general rules to predict the direction and sign of the acceleration when the cart is slowing down as it moves in the negative direction. Explain why the acceleration should have this sign in terms of the velocity and how the velocity is changing.

(b) The diagram below shows the positions of the cart at equal time intervals for slowing down while moving in the negative direction. At each indicated time, sketch, and label, a vector above the cart which might represent the velocity of the cart at that time while it is moving in the negative direction and slowing down.



(c) Show below how you would find the vector representing the change in velocity between the times $t_1 = 1$ s and $t_2 = 2$ s as you did in Activity 2(b). Based on the direction of the resultant vector and the direction of the positive x axis, what is the sign of the acceleration? Does this agree with the prediction you made in part (a)?

Activity 6: Acceleration and Turning Around

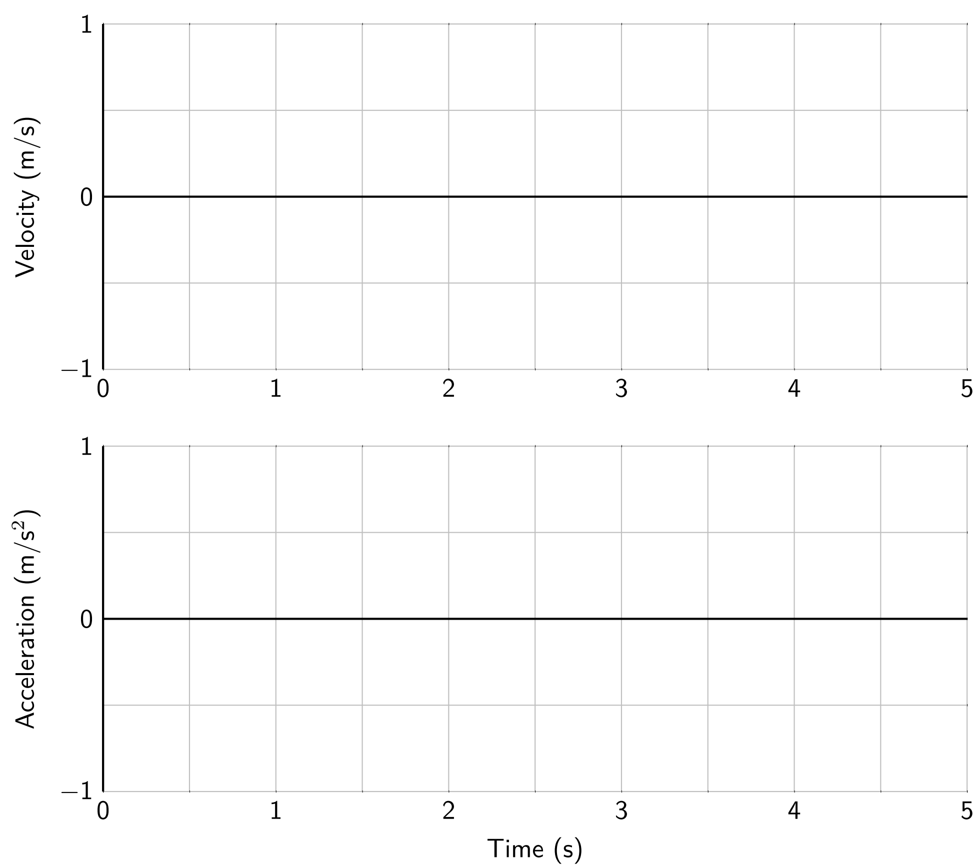
In Lab 4, and in Activity 1 in this session, you looked at velocity *vs.* time and acceleration *vs.* time graphs for a cart moving in one direction with a changing velocity. In this investigation you will look at what happens when the cart slows down, turns around and then speeds up. (This is a combination of Activities 1 and 3.)

To practice this motion you should position the cart near the lowered end of the track and and give the cart a gentle push in the positive direction (defined by the x-axis printed on the top of the cart) up the incline. It should move up the track, slow down, reverse direction and then move back down the incline. Be sure that the cart does not hit the end of the track before it turns around. Try it, but don't record the data (yet).

(a) For each part of the motion (up the incline, at the turning point, and down the incline) predict in the table that follows whether the cart's velocity is positive, zero or negative. Also indicate whether the acceleration is positive, zero or negative.

	Moving Away	Turning Around	Moving Toward
Velocity			
Acceleration			

(b) Sketch the predicted shapes of the velocity *vs.* time and acceleration *vs.* time graphs of this entire motion on the axes that follow using dashed lines.



(c) Test your predictions by making graphs of the motion. Use the procedures you used in the slowing down and speeding up activities. You may have to try a few times to get a good run. When you get a good run, sketch both graphs on the axes above using solid lines.

(d) Did the cart have a zero velocity at any point in the motion? Does this agree with your prediction? How much time did it spend at zero velocity?

(e) According to your acceleration graph, what is the acceleration at the instant the cart comes to rest? Is it positive, negative or zero? Does this agree with your prediction?

(f) Explain the observed sign of the acceleration when the cart changes direction. (Hint: Remember that acceleration is the rate of change of velocity.)

(g) If your instructor requests it, print a copy of the position, velocity, and acceleration graphs for each person.

(h) Notice that the slope of the velocity graph is not quite the same for positive velocities as it is for negative velocities. (This difference can also be seen on the acceleration graph.) What accounts for this difference?

Activity 7: The Rise and Fall of a Ball

Suppose you throw a ball up into the air. It moves upward, reaches its highest point and then moves back down toward your hand. We will now consider what can be said about the directions of its velocity and acceleration vectors at various points.

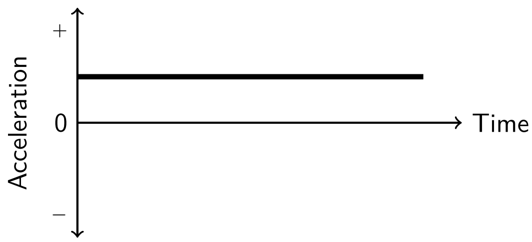
(a) Consider the ball toss carefully. Assume that upward is the positive direction. Indicate in the table that follows whether the velocity is positive, zero or negative during each of the three parts of the motion. Also indicate if the acceleration is positive, zero or negative. Hint: Remember, to find the acceleration you must look at the change in velocity.

	Moving Up (After Release)	At Highest Point	Moving Down
Velocity			
Acceleration			

(b) In what ways is the motion of the ball similar to the motion of the cart which you just observed?

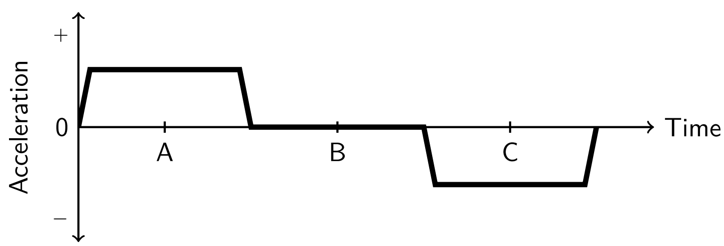
Homework

1. An object moving along a line (the + position axis) has the acceleration-time graph below. How might the object move to create this graph if it is moving *toward the origin*?



2. Sketch on the axes below a velocity-time graph that goes with the above acceleration-time graph.

3. How would an object move to create each of the three *labeled* parts of the acceleration-time graph shown below? (Consider the labeled horizontal line segments only, not the connectors between them.)



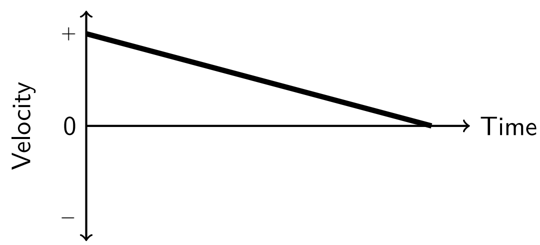
A:

B:

C:

4. Sketch below a velocity-time graph which might go with the acceleration-time graph in question (3). (Again, consider the straight line segments only.)

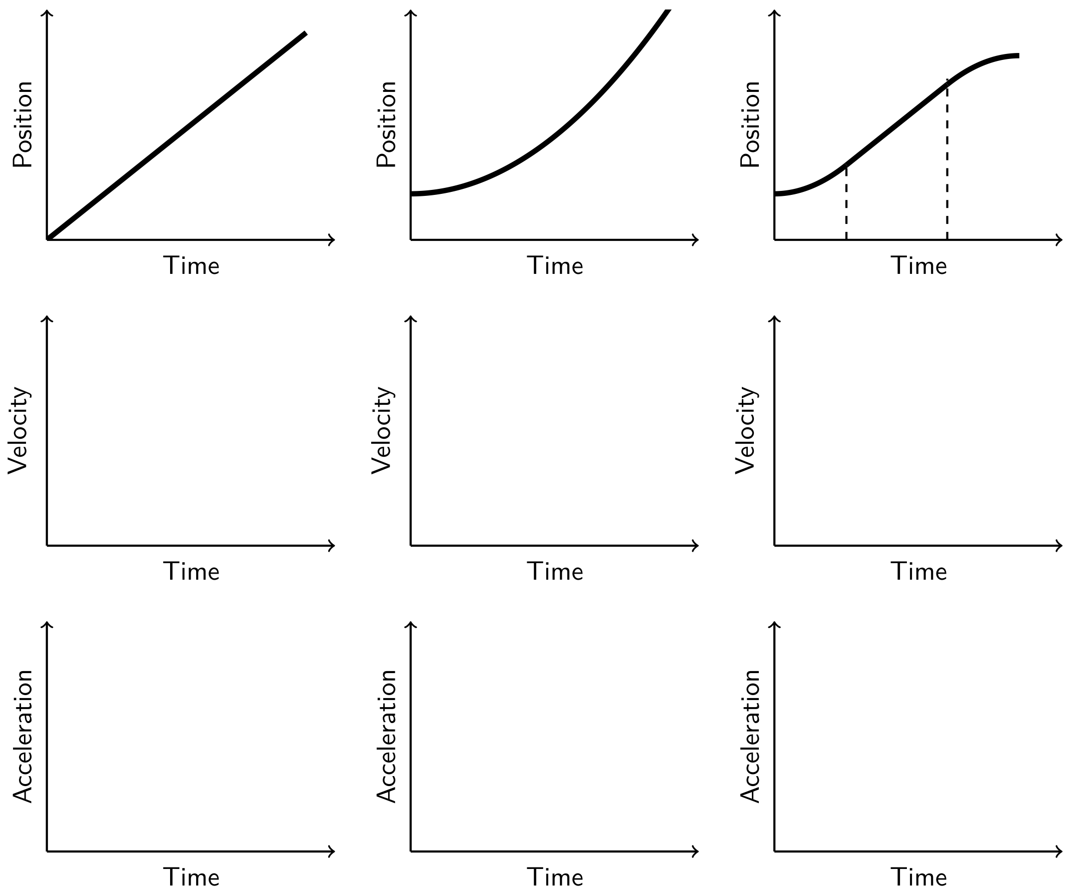
5. Sketch the shape of the acceleration-time graph that goes with the velocity-time graph shown below.



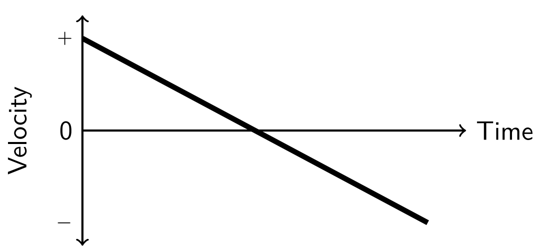
6. A car moves along a line [the + position axis]. Fill in the table below with the sign (+ or -) of the velocity and acceleration of the car for each of the motions described.

	Position	Velocity	Acceleration Speeding Up	Acceleration Slowing Down
Car Moves Away from the Origin	+			
Car Moves Toward the Origin	+			

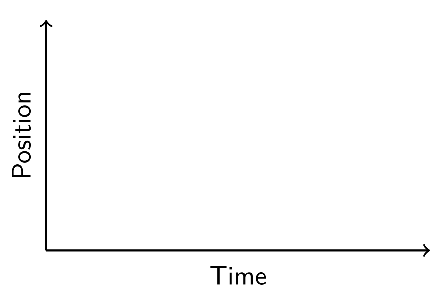
7. For each of the position-time graphs shown, sketch below it the corresponding velocity-time and acceleration-time graphs.



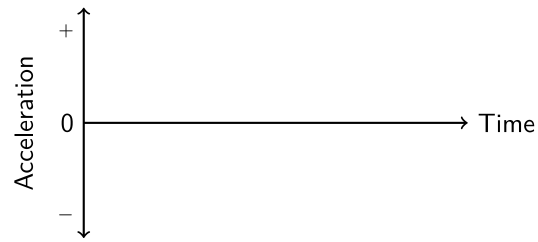
8. Describe how you would move to produce the velocity-time graph shown below.



9. Sketch a position-time graph corresponding to the velocity-time graph above.

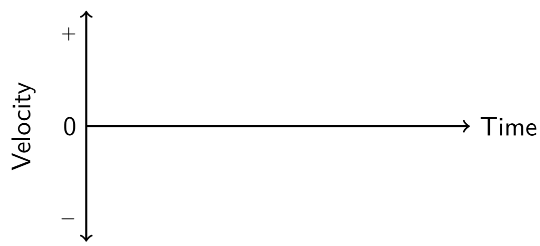


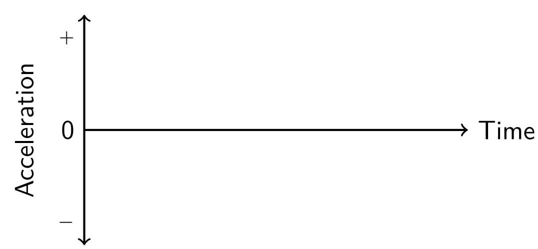
10. Sketch an acceleration-time graph corresponding to the velocity-time graph above.



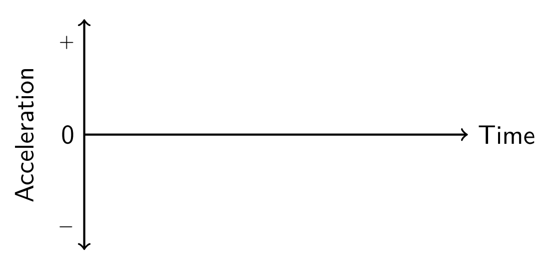
A car can move in either direction along a line (the + position axis). Sketch velocity-time and acceleration-time graphs that correspond to each of the following descriptions of the car's motion.

11. The car is moving toward the origin at a constant velocity.

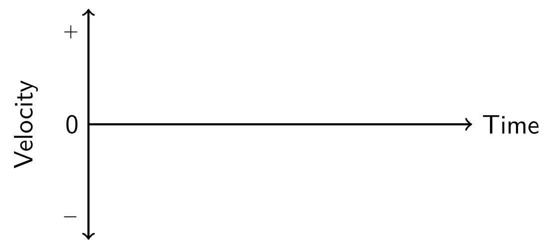




12. The car starts from rest and moves toward the origin, speeding up at a steady rate.

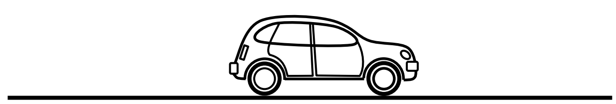


13. A ball is tossed in the air. It moves upward, reaches its highest point and falls back downward. Sketch a velocity-time and an acceleration-time graph for the ball from the moment it leaves the thrower's hand until the moment just before it reaches their hand again. Consider the positive direction to be upward.

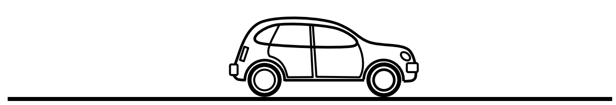


14. Each of the pictures below represents a car driving down a road. The motion of the car is described. In each case, draw velocity and acceleration vectors above the car which might represent the described motion. Also specify the sign of the velocity and the sign of the acceleration. (The positive direction is toward the right.)

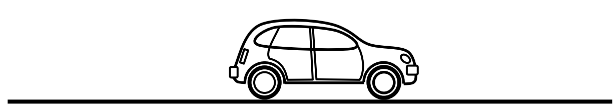
(a) The driver has stepped on the accelerator and the car is just starting to move forward.



(b) The car is moving forward. The brakes have been applied. The car is slowing down, but has not yet come to rest.

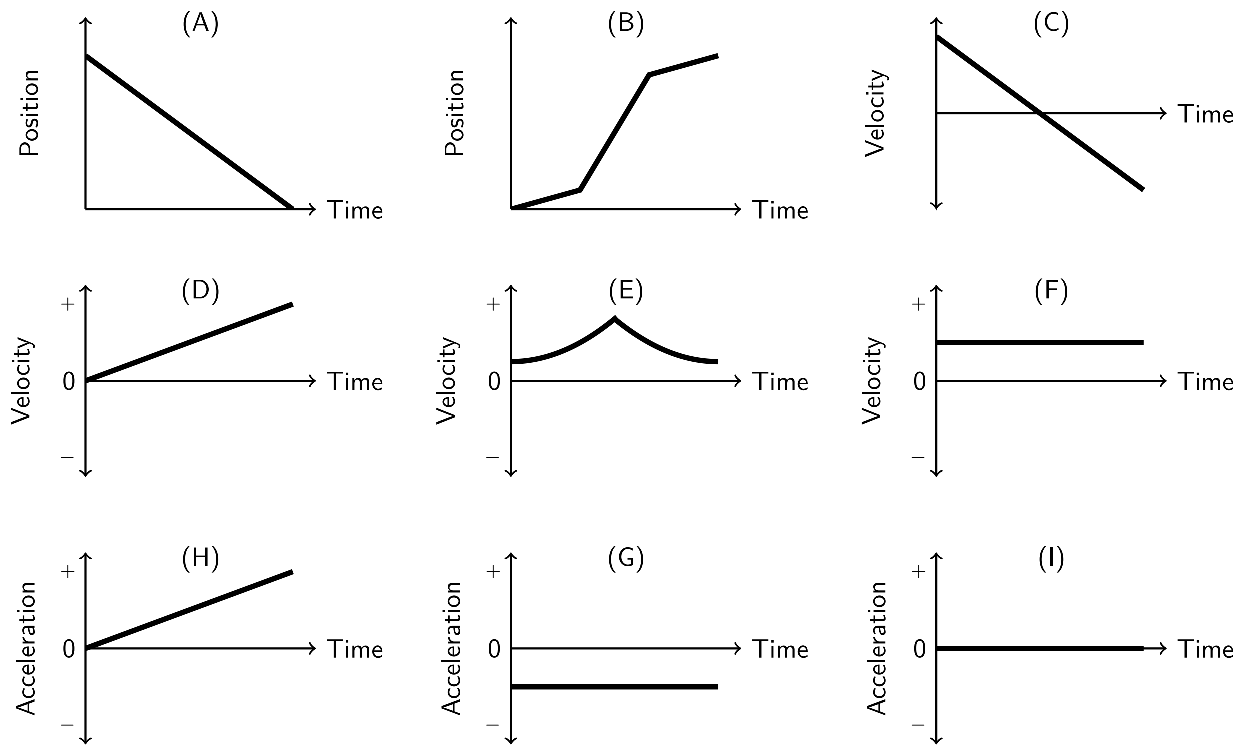


(c) The car is moving backward. The brakes have been applied. The car is slowing down, but has not yet come to rest.



The following graphs represent the motions of objects along the positive position axis. Notice that the motion of the objects is represented by position, velocity, or acceleration graphs.

Answer the following questions. You may use a graph more than once or not at all, and there may be more correct choices than blanks. If none of the graphs is correct, answer none.



15. Pick one graph that gives enough information to indicate that the velocity is always negative. ________

Pick three graphs that represent the motion of an object whose velocity is constant.

16. ________ 17. ________ 18. ________

19. Pick one graph that definitely indicates an object has reversed direction. ________

20. Pick one graph that might possibly be that of an object standing still. ________

Pick 3 graphs that represent the motion of objects whose acceleration is changing.

21. ________ 22. ________ 23. ________

Pick a velocity graph and an acceleration graph that could describe the motion of the same object during the time shown.

24. Velocity graph. ________ 25. Acceleration graph. ________

Lab 6 Measurement, Uncertainty, and Variation[1]

Name: ______________________ Lab Partner(s): ______________________

Objectives

- To explore the mathematical meaning of the standard deviation and standard error associated with a set of measurements.
- To investigate random and systematic variations associated with a set of measurements.

Statistics - The Inevitability of Uncertainty

With care and attention, it is commonly believed that both mistakes and systematic errors can be eliminated completely. However, inherent uncertainties do not result from mistakes or errors. Instead, they can be attributed in part to the impossibility of building measuring equipment that is precise to an infinite number of significant figures. The ruler provides us with an example of this. It can be made better and better, but it always has an ultimate limit of precision.

Another cause of inherent uncertainties is the large number of random variations affecting any phenomenon being studied. For instance, if you repeatedly drop a baseball from the level of the lab table and measure the time of each fall, the measurements will most probably not all be the same. Even if the stop watch was gated electronically so as to be as precise as possible, there would be small fluctuations in the flow of currents through the circuits as a result of random thermal motion of atoms and molecules that make up the wires and circuit elements. This could change the stop watch reading from measurement to measurement. The sweaty palm of the experimenter could cause the ball to stick to the hand for an extra fraction of a second, slight air currents in the room could change the ball's time of fall, vibrations could cause the floor to oscillate up and down an imperceptible distance, and so on.

Repeated Time-of-Fall Data

In the first two activities, you and your partners will take repeated data on the time of fall of a ball and study how the data vary from some average value for the time-of-fall.

Apparatus

- A ball
- A stop watch
- A 2-meter stick

Activity 1: Timing a Falling Ball

(a) Drop the ball so it falls through a height of exactly 2.0 m at least 10 times in rapid succession and measure the time of fall in each case. Be as exact as possible about the height from which you drop the ball. Record your 10 time values here:

[1]1990-93 Dept. of Physics and Astronomy, Dickinson College. Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Portions of this material may have been modified locally and may not have been classroom tested at Dickinson College.

(b) Enter your data as a single column in Excel and find the average (mean) of the data. See **Appendix D** for instructions. Report the mean value in the space below (including units).

The Standard Deviation as a Measure of Uncertainty

How certain are we that the average fall-time determined in the previous activity is accurate? The average of a number of measurements does not tell the whole story. If all the times you measured were the same, the average would seem to be very precise. If each of the measurements varied from the others by a large amount, we would be less certain of the meaning of the average time. We need criteria for determining the certainty of our data. Statisticians often use a quantity called the standard deviation as a measure of the level of uncertainty in data. The standard deviation is usually represented by the Greek letter σ (sigma). A customary way of expressing an experimentally determined value is: Mean $\pm$ σ. The formal mathematical definition of σ can be found in **Appendix E**.

In the next activity you will use Excel (see **Appendix D**) to calculate the value of the standard deviation for the repeated fall-time data you just obtained and explore how the standard deviation is related to variation in your data. In particular, you will try to answer this question: What percentage of your data lies within one standard deviation of the average you calculated?

Activity 2: Standard Deviation

(a) Report the value for the standard deviation of your data in the space below (including units).

(b) In the space below, write the time of fall from your data as $t = \langle t \rangle \pm \sigma$, (including units).

(c) Determine the number of your data points that lie within $\pm\sigma$ of the average and write the result in the space below. Also, calculate the percentage of data points lying within a standard deviation of the average and report that result.

(d) Combine your results with those obtained by the other groups in the class and create a table in the space below with the following column headings: Lab Station, $\langle t \rangle$ (s), σ (s), %Data $\pm\sigma$.

(e) Study the last column, which represents the percentage of data points lying within one standard deviation of the average. What does the standard deviation, σ, tell you about the approximate probability that another measurement will lie within $\pm\sigma$ of the average?

Systematic Error - How About the Accuracy of Your Timing Device and Timing Methods?

As the result of problems with your measuring instrument or the procedures you are using, each of your measurements may tend to be consistently too high or too low. If this is the case, you probably have a source of systematic error. There are several types of systematic error.

Most of us have set a watch or clock only to see it gain or lose a certain amount of time each day or week. In ordinary language we would say that such a time keeping device is inaccurate. In scientific terms, we would say that it is subject to systematic error. In the case of a stopwatch or digital timer that doesn't run continuously like a clock, we have to ask an additional set of questions. Does it start up immediately? Does it stop exactly when the event is over? Is there some delay in the start and stop time? A delay in starting or stopping a timer could also cause systematic error.

Finally, systematic error can be present as a result of the methods you and your partner are using for making the measurement. For example, are you starting the timer exactly at the beginning of the event being measured and stopping it exactly at the end? Are you dropping the ball from a little above the exact starting point each time? A little below?

Activity 3: Is There Systematic Error in the Data?

We will now use your data set to determine the acceleration due to gravity g *and* an uncertainty in the value obtained. As shown in your text, the distance h that a ball falls in time t (starting from rest) near the earth's surface is given by

$$h = \frac{1}{2}gt^2$$

where g is the acceleration due to gravity. In this idealized equation the effects of air resistance have been neglected.

(a) Calculate your value of g from the above equation.

(b) *Assume* an uncertainty δh in the height h from which you dropped the ball. Write h as $h \pm \delta h$:

(c) Assume the uncertainty in your average time of fall is the standard deviation σ that you found in Activity 2; i.e. $\delta t = \sigma$. Write the time of fall as $t = \langle t \rangle \pm \delta t$:

(d) Next, we'll calculate the uncertainty in g. Since g depends on two independent measurements that have uncertainty, you need to find the uncertainty in g from each one of the two measurements separately, as explained in Appendix F. Then, combine those uncertainties "in quadrature," to get the total uncertainty. The following steps will guide you to carry out this calculation.

1. First, assume that your measurement of h is exact, but that the value of t is the worst case of $t + \delta t$. Calculate the resulting worst-case value for g. The difference between this value and the best-guess value is the uncertainty in the value of g caused by the measurement of t.

$$\delta g_{\text{due to } t} =$$

2. Next, assume that your measurement of t is exact, but that the value of h is the worst case of $h + \delta h$. Calculate the resulting worst-case value for g, and subsequently, the uncertainty in the value of g caused by the measurement of h.

$$\delta g_{\text{due to}\,h} =$$

3. Now, combine the two uncertainties calculated in the last two steps by adding them "in quadrature" (which looks an awful lot like finding the hypotenuse of a right triangle from the two legs):

$$\delta g = \sqrt{(\delta g_{\text{due to}\,t})^2 + (\delta g_{\text{due to}\,h})^2} =$$

Finally, write g as $g = g \pm \delta g$:

Does the "accepted value" of g (9.80 m/s^2) fall within the range of values you have determined for g? If not, you probably have a source of systematic error.

(e) If you seem to have systematic error, explain whether the measured times tend to be too short or too long and list some of the possible causes of it in the space below.

Activity 4: Visualizing Your Data

In Activities 1-2 you extracted the mean and standard deviation for your data set. These two numbers characterize your data, but they contain far less information than the full data set. To pull more understanding from your data we want to investigate a common method for creating a pictorial representation of your data: the histogram. A histogram is a summary graph showing a count of your data points falling in various ranges. The effect is a rough approximation of the frequency distribution of the data showing you how often different outcomes might occur. You may be familiar with histograms from seeing distributions of grades on an exam. The histogram you are about to make is analogous to such a grade distribution except you will be using your measured fall times instead of student grades.

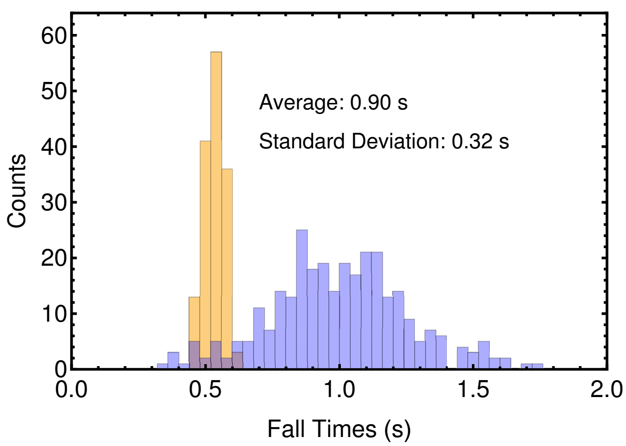
(a) Follow the instructions in **Appendix D** to make a histogram of your data. One of the choices you have to make in producing the histogram is the size and position of the 'bins'. The horizontal range of your data will be broken up into a series of adjacent sub-ranges that will be used to sort your data. The bin size is simply the length of each sub-range. We will typically use a constant bin size. Use simple numbers for the bin limits, and be sure they cover the entire range of your data when you follow the directions in **Appendix D**. The horizontal label "bin" must be removed and replaced by a label describing whatever you are plotting (including units). Once you have made the histogram, include an appropriate title, print it, and attach it to this unit.

(b) How many "peaks" are in your histogram? Explain your reasoning.

(c) What does the histogram of the fall-times data tell you?

(d) Do the average and standard deviation capture the full description of your data? Why or why not.

(e) Consider the histogram shown below for two sets of possible fall times from different classes. The average and standard deviation of the data is (0.95 ± 0.36)' s. How many peaks are in this histogram? Do the average and standard deviation capture the full description of the data? Why or why not.



(f) What advantages do histograms have over just calculating the average and standard deviation?

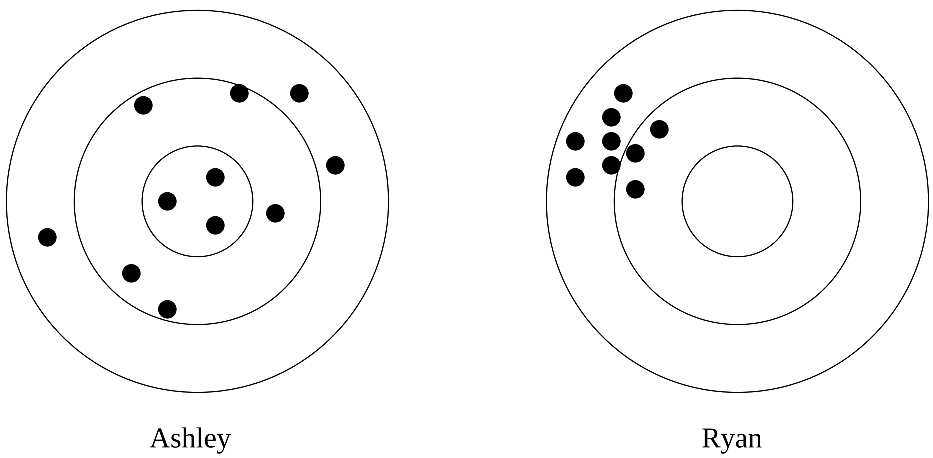
Homework

1. Suppose you made the following five length measurements of the width of a piece of $8\frac{1}{2}'' \times 11''$ paper which has been cut carefully by a manufacturer using an unfamiliar centimeter rule: 21.33 cm, 21.52 cm, 21.47 cm, 21.21 cm, 21.45 cm. (a) Find the mean and standard deviation of the measurements. The formal mathematical definition of standard deviation is given in Appendix E.

(b) Is there any evidence of uncertainty in the measurements or are they precise? Explain.

(c) Is there any evidence of systematic error in the measurements? If so, what might cause this? Explain.

2. Suppose Ashley and Ryan each throw darts at targets as shown below. Each of them is trying very hard to hit the bulls eye each time. Discuss in essay form which of the two students has the least amount of random error associated with his or her throws and is thus more precise. Is one of the students less accurate is the sense of having a systematic error associated with his or her throws? What factors like eyesight and coordination might cause one to be more precise and another more accurate?



Lab 7 Projectile Motion[1]

Name: ______________________ Lab Partner(s): ______________________

Objectives

To understand the experimental and theoretical basis for describing projectile motion as the superposition of two independent motions: (1) a body falling in the vertical direction, and (2) a body moving in the horizontal direction with no forces.

Apparatus

- A tennis ball.
- A movie scaling ruler or meter stick.
- A video analysis system (*Tracker*).
- Graphing and curve fitting software (*Excel*).

Activity 1: Predicting the Two-Dimensional Motion of a Tossed Ball

(a) Toss a tennis ball up at an angle of about 60° with the horizontal a couple of times. Sketch the motion and describe it in words below. What is the shape of the trajectory?

(b) Let's consider the horizontal and vertical components of the motion separately. What do you think is the horizontal motion of the ball? Is it motion with constant velocity, constant acceleration, or some other kind of motion? (Hint: What is the force acting on the ball in the horizontal direction after it is released?)

(c) What do you think is the vertical motion of the ball? Is it motion with constant velocity, constant acceleration, or some other kind of motion? (Hint: What is the force acting on the ball in the vertical direction after it is released?)

The two-dimensional motion of a tossed ball is too fast to observe carefully by eye without the aid of special instruments. In the next activity we will use a video analysis system to study the motion of a small ball launched at an angle of about 60° with respect to the horizontal. You are to use the video analysis software and mathematical modeling techniques to find the equations that describe: (a) the trajectory (y *vs.* x), (b) the horizontal motion (x *vs.* t), and (c) the vertical motion (y *vs.* t) of the projectile.

[1]1990-93 Dept. of Physics and Astronomy, Dickinson College. Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Portions of this material have been modified locally and may not have been classroom tested at Dickinson College.

Activity 2: Analyzing Projectile Motion

(a) Make a movie of a tennis ball in flight by following these steps.

1. Open **Camera** and turn on the video camera as explained in **Appendix B: Video Analysis Using Tracker**. Center the field of view of the camera on the region where you will toss the ball. This region should be about 2 meters from the camera to get a large enough area for the flight of the ball. Place a ruler or meter stick somewhere in the field of view close to the plane of the motion of the ball where it won't interfere with the motion. This ruler will be used later to determine the scale.
2. Make a movie of the tennis ball flying through the air with a significant component of its initial velocity in the horizontal direction (i.e., don't toss it straight up). Make sure most of the complete trajectory is visible to the camera. See **Appendix B: Video Analysis Using Tracker** for details on making the movie.

(b) Determine the position of the ball as a function of time using *Tracker*. To do this, track the position of the ball throughout its trajectory. The resulting file will contain three columns with the values of time, x-position and y-position.

(c) Determine the equation that describes the trajectory of the projectile by plotting the vertical position y versus the horizontal position x. Write the equation for the trajectory of the projectile in the space below. Be sure to include the proper units. What is the shape of the trajectory? Does the result agree with your earlier prediction?

(d) Determine the equation that describes the horizontal motion of the projectile by plotting the horizontal position (x) versus time (t) in *Tracker*. What kind of motion is it? What would you expect for the horizontal acceleration? Does the result agree with your earlier prediction? **Note:** As in the previous experiment, numbers from your graph should be rounded off to no more than 3 significant figures.

1. The equation for the horizontal component of the motion with proper units is: $x =$
2. The horizontal component of the acceleration with proper sign and units is: $a_x =$
3. The horizontal component of the initial velocity with proper sign and units is: $v_{0x} =$
4. The initial x position with proper units is: $x_0 =$

(e) Determine the equation that describes the vertical motion of the projectile by plotting the vertical position (y) versus time (t) in *Tracker*. Print the graph and attach a copy to this unit. What kind of motion is it? What would you expect for the vertical acceleration? Does the result agree with your earlier prediction?

1. The equation for the vertical component of the motion with proper units is: $y =$
2. The vertical component of the acceleration with proper sign and units is: $a_y =$

3. The vertical component of the initial velocity with proper sign and units is: $v_{0y} =$

4. The initial y position with proper units is: $y_0 =$

(f) Does it appear that projectile motion is simply the superposition of two types of motion that we have already studied? Explain.

(g) Go around to the other groups in the lab and ask them for their measured values of the horizontal and vertical accelerations. Make a histogram of your results for each component and calculate the average and standard deviation of each one. For information on making histograms, see **Appendix D**. For information on calculating the average and standard deviation, see **Appendix E**. Record the average and standard deviation here. Attach the histogram to the unit.

(h) What is your expectation for the vertical acceleration of the ball? Is your data consistent with your expectation? Is the acceleration the same for the entire class? Use the average and standard deviation for the class to quantitatively answer these questions.

(i) What is your expectation for the horizontal acceleration of the ball? Is your data consistent with your expectation? Is this acceleration the same for the entire class? Use the average and standard deviation for the class to quantitatively answer these questions.

(j) What do the histograms of the class data for each component of the acceleration tell you? Be quantitative in your answer.

Lab 8 Uniform Circular Motion[1]

Name: ______________________ Lab Partner(s): ______________________

Objective

To explore the phenomenon of uniform circular motion and the acceleration needed to maintain it.

Overview

You have recently studied projectile motion. In this unit we are going to explore another phenomenon in two dimensions, uniform circular motion. In particular, you will develop a mathematical description of the centripetal acceleration that keeps an object moving in a circle. You will start by watching a toy "airplane" suspended from a string fly in a circle.

Apparatus

- The "airplane."
- A movie scaling ruler or meter stick.
- A video analysis system (*Tracker*).
- Graphing software (*Excel*).

Activity 1: Observing an Airplane Undergoing Circular Motion

(a) Put on your safety glasses and wear them until your instructor announces they can be removed. Turn on the engine of the airplane suspended from the string. Be careful to stay clear of the propeller. Launch the airplane into circular motion by giving it a gentle push. If it doesn't quickly settle into steady circular motion, catch it and try launching it in the opposite direction. Sketch the motion and describe it in words below.

(b) How would you describe the speed of the airplane? How would you describe the velocity? Would you say that this is accelerated motion? Why?

(c) What is the definition of acceleration? (Remember that acceleration is a vector!)

(d) Are velocity and speed the same thing? Is the velocity of the airplane constant? (Hint: Velocity is a vector quantity!)

[1]1990-93 Dept. of Physics and Astronomy, Dickinson College. Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Portions of this material have been modified locally and may not have been classroom tested at Dickinson College.

(e) In light of your answers to (c) and (d), would you like to change your answer to part (b)? Explain.

The two-dimensional motion of the airplane is too fast to observe carefully so we will use a video system to record the motion of the airplane and to analyze the movie frame-by-frame. You will use the video analysis system to investigate the direction of the velocity of the circling airplane and the direction of its acceleration.

Activity 2: Analyzing Circular Motion

(a) Make a movie of the airplane in flight by following these steps.

1. Open **Camera** and turn on the video camera as explained in **Appendix B: Video Analysis Using Tracker**. Center the field of view on the region where the plane will fly. This region should be at least 1 meter from the camera to get a large enough area for the flight of the plane. Mount a ruler or meter stick somewhere in the field of view where it won't interfere with the motion. This ruler will be used later to determine the scale.

2. Launch the airplane into circular motion and wait until it settles into steady, circular flight. Record several revolutions of the airplane and save the movie as the file Airplane. See **Appendix B: Video Analysis Using Tracker** for details on making the movie.

(b) Determine the position of the airplane during one complete revolution using *Tracker*. The resulting file should contain three columns with the values of time, x-position, and y-position for one complete revolution.

(c) Within *Tracker*, make a graph of the trajectory of the airplane during one full revolution. This can be done by plotting y vs x. When you make your plot make sure the x and y axes cover the same size range; otherwise you will distort the path of the airplane. Double click on the graph to produce a larger version. Print the graph and attach a copy to this unit.

1. Is the motion circular? What is your evidence?

2. Draw vectors for the position vector and the velocity vector at one point on the trajectory. How are the position vector and the velocity vector related? How is the velocity vector related to the path of the airplane?

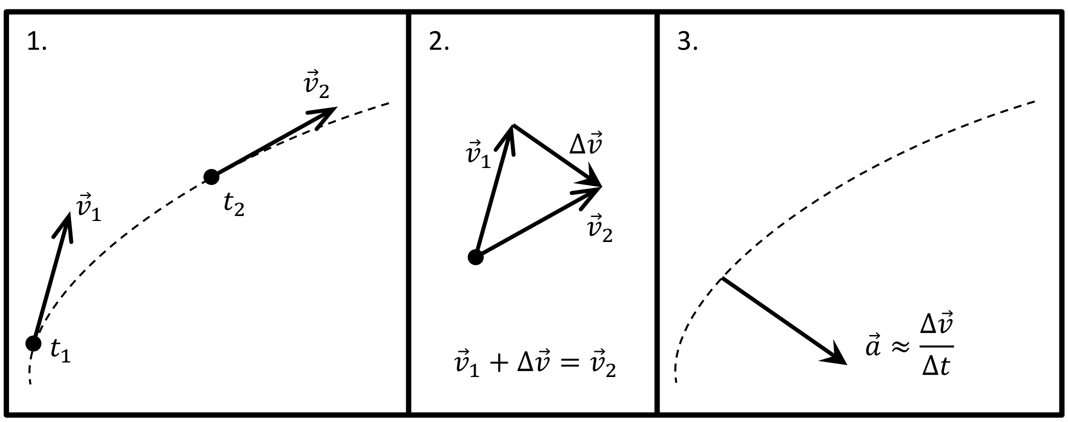
3. What would happen to the path of the plane if you cut the string? Explain.

(d) Launch the airplane into uniform circular motion. You are going to investigate what happens to the trajectory after the string is cut. Be sure you are wearing your eye protection. After the plane settles into steady, circular flight start recording a movie of the plane. After it completes one revolution use the scissors to cut the string just above the horizontal metal bar. BE CAREFUL to avoid letting the plane strike anyone or any object except the floor. Halt recording and save the movie.

(e) Make a plot of the trajectory of the airplane after the string was cut and for at least one full revolution before. Is the trajectory circular before the string was cut? Does the trajectory of the plane after the string was cut agree with the prediction you made earlier? Explain. Print the graph and attach a copy to the unit.

Using Vectors to Diagram How Velocity Changes

By now you should have concluded that since the direction of the motion of an object undergoing uniform circular motion is constantly changing, its velocity is also changing and thus it is accelerating. We would like you to figure out how to calculate the direction of the acceleration and its magnitude as a function of the speed v of an object such as a ball as it revolves and as a function of the radius of the circle in which it revolves. In order to use vectors to find the direction of velocity change in circular motion, let's review some rules for adding velocity vectors.



1. To Draw Velocities: Draw an arrow representing the velocity, $\vec{v}_1$, of the object at time t_1. Draw another arrow representing the velocity, $\vec{v}_2$, of the object at time t_2.

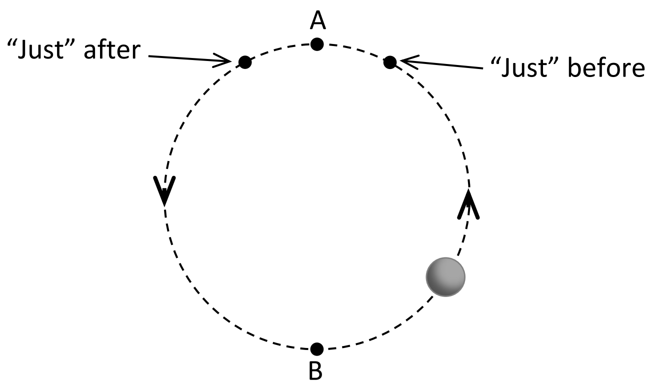
2. To Draw Velocity Change: Find the change in the velocity $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$ during the time interval described by $\Delta t = t_2 - t_1$. Start by using the rules of vector sums to rearrange the terms so that $\vec{v}_1 + \Delta\vec{v} = \vec{v}_2$. Next place the tails of the two velocity vectors together halfway between the original and final location of the object. The change in velocity is the vector that points from the head of the first velocity vector to the head of the second velocity vector.

3. To Draw Acceleration: The acceleration equals the velocity change $\Delta\vec{v}$ divided by the time interval Δt needed for the change. Thus, $\vec{a}$ is in the same direction as $\Delta\vec{v}$ but is a different length (unless $\Delta t = 1$). Thus, even if you do not know the time interval, you can still determine the direction of the acceleration because it points in the same direction as $\Delta\vec{v}$.

The acceleration associated with uniform circular motion is known as centripetal acceleration. You will use the vector diagram technique described above to find its direction.

Activity 3: The Direction of Acceleration in Circular Motion

(a) Determine the direction of motion of the ball shown below if it is moving counter-clockwise at a constant speed. Note that the direction of the ball's velocity is always tangential to the circle as it moves around. Draw an arrow representing the direction and magnitude of the ball's velocity as it passes the dot just before it reaches point A. Label this vector $\vec{v}_1$.



(b) Next, use the same diagram to draw the arrow representing the velocity of the ball when it is at the dot just after it passes point A. Label this vector $\vec{v}_2$.

(c) Find the direction and magnitude of the change in velocity as follows. In the space below, make an exact copy of both vectors, placing the tails of the two vectors together. Next, draw the vector that must be added to vector $\vec{v}_1$ to add up to vector $\vec{v}_2$; label this vector $\Delta\vec{v}$. Be sure that vectors $\vec{v}_1$ and $\vec{v}_2$ have the same magnitude and direction in this drawing that they had in your drawing in part (a)!

(d) Now, draw an exact copy of $\Delta\vec{v}$ on your sketch in part (a). Place the tail of this copy at point A. Again, make sure that your copy has the exact magnitude and direction as the original $\Delta\vec{v}$ in part (c).

(e) Now that you know the direction of the change in velocity, what is the direction of the centripetal acceleration, $\vec{a}_c$?

(f) If you redid the analysis for point B at the opposite end of the circle, what do you think the direction of the centripetal acceleration, $\vec{a}_c$, would be now?

(g) As the ball moves on around the circle, what is the direction of its acceleration?

Using Mathematics to Derive How Centripetal Acceleration Depends on Radius and Speed

You haven't done any experiments yet to see how centripetal acceleration depends on the radius of the circle and the speed of the object. You can use the rules of mathematics and the definition of acceleration to derive the relationship between speed, radius, and magnitude of centripetal acceleration.

Activity 4: How Does a_c Depend on v and r?

(a) Do you expect you would need more centripetal acceleration or less centripetal acceleration to cause an object moving at a certain speed to rotate in a smaller circle? In other words, would the magnitude, a_c, have to increase or decrease as r decreases if circular motion is to be maintained? Explain.

(b) Do you expect you would need more centripetal acceleration or less centripetal acceleration to cause an object to rotate at a given radius r if the speed v is increased? In other words, would the magnitude, a_c, have to increase or decrease as v increases if circular motion is to be maintained? Explain.

You should have guessed that it requires more acceleration to move an object of a certain speed in a circle of smaller radius and that it also takes more acceleration to move an object that has a higher speed in a circle of a given radius. Let's use the definition of acceleration in two dimensions and some accepted mathematical relationships to show that the magnitude of centripetal acceleration should actually be given by the equation

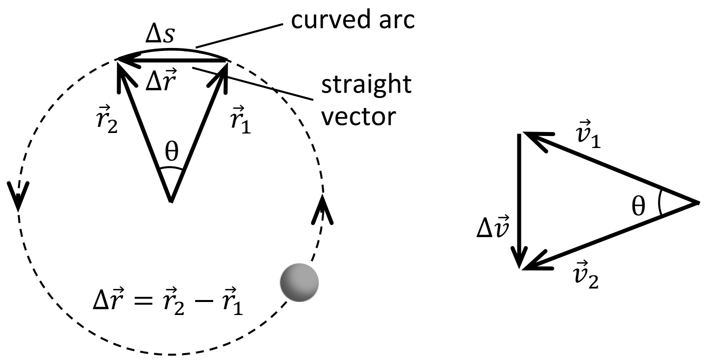
$$a_c = \frac{v^2}{r} \qquad [Eq.\, 1]$$

In order to do this derivation you will want to use the following definition for average acceleration:

$$\vec{a}_{\text{avg}} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta \vec{v}}{\Delta t} \qquad [Eq.\, 2]$$

Activity 5: Finding the Equation for a_c

(a) Refer to the diagram below. Explain why, at the two points shown on the circle, the angle between the position vectors at times t_1 and t_2 is the same as the angle between the velocity vectors at times t_1 and t_2. Hint: In circular motion, velocity vectors are always perpendicular to their position vectors.



(b) Since the angles are the same and since the magnitudes of the position vectors never change (i.e., $r = r_1 = r_2$) and the magnitudes of the velocities never change (i.e., $v = v_1 = v_2$), use the properties of similar triangles to explain why $\frac{\Delta v}{v} = \frac{\Delta r}{r}$.

(c) Now use the equation in part (b) and the definition of a_{avg} to show that $a_{\text{avg}} = a_c = \frac{\Delta v}{\Delta t} = \frac{(\Delta r)}{(\Delta t)} \frac{v}{r}$.

(d) The speed of the object as it rotates around the circle is given by $v = \frac{\Delta s}{\Delta t}$. Is the change in arc length, Δs, larger or smaller than the magnitude of the change in the position vector, Δr? Explain why the arc length change and the change in the position vector are approximately the same when Δt is very small (so that the angle θ becomes very small) i.e., why is $\Delta s \simeq \Delta r$?

(e) If $\Delta s \simeq \Delta r$, then what is the equation for the speed in terms of Δr and Δt? (Start with the formula for v in part (d).)

(f) Using the equation in part (c), and your result from part (e), show that as $\Delta t \to 0$, the instantaneous value of the centripetal acceleration is given by Eq. 1.

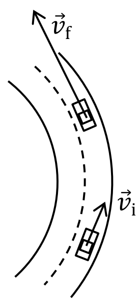
Lab 9 Tangential Acceleration and Centripetal Acceleration

Name: ____________________ Lab Partner(s): ______________________________

Activity 1: Drawing Vectors

The picture below shows a top view of a car speeding up while taking a curve on a road.

(a) Use graphical techniques (no calculations) to draw a vector representing the change in velocity $\Delta\vec{v}$ during the time shown.

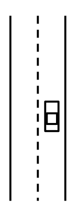


(b) Does the average acceleration $\vec{a}_{\text{avg}}$ during this time point exactly towards the center of the curve?

(c) You may have done previous exercises that involved a car driving on a circular track, in which the average acceleration always pointed towards the center of the curve. Why is the case pictured above different?

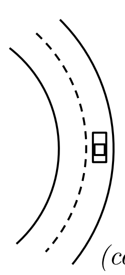
Activity 2: An Example with Numbers

(a) Suppose you are driving north on a straight road. You accelerate uniformly from rest to a speed of 15 m/s over a time $\Delta t = 5$ seconds. What is the magnitude and direction of your accleration $\vec{a}$ during this time? Also draw your acceleration vector $\vec{a}$ on the picture below.



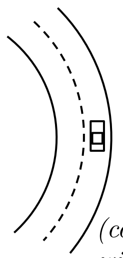
(causing acceleration with the gas pedal...)

(b) Now suppose you are driving at a constant speed $v = 10$ m/s around a curve of radius $R = 25$ m. What is the magnitude and direction of your acceleration $\vec{a}$ at exactly the moment pictured below? Also draw your acceleration vector $\vec{a}$ on the picture.



(causing acceleration with the steering wheel...)

(c) Finally, suppose that you increase your speed just like you did in part (a), but you do it *while* taking the same curve in the road as you did in part (b). You are now combining the accelerations from parts (a) and (b), doing both at the same time. Draw your acceleration vector $\vec{a}$ on the picture below. *(Hint: what does it mean to "combine" two acceleration vectors?)*



*(causing acceleration with **both** the gas pedal **and** the steering wheel...)*

Let's use the same numbers as we did before: you increased your speed from from 0 m/s to 15 m/s over $\Delta t = 5$ s, and at the moment shown in the picture above, you happen to have a speed of exactly $v = 10$ m/s.

(d) What is the *tangential* component a_t of your acceleration? (That is, the component of your acceleration vector $\vec{a}$ that is along your direction of motion.)

(e) What is the *centripetal* component a_c of your acceleration? (That is, the component of your acceleration vector $\vec{a}$ that is towards the center or the curve.)

(f) What are the magnitude and direction of your acceleration $\vec{a}$?

Lab 10 Force and Motion I[1]

Name: ______________________ Lab Partner(s): ______________________

Objectives

- To understand the relationship between forces applied to an object and its motions.
- To find a mathematical relationship between the force applied to an object and its acceleration.

Overview

In the previous labs, you used motion sensors and smart carts to display position-time, velocity-time and acceleration-time graphs for different objects. You were not concerned about how you got the objects to move, i.e., what forces (pushes or pulls) acted on the objects. From your experiences, you know that force and motion are related in some way. To start your bicycle moving, you must apply a force to the pedal. To start up your car, you must step on the gas pedal to get the engine to apply a force to the road through the tires.

But exactly how is force related to the quantities you used in the previous unit to describe motion: position, velocity and acceleration? In this unit you will pay attention to forces and how they affect motion.

Apparatus

- *Capstone* software (*V_A_F_Graphs.cap* experiment file)
- CS2000 compact scale
- Dynamics track
- Hanging masses
- Pulley with clamp
- String
- Wireless smart cart

Measuring Forces

In this investigation you will use a force sensor (built in to the smart cart) to measure forces. The force sensor puts out a voltage signal proportional to the force applied to the arm of the sensor. Physicists have defined a standard unit of force called the newton, abbreviated N. For your work on forces and the motions they cause, it will be more convenient to have the force sensor read directly in newtons rather than voltage.

Open the *V_A_F_Graphs.cap* file in the *Phys131* folder. Turn on the cart at your station and connect it to the computer via Bluetooth. At the beginning of each experiment, the measurements from the built-in acceleration and force sensors may not be zero when the acceleration or force is actually zero. To tare the sensors, select the desired sensor (either the **Smart Cart Acceleration Sensor** or the **Smart Cart Force Sensor**) in the **Controls** palette and then click **Zero Sensor Now**. When taring the sensors, the cart should be at rest and there should be no applied force on the hook.

Activity 1: Pushing and Pulling a Cart

In this activity you will move a cart by pushing and pulling it with your hand and measure the force exerted on it, as well as the cart's velocity and acceleration.

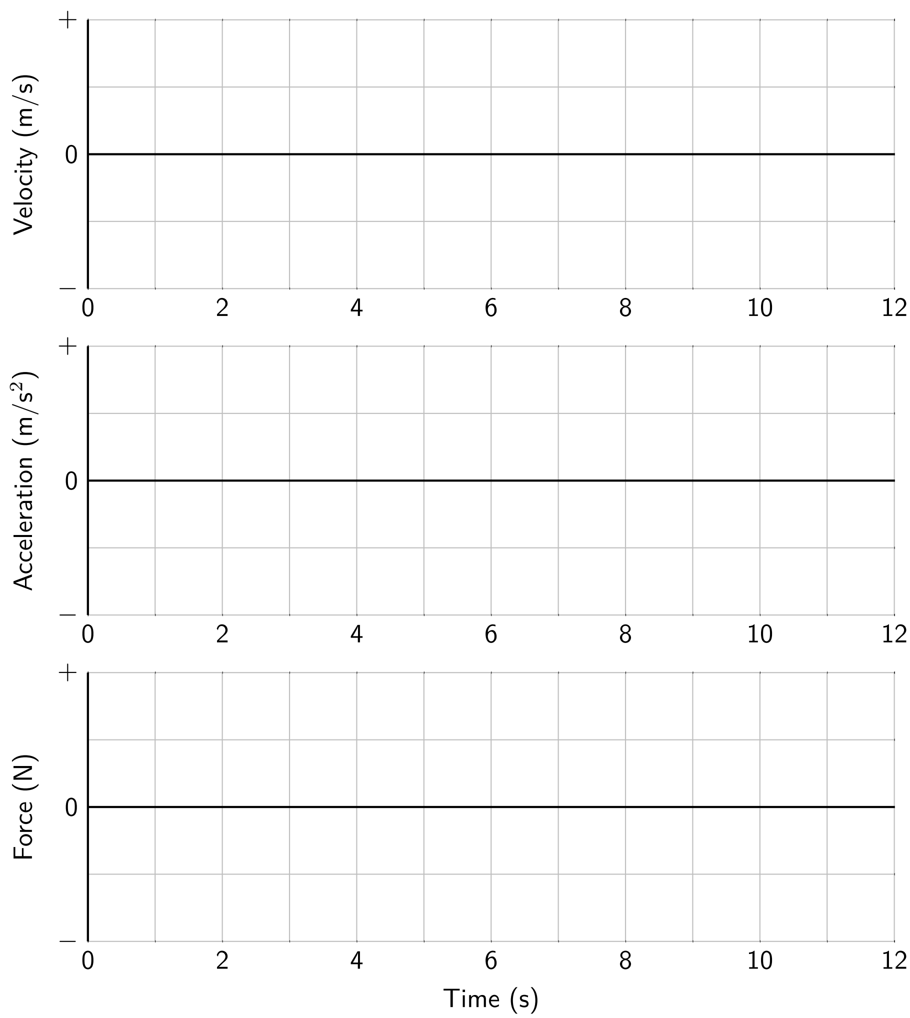
(a) Measure the mass of the cart (using the compact scale) and record the result below.

(b) Place the cart on the level track. Suppose you were to grasp the hook on the cart (which is attached to the force sensor) and move the cart forwards and backwards. Would either the velocity or the acceleration graph

[1] 1990-93 Dept. of Physics and Astronomy, Dickinson College. Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Portions of this material may have been modified locally and may not have been classroom tested at Dickinson College.

look like the force graph?

(c) To test your predictions, click the **Record** button, grasp the hook on the cart and push and pull the cart back and forth 3 or 4 times. Repeat until you get a good run, and adjust the sampling rate and scale of the axes if necessary. Sketch your graphs on the axes below.



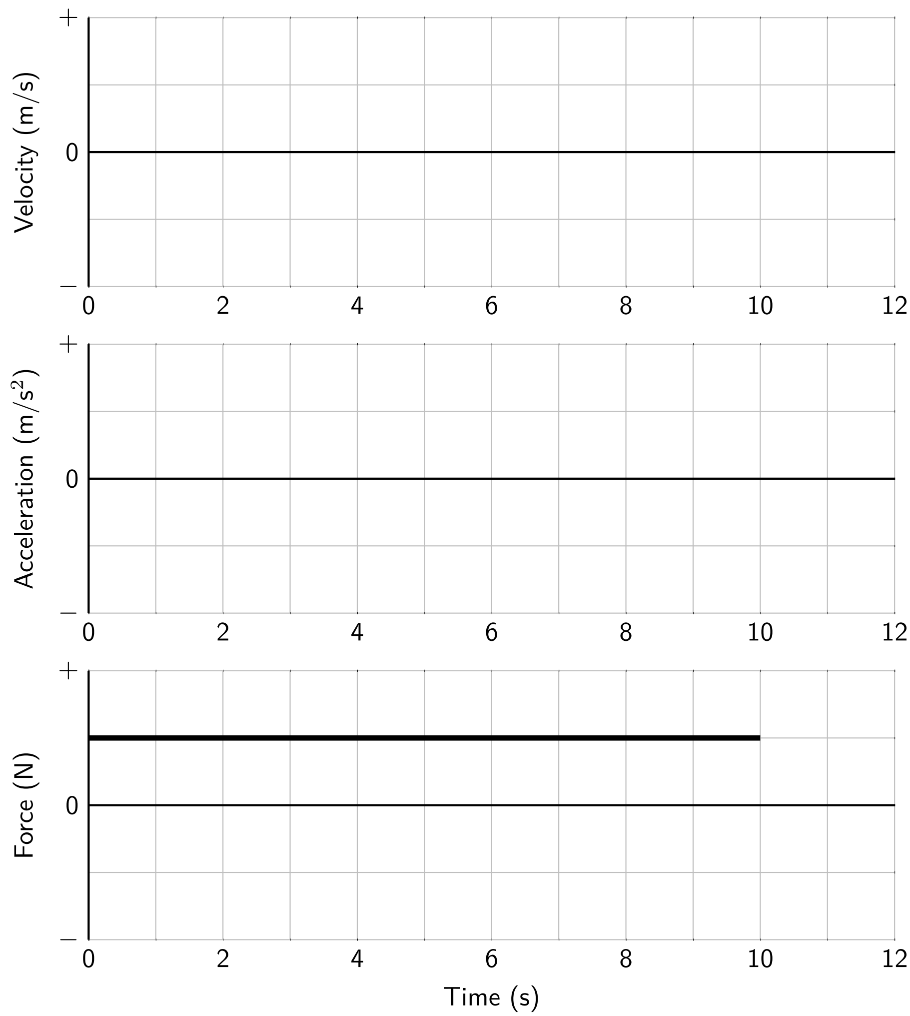
(d) Does either graph—velocity or acceleration—resemble the force graph? Which one? Explain.

(e) Based on your observations, does it appear that either the velocity or acceleration of the cart might be related to the applied force? Explain.

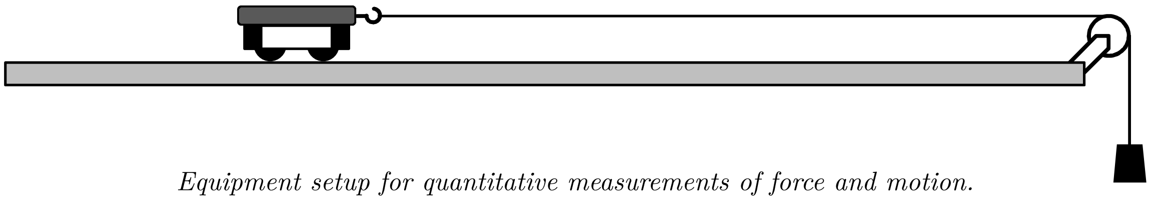
Activity 2: Speeding Up

You have seen in the previous activity that force and acceleration seem to be related. But just what is the relationship between force and acceleration?

(a) Suppose you have a cart with very little friction, and that you pull this cart with a constant force as shown below on the force *vs.* time graph. Predict with sketches on the axes below the velocity *vs.* time and acceleration *vs.* time graphs of the cart's motion.



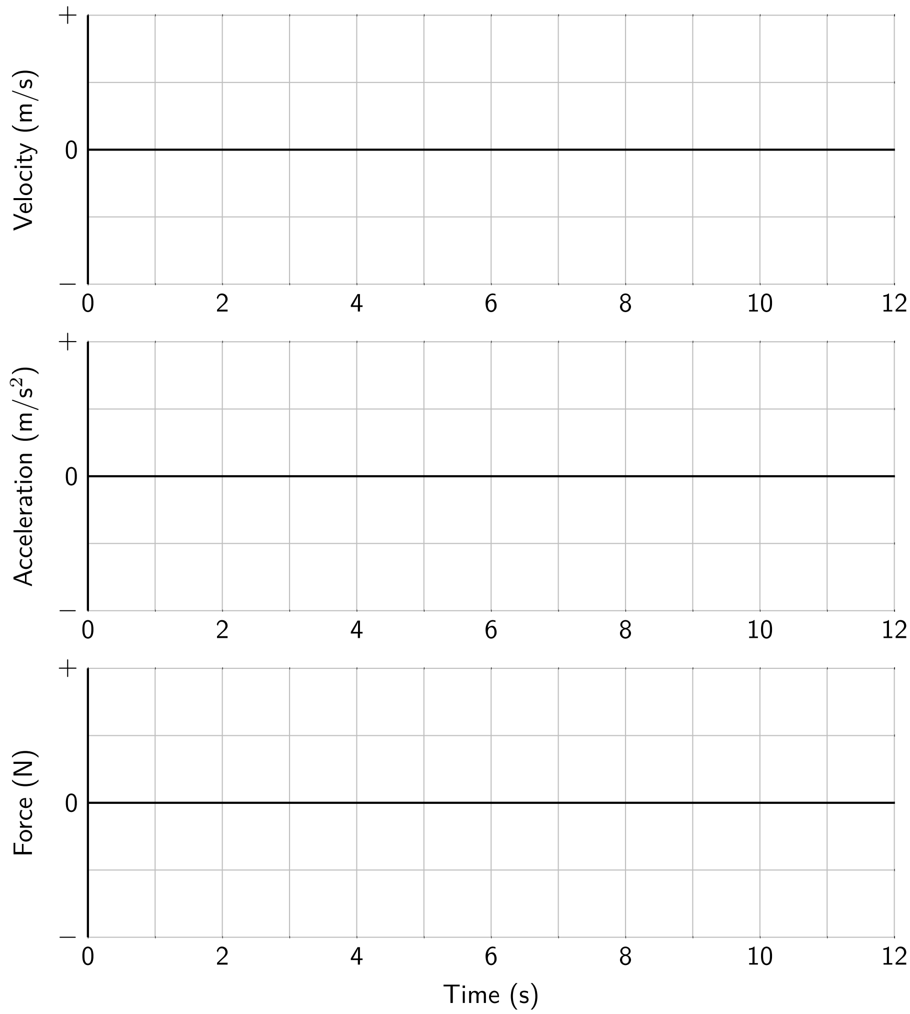
(b) Describe in words the predicted shape of the velocity *vs.* time and acceleration *vs.* time graphs for the cart.



Equipment setup for quantitative measurements of force and motion.

(c) Test your predictions. Set up the pulley, cart, and string as shown in the figure above. The cart should be the same mass as before. At the start of each trial, tare the cart's built-in force sensor when there is no applied force on the hook.

Take one of the masses (say, the 50-g one) from the tray and hang it from the end of the string. Release the cart from rest and create graphs of its motion as it moves in the positive direction (defined by the x-axis printed on the top of the cart). Stop the cart before it hits the end of the track. Sketch the graphs neatly on the axes below and indicate the scale on the axes.



(d) Is the force which is applied to the cart by the string constant, increasing or decreasing? Explain based on your graph.

(e) How does the acceleration graph vary in time? Does this agree with your prediction? What kind of acceleration corresponds to a constant applied force?

(f) How does the velocity graph vary in time? Does this agree with your prediction? What kind of velocity corresponds to a constant applied force?

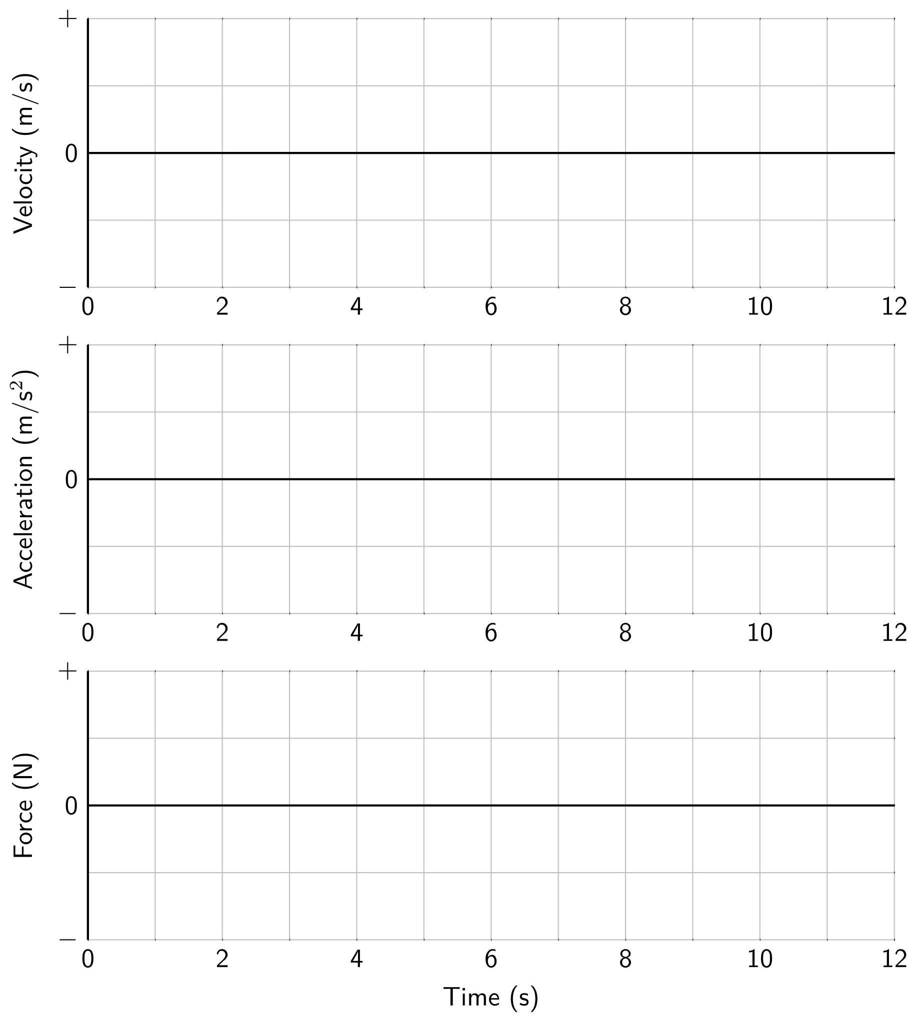
(g) Use the **Highlight** and **Statistics** functions to determine the average force and the average acceleration and record them below. Find the mean values only during the time interval when the force and acceleration are nearly constant. See **Appendix A** for details on the use of the **Highlight** and **Statistics** functions of Capstone.

Activity 3: Acceleration from Different Forces

In the previous activity you examined the motion of an object with a constant force applied to it. But, what is the relationship between acceleration and force? If you apply a larger force to the same object (same mass as before) how will the acceleration change? In this activity you will try to answer these questions by applying a different force to the object, and measuring the corresponding acceleration.

(a) Suppose you pulled the cart with a force about twice as large as before. What would happen to the acceleration of the cart? Explain.

(b) Test your prediction by replacing the 50-g hanging mass with a 100-g mass. Tare the force sensor with no force acting on the hook. Now create graphs of the motion as before. Repeat until you have a good run. Sketch the results on the axes that follow. Don't forget to put the scale on the axes.



(c) Use the **Highlight** and **Statistics** functions to find the average force and the average acceleration and record them below. Find the mean values only during the time interval when the force and acceleration are nearly constant.

(d) How did the force applied to the cart compare to the force in Activity 2?

(e) How did the acceleration of the cart compare to that caused by the force in Activity 2? Did this agree with your prediction? Explain.

Activity 4: The Relationship Between Acceleration and Force

If you accelerate the same object (same mass) with different forces, you can plot a graph of force *vs.* acceleration. You can then find the mathematical relationship between acceleration and force.

(a) Accelerate the cart with a force roughly midway between the other two forces tried. Use a hanging mass about midway between those used in the last two activities. Record the mass below.

(b) Graph velocity, acceleration and force. Sketch the graphs on the axes in Activity 3 using dashed lines.

(c) Find the mean acceleration and force, as before, and record the values in the table below (in the Activity 4 line). Also, enter the values from the previous two activities in the table. Use different combinations of the masses to get other applied forces and enter the results in the table.

	Average Force (N)	Average Acceleration (m/s^2)	Mass on string
Activity 4			
Activity 3			
Activity 2			
Activity 4			
Activity 4			

(d) Using *Excel*, plot the average force applied to the cart as a function of the average acceleration of the cart by fitting the data with a linear function. Include a sixth data point (0,0) (since zero force means 0 acceleration). Label and print the graph showing the best fit, and add it to this unit.

(e) Does there appear to be a simple mathematical relationship between the acceleration of the cart (with fixed mass) and the force applied to the cart (measured by the force sensor)? Write down the equation you found and describe the mathematical relationship in words. What is the slope of the graph?

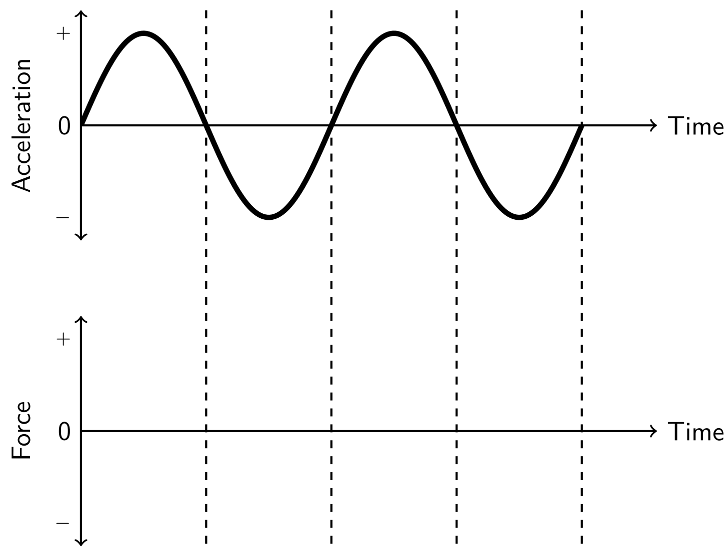
(f) Use the LINEST function in *Excel* (see **Appendix D: Excel**) to determine the uncertainty in mass of the cart. Write your result as $m = \overline{m} \pm \Delta m$. Be sure to include proper units.

(g) Does your measurement of m from Activity 1 fall within the range indicated in (f) above? If not, what are some possible sources of systematic error?

Comment: The relationship which you have been examining between the acceleration of the cart and the applied force is known as Newton's Second Law, $F = ma$.

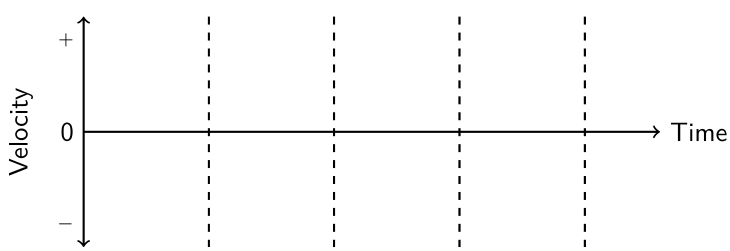
Homework

1. A force is applied which makes an object move with the acceleration shown below. Assuming that friction is negligible, sketch a force-time graph of the force on the object on the axes below.

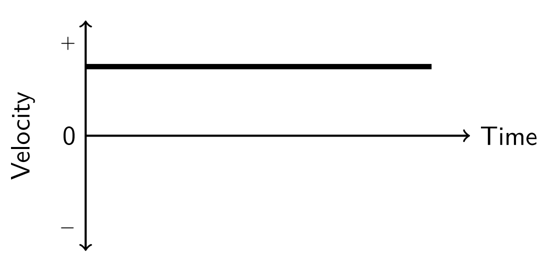


Explain your answer:

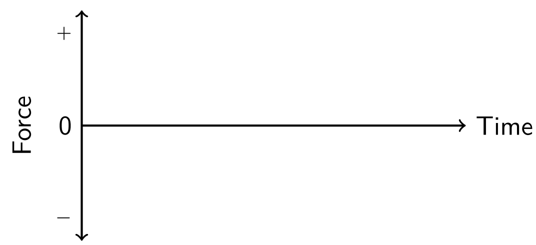
2. Roughly sketch the velocity-time graph for the object in question 1 on the axes below, beginning with a *negative* velocity. Remember that acceleration is the *derivative* of velocity.



3. A cart can move along a horizontal line (the + position axis). It moves with the velocity shown below.



Assuming that friction is so small that it can be neglected, sketch on the axes that follow the acceleration-time and force-time graphs of the cart's motion.



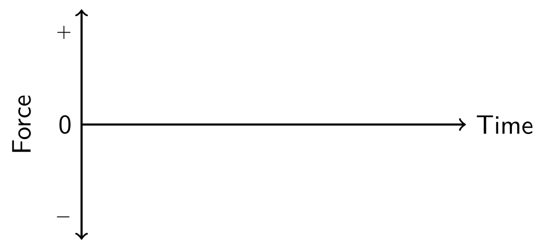
Explain both of your graphs.

Questions 4-6 refer to an object which can move in either direction along a horizontal line (the + position axis). Assume that friction is so small that it can be neglected. Sketch the shape of the graph of the force applied to the object which would produce the motion described.

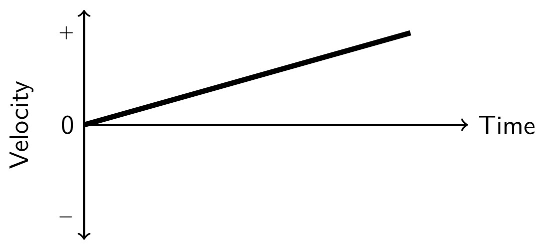
4. The object moves away from the origin while speeding up at a constant rate.

5. The object moves toward the origin while speeding up at a constant rate.

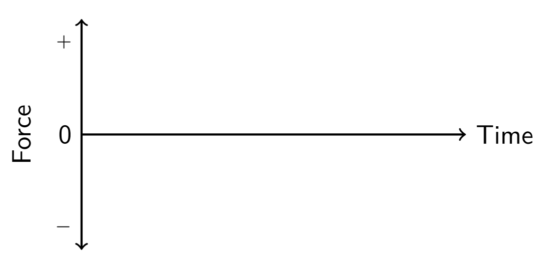
6. The object moves away from the origin with a constant velocity.



Questions 7 and 8 refer to an object which can move along a horizontal line (the + position axis). Assume that friction is so small that it can be ignored. The object's velocity-time graph is shown below.

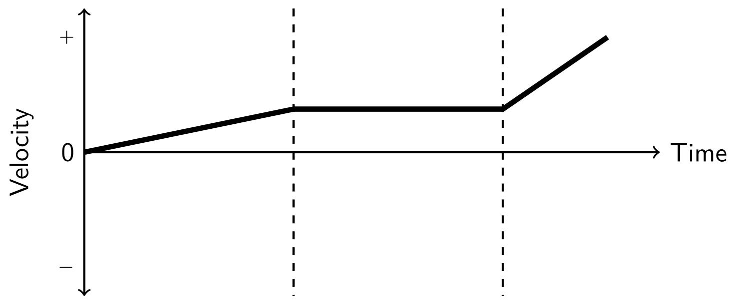


7. Sketch the shapes of the acceleration-time and force-time graphs on the axes below.

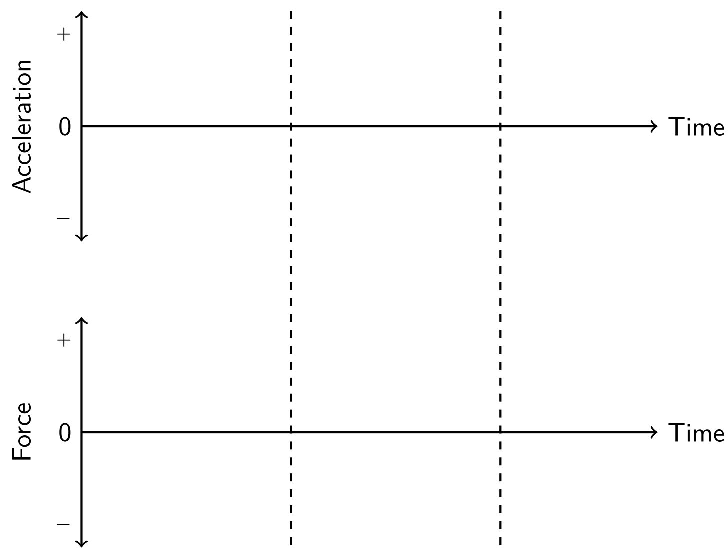


8. Suppose that the force applied to the object were twice as large. Sketch with dashed lines on the same axes above the force, acceleration, and velocity.

9. An object moves along a horizontal line (the + position axis). Assume that friction is so small that it can be ignored. The object's velocity-time graph is shown below.



Sketch the shapes of the acceleration and force graphs on the axes below.



Lab 11 Force and Motion II[1]

Name: ____________________ Lab Partner(s): ______________________________

Objective

To understand the relationship between the direction of the force applied to an object and the direction of the acceleration of the object.

Overview

In the previous lab you examined the one-dimensional motions of an object caused by a single force applied to the object. You have seen that when friction is so small that it can be ignored, a single constant applied force will cause an object to have a constant acceleration. (The object will speed up at a steady rate.)

Under these conditions, you have seen that the acceleration is proportional to the applied force, if the mass of the object is not changed. You saw that when a constant force is applied to a cart, the cart speeds up at a constant rate so that it has a constant acceleration. If the applied force is made larger, then the acceleration is proportionally larger. This allows you to define force more precisely not just in terms of the stretches of rubber bands and springs, but as the entity (the "thing") that causes acceleration.

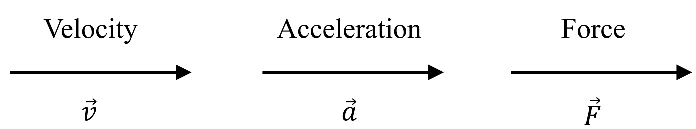
The goal of this lab is to continue to develop the relationship between force and acceleration, an important part of the first two of Newton's famous laws of motion. You will explore motions in which the applied force (and hence the acceleration of the object) is in a different direction than the object's velocity. In this case the object is slowing down in the sense that its speed is decreasing.

Apparatus

- *Capstone* software (*V_A_F_Graphs.cap* experiment file)
- Dynamics track
- Hanging masses
- Pulley with clamp
- String
- Wireless smart cart

Speeding Up and Slowing Down

So far you have looked at cases where the velocity, force and acceleration all have the same sign (all positive). That is, the vectors representing each of these three vector quantities all point in the same direction. For example, if the cart is moving toward the right and a force is exerted toward the right, then the cart will speed up. Thus the acceleration is also toward the right. The three vectors can be represented as:

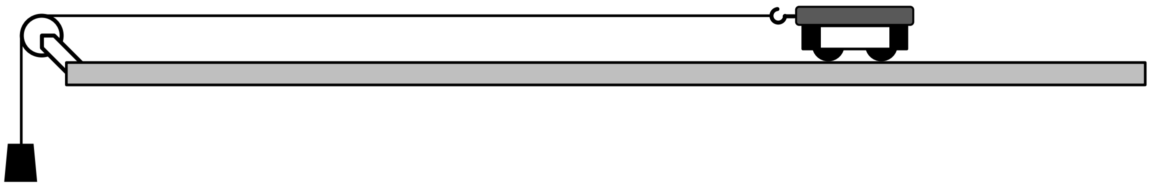


If the positive direction is toward the right, then you could also say that the velocity, acceleration and also force are all positive. In this investigation, you will examine the vectors representing velocity, force and acceleration for other motions of the cart. This will be an extension of your earlier observations of changing motion. 3

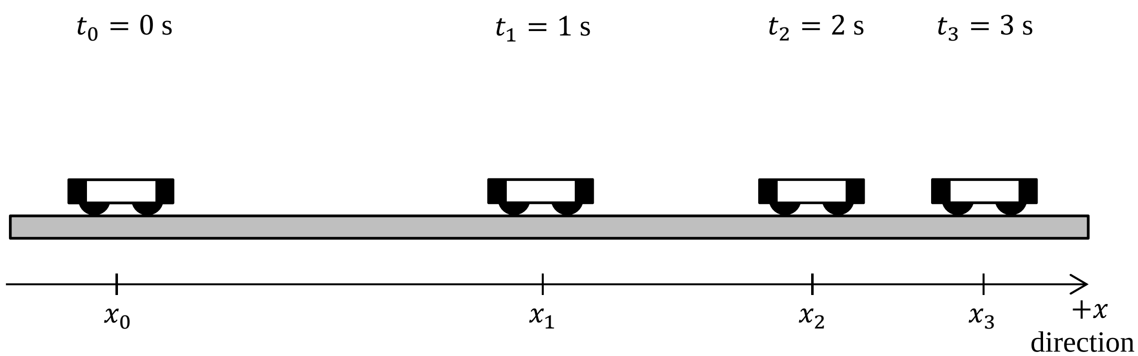
[1] 1990-93 Dept. of Physics and Astronomy, Dickinson College. Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Portions of this material have been modified locally and may not have been classroom tested at Dickinson College.

Activity 1: Slowing Down

Set up the pulley, cart, and string as shown in the figure below. Now, if you give the cart a push away from the pulley, it will slow down after it is released. In this activity you will examine the acceleration and the applied force.



(a) Suppose that you give the cart a push away from the pulley and release it. Draw on the diagram below vectors which might represent the velocity, force and acceleration of the cart at each time after it is released and is moving away from the pulley. Be sure to mark your arrows with $\vec{v}$, $\vec{a}$, or $\vec{F}$ as appropriate.



(b) If the positive direction is to the right, what are the signs of the velocity, force and acceleration after the cart is released and is moving toward the right?

(c) To test your predictions:

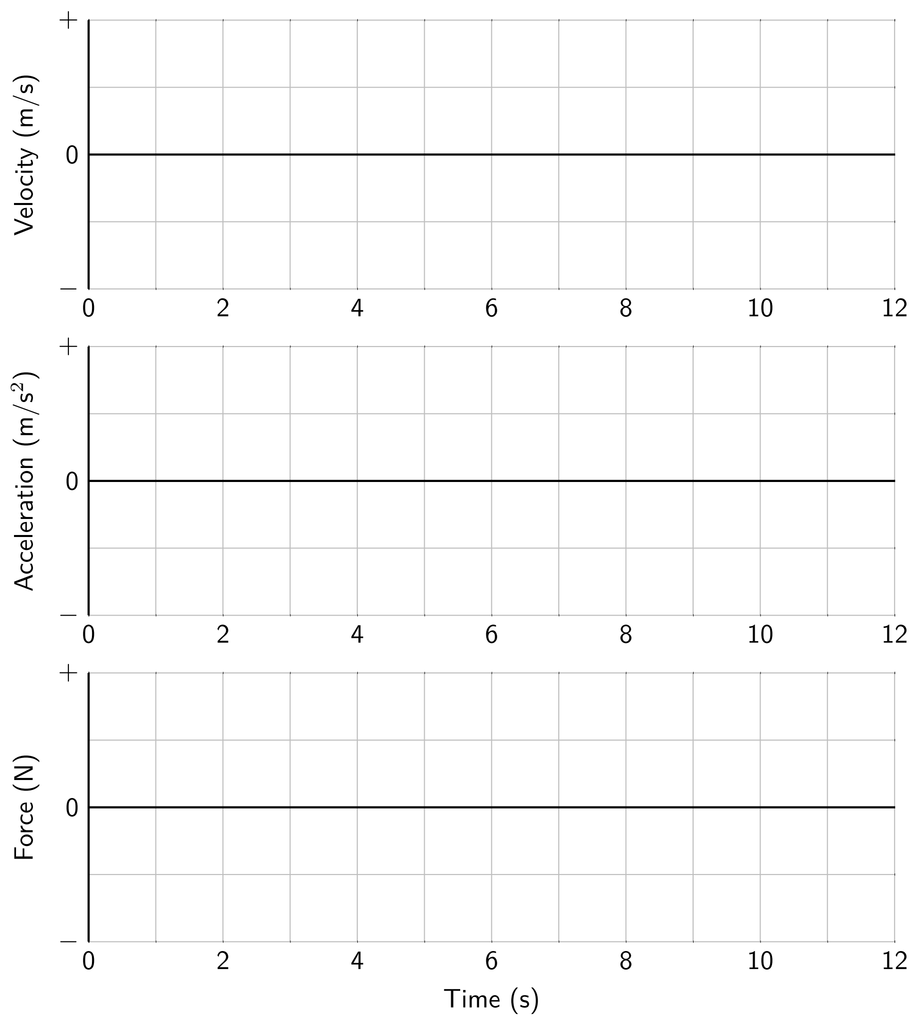
1. Open the file *V_A_F_Graphs.cap* in the *Phys131* folder. Turn on the cart at your station and connect it to the computer via Bluetooth. At the beginning of each experiment, the measurements from the built-in acceleration and force sensors may not be zero when the acceleration or force is actually zero. To tare the sensors, select the desired sensor (either the **Smart Cart Acceleration Sensor** or the **Smart Cart Force Sensor**) in the **Controls** palette and then click **Zero Sensor Now**. When taring the sensors, the cart should be at rest and there should be no applied force on the hook.

2. By default, the x-axis printed on the top of the cart defines the positive direction for the cart's built-in sensors. Therefore, by default, the positive direction points towards the pulley (or to the left in the figure above). To change this, so that the positive direction points away from the pulley (or to the right), select **Hardware Setup** in the **Tools** palette. To change the sign of the cart's velocity and acceleration measurements, select the **Properties** icon (the gear symbol) in the **Smart Cart Position Sensor** window and then select the **Change Sign** checkbox. To change the sign of the cart's built-in force sensor, select the **Properties** icon in the **Smart Cart Force Sensor** window and select the **Change Sign** checkbox.

3. Hang a 50-g mass from the end of the string.

4. Click the **Record** button. Then, give the cart a short push away from the pulley and let it go. Stop the cart before it reverses its direction. Repeat until you get a good run.

5. Sketch your velocity, acceleration and force graphs on the axes below. Indicate with an arrow the time when the push stopped.



(d) Did the signs of the velocity, force and acceleration agree with your predictions? If not, can you now explain the signs?

(e) Did the velocity and acceleration both have the same sign? Explain these signs based on the relationship between acceleration and velocity.

(f) Did the force and acceleration have the same sign? Were the force and acceleration in the same direction? Explain.

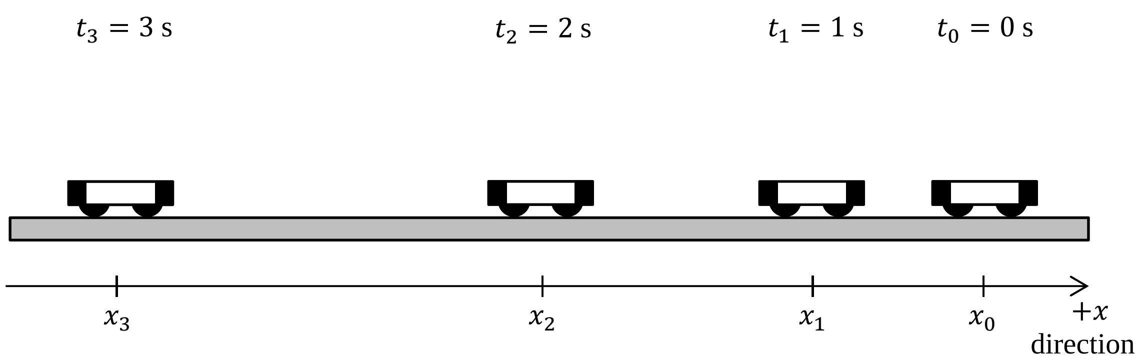
(g) Based on your observations, draw below vectors which might represent the velocity, force and acceleration for the cart at the same instant in time. Do these agree with your predictions? If not, can you now explain the directions of the vectors?

(h) After you released the cart, was the force applied by the falling mass constant, increasing or decreasing? Why is this kind of force necessary to cause the observed motion of the cart?

Activity 2: Speeding Up Toward the Motion Detector

Using the same setup as in the last activity, you can start with the cart at the opposite end of the table from the pulley and release it from rest. It will then be accelerated toward the pulley as a result of the force applied by the falling mass.

(a) Suppose that you release the cart from rest and let it move toward the pulley. Draw on the diagram below vectors which might represent the velocity, force and acceleration of the cart at each time after it is released and is moving toward the left. Be sure to mark your arrows with $\vec{v}$, $\vec{a}$, or $\vec{F}$ as appropriate.



(b) What are the signs of the velocity, force and acceleration after the cart is released and is moving toward the pulley? (The positive direction is toward the right.)

(c) Test your predictions. Use a hanging mass of 50 g. Start recording data and release the cart from rest as far away from the pulley as possible. Catch the cart before it hits the end of the track. Repeat until you get a good run. Sketch your graphs on the axes in the previous activity with dashed lines.

(d) Which of the signs – velocity, force and/or acceleration – are the same as in the previous activity (where the cart was slowing down and moving away), and which are different? Explain any differences in terms of the differences in the motion of the cart.

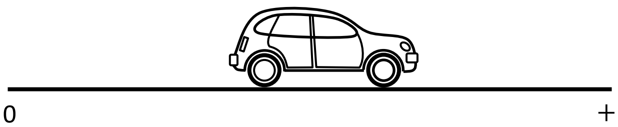
(e) Based on your observations, draw below vectors which might represent the velocity, force and acceleration for the cart at the same instant in time. Do these agree with your predictions? If not, can you now explain the directions of the vectors?

(f) Write down a simple rule in words which describes the relationship between the direction of the applied force and the direction of the acceleration for any motion of the cart.

(g) Is the direction of the velocity always the same as the direction of the force? Is the direction of the acceleration always the same as the direction of the force?

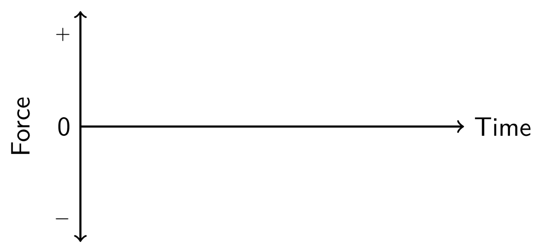
Homework

Questions 1-6 refer to a toy car which can move in either direction along a horizontal line (the + position axis).



Assume that friction is so small that it can be ignored. Sketch the shape of the graph of the applied force which would keep the car moving as described in each statement.

1. The toy car moves away from the origin with a constant velocity.

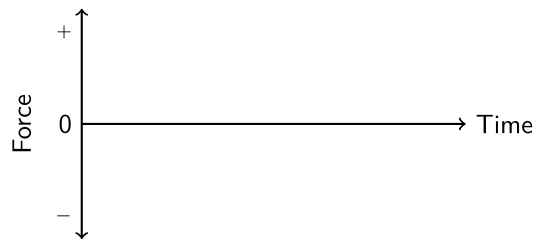


2. The toy car moves toward the origin with a constant velocity.

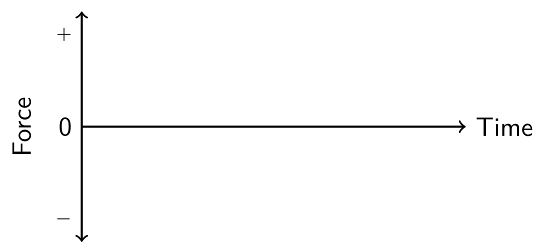
3. The toy car moves away from the origin with a steadily decreasing velocity (a constant acceleration).

4. The toy car moves away from the origin, speeds up and then slows down.

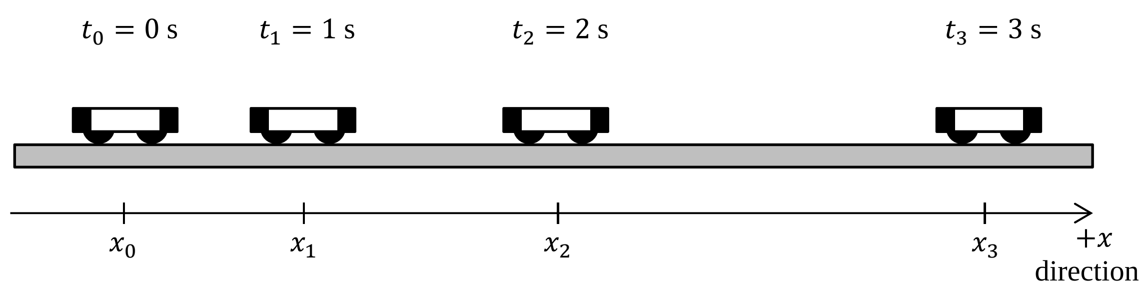
5. The toy car moves toward the origin with a steadily increasing speed (a constant acceleration).



6. The toy car is given a push away form the origin and released. It continues to move with a constant velocity. Sketch the force after the car is released.



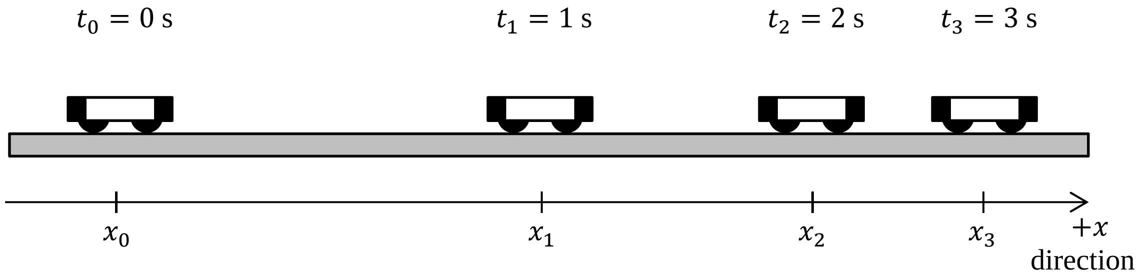
7. A cart is moving toward the right and speeding up, as shown in the diagram below. Draw arrows above the cart representing the magnitudes and directions of the net (combined) forces you think are needed on the cart at the times shown to maintain its motion with a steadily increasing velocity.



Explain the reasons for your answers.

8. If the positive direction is toward the right, what is the sign of the force at $t = 2$ sec in question 7? Explain.

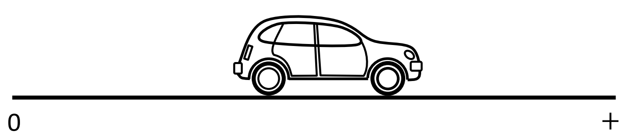
9. A cart is moving toward the right and slowing down, as shown in the diagrams below. Draw arrows above the cart representing the magnitudes and directions of the net (combined) forces you think are needed on the cart at $t = 0$ s, $t = 1$ s, etc. to maintain its motion with a steadily decreasing velocity.



Explain the reasons for your answers.

10. If the positive direction is toward the right, what is the sign of the force at $t = 2$ sec in question 9? Explain.

11. A toy car can move in either direction along a horizontal line (the + position axis).



Assume that friction is so small that it can be ignored. A force toward the right of constant magnitude is applied to the car. Sketch on the axes below using a solid line the shape of the acceleration-time graph of the car.

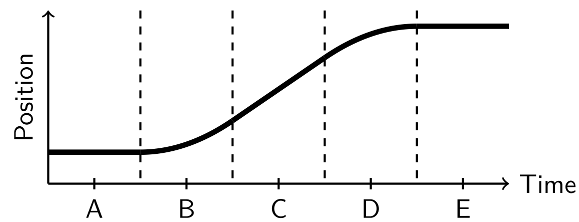
Explain the shape of your graph in terms of the applied force.

Lab 12 Gravity, Normal Force, and Elevators

Name: ____________________ Lab Partner(s): ____________________

Apparatus

- digital lab scale
- 200-g mass



Activity 1: Some predictions

(a) Imagine you are riding in an elevator, and your vertical position as a function of time is given in the graph above. You begin at rest, accelerate briefly to a constant velocity upwards for a while, then decelerate to a stop at the top floor. Are there any of the times A through E when you *feel* heavier than usual? Which time(s)?

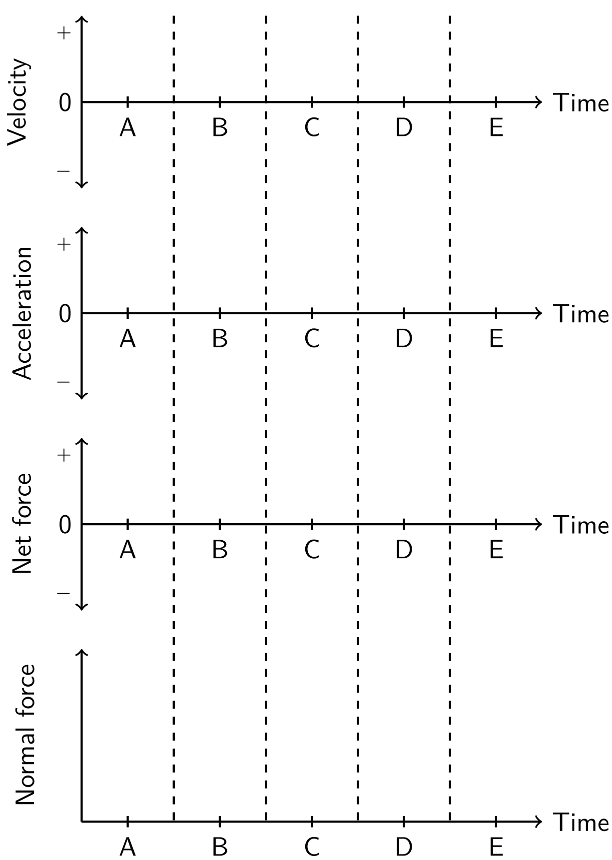
(b) Suppose your normal weight is exactly 100 pounds, and you happen to be standing on a bathroom scale in the elevator as it goes up. Take a wild guess: During which of the times A through E does the scale read:

> 100 pounds?

< 100 pounds?

= 100 pounds?

(c) Let's think through the last two questions by drawing some additional pictures and graphs. First, in the space below, draw a free body diagram showing only the forces acting *on you* as you ride the elevator. Next, fill in the axes to the right with sketches of your velocity, acceleration, F_{NET} and F_{normal} vs. time, being super careful to align your graphs to the position vs. time graph at the top of the page.

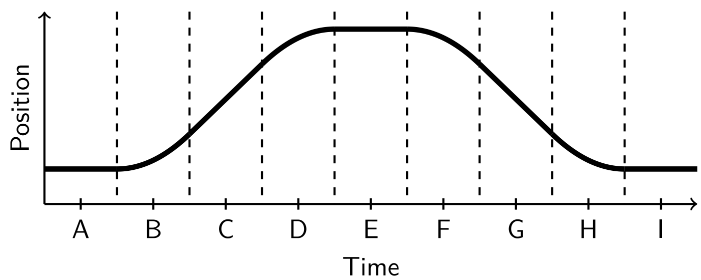


(d) Would you care to reconsider either of your answers to parts (a) or (b) now? Don't erase anything you wrote before; just note any additional thoughts or changes below.

Activity 2: An actual experiment, and coming back down

(*a*) Physics is an experimental science, and you do have at your desposal a scale, a 200-g mass, and an elevator. Go for it! (While you're there, you might consider the return trip back down in the elevator too, as depicted in the graph in the following part.)

(*b*) In the space following the graph below, draw any additional diagrams or graphs you need to explain exactly where you felt lighter than normal, and where you felt heavier than normal.



(*c*) Bottom line: If your normal weight was exactly 100 pounds, during which of the times A through I does the scale read:

$>$ 100 pounds?

$<$ 100 pounds?

$=$ 100 pounds?

Lab 13 Newton's 3rd Law, Tension, and Normal Forces[1]

Name: ______________________ Lab Partner(s): ______________________

Objectives

- To investigate Newton's 3rd law of motion.
- To explore the characteristics of two different types of forces: tension (in strings, ropes, springs, and chains), and normal forces (which support objects that are in contact with solid surfaces).

Apparatus

- Spring scales (2)
- Variety of masses
- Rubber band
- Various lengths of string
- Pulleys (2)

Activity 1: An Introduction to Newton's Third Law

In order to apply Newton's laws to complex situations with strings, pulleys, inclined planes and so forth, we need to consider a third force law formulated by Newton having to do with the forces of interaction between two objects.

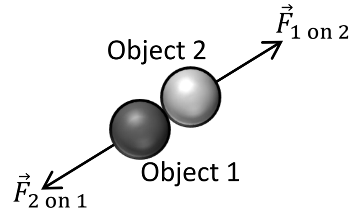
Grab a spring scale, and have your lab partner grab a second spring scale. If you hook the end of your scale and pull, the scale measures the force you are pulling with. Hook the end of your spring scale onto the end of your partner's spring scale so that the two of you are pulling on each other. See if you can find a way for each of you to pull so that your two spring scales give different readings. (You might try holding still, or pulling your partner's arm closer to yours, for instance.) Describe your conclusions below.

In contemporary English, Newton's third law can be stated as follows.

> *Newton's Third Law:*
>
> *If one object exerts a force on a second object, then the second object exerts a force back on the first object which is equal in magnitude and opposite in direction to that exerted on it by the first object.*

In mathematical terms, we can describe the interaction forces between two objects use vector notation:

$$\vec{F}_{1\text{ on }2} = -\vec{F}_{2\text{ on }1}$$



[1]1990-93 Dept. of Physics and Astronomy, Dickinson College. Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Portions of this material may have been modified locally and may not have been classroom tested at Dickinson College.

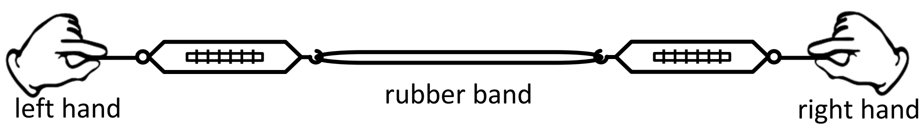
Activity 2: Tension Forces

When you pull on one end of a rope attached to a crate, a force is transmitted down the rope to the crate. If you pull hard enough the crate may begin to slide. Tension is the name given to forces transmitted in this way along devices that can stretch such as strings, ropes, rubber bands, springs, and wires.

(a) Pull on the two ends of a rubber band, stretching it between your two hands. What is the direction of the force applied by the rubber band on your right hand? On your left hand?

(b) Does the magnitude of the forces applied by the rubber band on each hand feel the same?

(c) Now add a spring scale to each side, to measure the forces at both ends. Are they indeed the same?



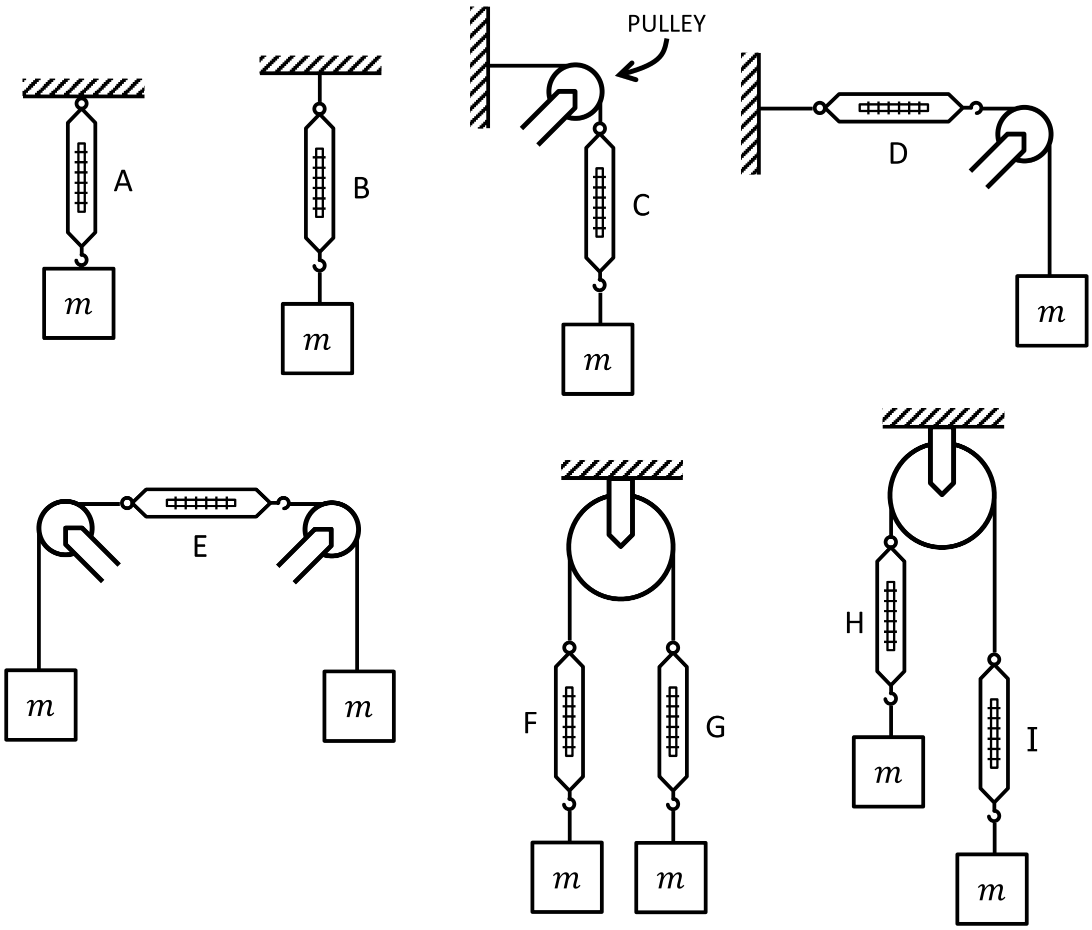
(d) Repeat part (c) with a string instead of a rubber band. Does the string stretch? (Look carefully!) Are the forces still the same on both sides?

(e) If you pull by the same amount on the string as you did on the rubber band, does substituting the string for the rubber band change anything about the directions and magnitudes of the tension forces exerted on each hand?

(f) If the forces caused by the string, known as tension force, on your left and right hands respectively are given by T_1 and T_2, what is the equation that relates these two forces?

Activity 3: Tension Forces when a String Changes Direction

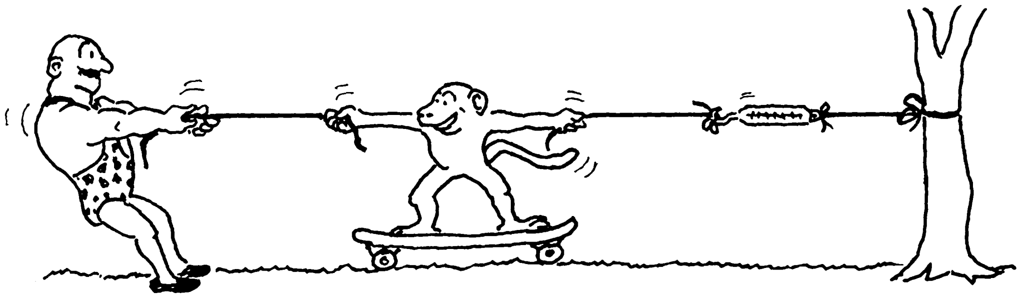
Suppose you were to hang equal masses of $m = 0.5$ kg in the various configurations shown below.



(a) For each configuration shown, predict the reading in newtons on each of the spring scales; these readings indicate the forces that are transmitted by the tensions at various places along the string. Then measure all of the forces and record their values. Note: Remember that $m = 0.5$ kg.

Predicted Force Magnitudes	Measured Force Magnitudes
$F_A =$ ________ N	$F_A =$ ________ N
$F_B =$ ________ N	$F_B =$ ________ N
$F_C =$ ________ N	$F_C =$ ________ N
$F_D =$ ________ N	$F_D =$ ________ N
$F_E =$ ________ N	$F_E =$ ________ N
$F_F =$ ________ N	$F_F =$ ________ N
$F_G =$ ________ N	$F_G =$ ________ N
$F_H =$ ________ N	$F_H =$ ________ N
$F_I =$ ________ N	$F_I =$ ________ N

(b) Based on Newton's third law and the observations you just made, answer the following questions using vector notation. If the muscle man in the diagram below is pulling to the left on a rope with a force of $\vec{F} = -(150\text{ N})\hat{\imath}$.



(1) What is the magnitude and direction of the force that the rope is exerting on the man? ______________

(2) What force is the left-hand rope exerting on the monkey's right arm? ______________

(3) What force is the spring scale experiencing on its left end? ______________

(4) What force is the spring scale experiencing on its right end? ______________

(5) What is the reading on the spring scale? ______________

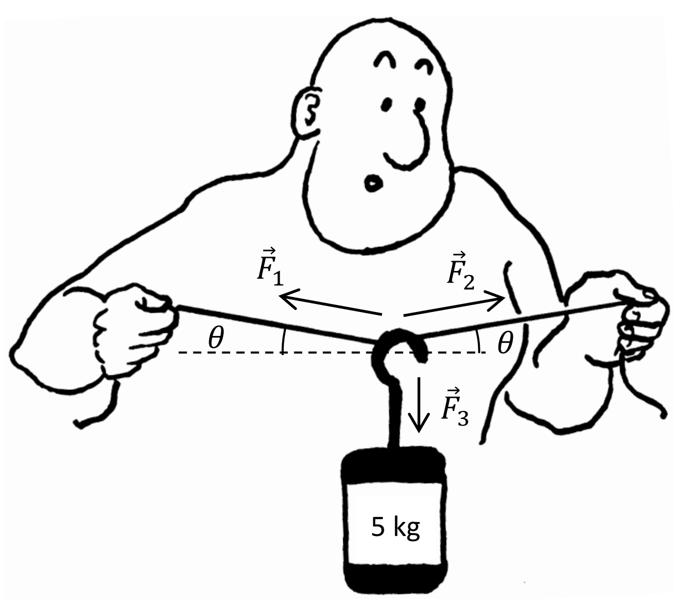
(6) What force is the rope exerting on the tree? ______________

(7) What force is the tree exerting on the rope? ______________

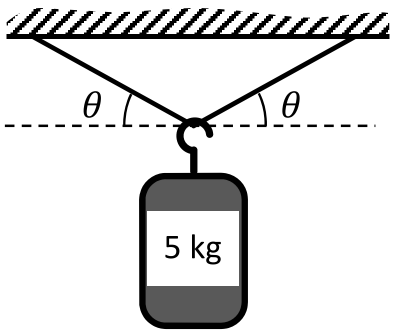
(c) Summarize what your observations reveal about the nature of tension forces everywhere along a string.

Activity 4: Can a String Support Lateral Forces?

Take a look at the diagram below. Can the strongest member of your group stretch a string or rope so that it is perfectly horizontal when a 5 kg mass is hanging from it? In other words, can the string provide a force that just balances the force exerted by the mass?



(a) Draw a vector diagram showing the directions of the forces exerted by the strings on the mass hook in the diagram below. What would happen to the direction of the forces as θ goes to zero? Do you think it will be possible to support the mass when $\theta = 0$? Why?

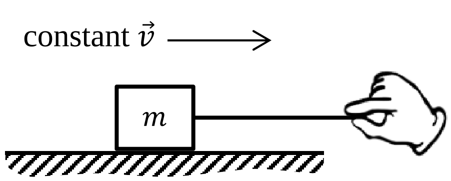


(b) Now, try holding a mass horizontally with a string. Explain why it is not possible to hold the string horizontal.

Activity 5: Normal Forces

A book resting on a table does not move; neither does a person pushing against a wall. According to Newton's first law the net force on the book and on the person's hand must be zero. There must be another type of force that explains why books don't fall through tables and hands don't usually punch through walls. The forces exerted by these surfaces always seem to act in a direction perpendicular to that surface; such a force is known as a *normal* force. (Here "normal" means perpendicular.)

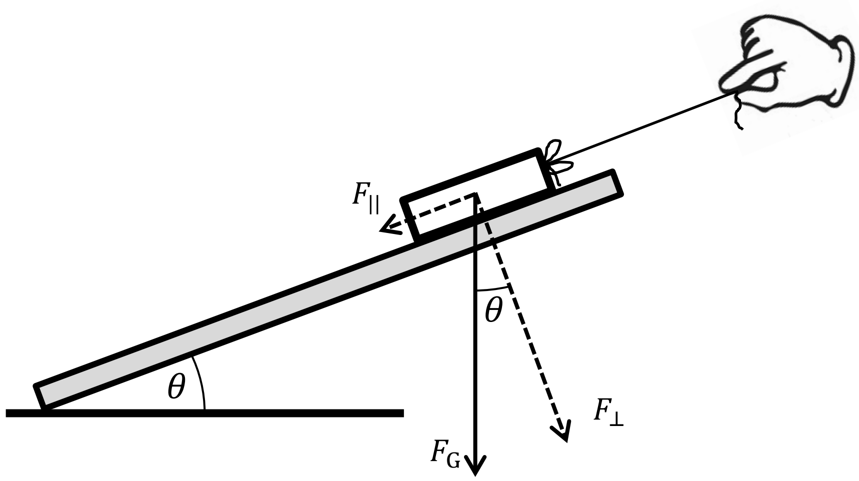
(a) The diagram below shows a block sliding along a table near the surface of the earth at a constant velocity. According to Newton's first law, what is the net force on the block? In other words, what is the vector sum of all the forces on the block?



(b) The net force is made up of four forces. In what direction does each one act? Draw a diagram indicating the direction of each of the forces.

Activity 6: Components of F_G on an Incline

Suppose that a block of mass m is perched on an incline of angle θ as shown in the diagram below. Also suppose that you know the angle of the incline and the magnitude and direction of the gravitational force vector. What do you predict the magnitude of the component of the force vector will be parallel to the plane? Perpendicular to the plane?



(a) The angle that the incline makes with the horizontal and the angle between $F_\perp$ and F_G are the same. Explain why.

(b) Choose a coordinate system with the x-axis parallel to the plane with the positive direction up the plane. Using normal mathematical techniques for finding the components of a vector, find the values of $F_{||}$ and $F_\perp$ as a function of F_G and the angle of the incline θ.

(c) Find an equation for the magnitude of the normal force exerted on the block by the surface of the incline. Hint: Use Newton's first law and the knowledge that the block is not moving in a direction perpendicular to the plane.

Lab 14 Friction and Applying the Laws of Motion[1]

Name: ______________________ Lab Partner(s): ______________________

Objectives

- To explore the characteristics of friction.
- To learn to use free-body diagrams to make predictions about the behavior of systems which are acted on by multiple forces in two and three dimensions.

Predicting and Measuring Friction Factors

If Newton's laws are to be used to describe the sliding of a block in contact with a flat surface, we must postulate the existence of a frictional force that crops up to oppose the applied force. There are two kinds of frictional forces: static friction and kinetic or sliding friction, which is the friction between surfaces in relative motion. We will concentrate on the study of kinetic friction for a sliding block.

Apparatus

- Wooden block with hook
- 5.0 newton spring scale
- Variety of masses
- CS2000 compact scale

Activity 1: Prediction of Friction Factors

(a) List several parameters that might influence the magnitude of the kinetic frictional force.

(b) Describe how you might do an experiment to determine the effect of mass on the magnitude of the frictional force.

Measuring the Effect of Mass on Kinetic Friction

Let's determine how mass actually influences the frictional force. Perform the experiment that you designed in the previous activity to determine how the frictional force varies with mass.

[1]1990-93 Dept. of Physics and Astronomy, Dickinson College. Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Portions of this material may have been modified locally and may not have been classroom tested at Dickinson College.

Activity 2: Friction Data and Analysis

(a) Create a data table for the frictional force as a function of mass in the space below. You should make at least 5-7 measurements.

(b) Graph the frictional force (vertical axis) versus the the magnitude of the normal force $n = mg$ (horizontal axis) and find the best fit to the data, using *Excel* with a linear function. Print the graph and include a copy with this unit. Write the equation (with appropriate units) that describes the data in the space below.

(c) Look up kinetic friction in your text. Read about the coefficient of sliding friction, μ_k, and figure out how to determine μ_k from the slope of your graph. Use the LINEST function in *Excel* (see **Appendix D: Excel**) to determine the slope and its uncertainty. Calculate μ_k and its uncertainty for the block sliding on the table. How does your range of values compare with the appropriate value in the table in your text? Do they agree?

Theories of Friction

No material is perfectly "smooth and flat." Any surface when examined under a microscope is full of irregularities. It is usually assumed that sliding friction forces result from the rubbing of rough surfaces, i.e., from the interlocking of surface bumps during the sliding process. How reasonable is this explanation for sliding friction?

Activity 3: What Surfaces Have High Friction?

(a) Which kinds of surfaces do you think will have the most friction rough ones or smooth ones? Why?

(b) Examine the table of coefficients of friction in a text. Do the values listed in this table support your predictions? Are you surprised?

The fact that smooth surfaces sometimes have more sliding friction associated with them than rough surfaces has led to the modern view that other factors such as adhesion (i.e., the attraction between molecules on sliding surfaces) also play a major role in friction. Predicting the coefficient of sliding friction for different types of surfaces is not always possible and there is much yet to be learned about the nature of the forces that govern sliding friction. Most authors of introductory physics texts still tend, incorrectly, to equate smooth surfaces with "frictionless ones" and to claim that the rubbing of rough surfaces is the cause of friction.

Free-Body Diagrams: Putting It All Together

You have made a series of observations which hopefully led you to reconstruct Newton's three laws of motion and some of its ramifications for yourself. In summary the laws are:

Newton's First Law: If the net force acting on an object is zero its acceleration is zero. [If $\sum \vec{F} = 0$ then $\vec{a} = 0$ so that $\vec{v} =$ constant or zero.]

Newton's Second Law: If the net force on an object is NOT zero, then it will be accelerated according to $\sum \vec{F} = m\vec{a}$.

Newton's Third Law: Any two objects that interact exert forces on each other which are equal in magnitude and opposite in direction. [$\vec{F}_{1\,\text{on}\,2} = -\vec{F}_{2\,\text{on}\,1}$]

These three laws are incredibly powerful because an understanding of them allows you to either: (1) use a complete knowledge of forces on a system of objects to predict motions in the system or (2) identify the forces on a system of objects based on observations of its motions.

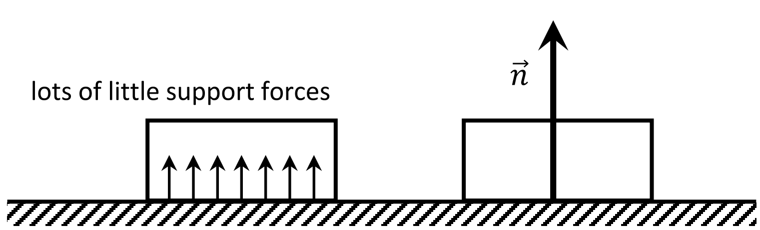
Using Free-Body Diagrams to Predict Motions and Calculate Forces

The key to the effective application of Newton's laws is to identify and diagram all the forces acting on each object in a system of interest. The next step is to define a coordinate system and break the forces down into components to take advantage of the fact that if $\sum \vec{F} = m\vec{a}$ then $\sum F_x = ma_x$ and $\sum F_y = ma_y$.

A free-body diagram consists of a set of arrows representing all the forces on an object, but NOT the forces that the object exerts on other objects. To create a free-body diagram you should do as follows:

1. Draw arrows to represent all force acting on the object or objects in the system of interest.
2. Place the tail of each arrow at the point where the force acts on the object.
3. Each arrow should point in the direction of the force it represents.
4. The relative lengths of the arrows should, if possible, be made to correspond to the magnitudes of the forces.
5. A set of coordinate axis should be chosen and indicated and all of the arrows should be labeled using standard notation to indicate the type of force involved.

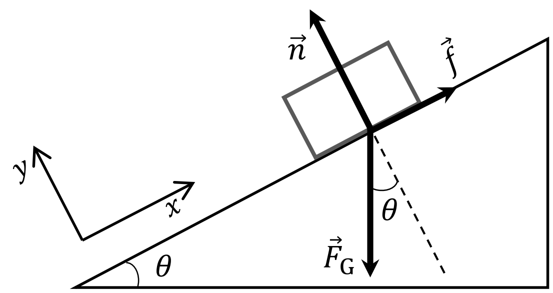
Important Note: The idea of using a single force vector to summarize external forces that act in the same direction is a useful simplification which is not real. For example, when a block rests on a table, we will say that the table exerts a normal force on the block. It is conventional to draw a single upward arrow at the point where the middle of the bottom surface of the block touches the table. This arrow actually represents the sum of all the smaller forces at each point where the block touches the table. This is shown in the diagram below:



An Example of a Free-Body Diagram

Consider a block of mass m sliding on a rough inclined plane as shown in the diagram below. It has three forces on it: (1) a gravitational force, (2) a normal force perpendicular to the surface of the plane, and (3) the friction force opposing its motion down the plane.

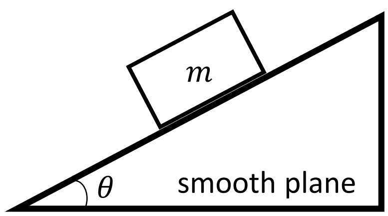
Since the block is sliding it could either be moving at a constant velocity or with a constant acceleration. Thus, it is possible that the vector sum of forces on it is not equal to zero in some cases. For example, if there is accelerated motion, then: $\sum \vec{F} = \vec{n} + \vec{F}_{\mathrm{G}} + \vec{f} = m\vec{a}$



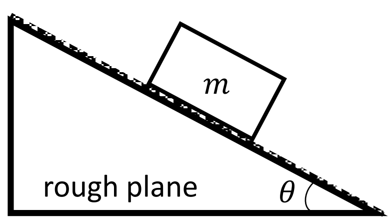
Activity 4: Some Free-Body Diagrams

Based on the example above, try your hand at drawing the free-body diagrams for the situations described below. In each case, write the equation for the vector sum of forces. As always, be sure to put arrows over symbols representing vector quantities.

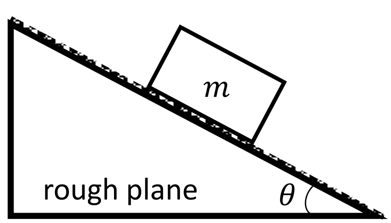
(a) A block slides freely down a smooth inclined plane.



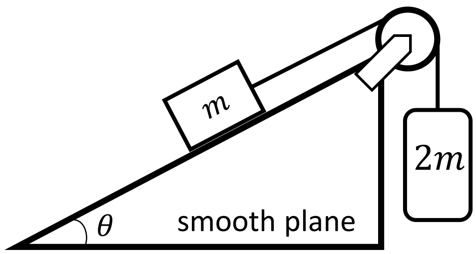
(b) A block is on a rough plane but is not moving due to static friction.



(c) A block is on a rough plane but the coefficient of friction between the block and the plane is small. The block is sliding down the plane at a constant velocity so that kinetic friction is acting.

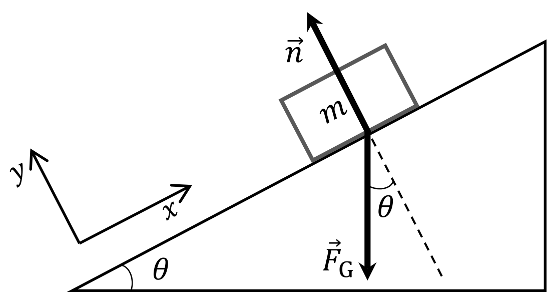


(d) The block is on a frictionless plane but is attached to a hanging mass of $2m$ by one of our famous "massless" strings over a pulley. Construct two free-body diagrams, one showing the forces on the mass m, and one showing the forces on the mass $2m$.



Breaking the Forces into Components: An Example

Consider the case of the block sliding down a "smooth" plane with a negligible amount of friction. The free-body diagram and coordinate system chosen for analysis are shown in the figure below.



Taking components: $(F_G)_x = -F_G \sin\theta$ and $(F_G)_y = -F_G \cos\theta$.

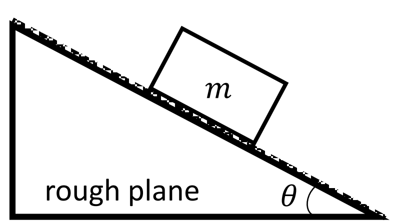
There is no motion perpendicular to the surface so $\sum F_y = 0 = n + (F_G)_y$ so that $n = -(F_G)_y = F_G \cos\theta$.

There is no balancing force for the x-component of F_G so according to Newton's second law $\sum F_x = ma_x = -F_G \sin\theta = -mg \sin\theta$ and therefore $a_x = -g \sin\theta$.

Activity 5: Solving an Inclined Plane Problem

(a) Consider a block sliding down a rough inclined plane at a **constant velocity**. What is the net force on it?

(b) Refer to the free-body diagram you created for this situation in the last activity (part c of Activity 4). Break the forces up into components and apply Newton's first law to find the equation for the coefficient of kinetic friction as a function of m, θ, and g. Remember that velocity is **constant**.



Lab 15 The Electrical and Gravitational Forces[1]

Name: ____________________ Lab Partner(s): ____________________

> I began to think of gravity extending to the orb of the moon, and . . . I deduced that the forces which keep the planets in their orbs must be reciprocally as the squares of their distances from the centers about which they revolve: and thereby compared the force requisite to keep the moon in her orb with the force of gravity at the surface of the earth, and found them to answer pretty nearly. All this was in the two plague years of 1665 and 1666, for in those days I was in the prime of my age for invention, and minded mathematics and philosophy more than at any time since. — Isaac Newton

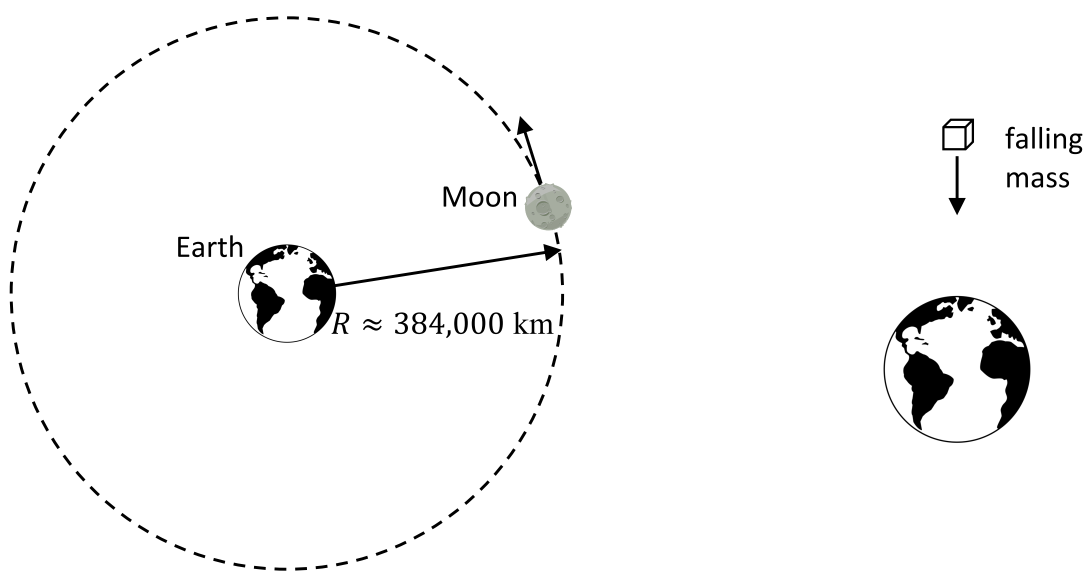
Objective

To understand the similarities of the gravitational and electrical forces.

Overview

The enterprise of physics is concerned ultimately with mathematically describing the fundamental forces of nature. Nature offers us several fundamental forces, which include a strong force that holds the nuclei of atoms together, a weak force that helps us describe certain kinds of radioactive decay in the nucleus, the force of gravity, and the electromagnetic force.

Two kinds of force dominate our everyday reality: the gravitational force acting between masses and the Coulomb force acting between electrical charges. The gravitational force allows us to describe mathematically how objects near the surface of the earth are attracted toward the earth and how the moon revolves around the earth and planets revolve around the sun. The genius of Newton was to realize that objects as diverse as falling apples and revolving planets are both moving under the action of the same gravitational force.



Similarly, the Coulomb force allows us to describe how one charge "falls" toward another or how an electron orbits a proton in a hydrogen atom.

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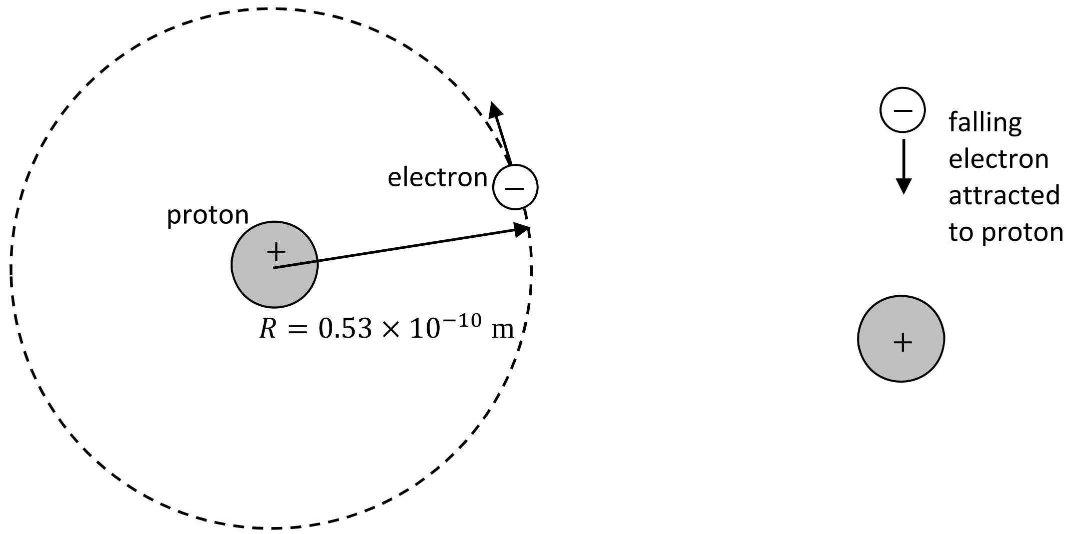
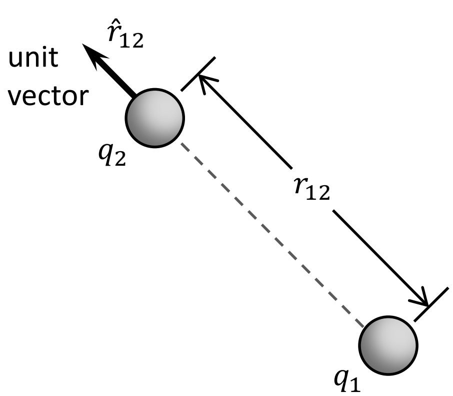


The fact that both the Coulomb and the gravitational forces lead to objects falling and to objects orbiting around each other suggests that these forces might have the same mathematical form.

In this unit we will explore the mathematical symmetry between electrical and gravitational forces for two reasons. First, it is beautiful to behold the unity that nature offers us as we use the same type of mathematics to predict the motion of planets and galaxies, the falling of objects, the flow of electrons in circuits, and the nature of the hydrogen atom and of other chemical elements. Second, what you have already learned about the influence of the gravitational force on a mass can be applied to aid your understanding of the forces on charged particles.

Activity 1: Comparison of Electrical and Gravitational Forces

Let's start our discussion of this comparison with the familiar expression of the Coulomb force exerted by charge 1 on charge 2.

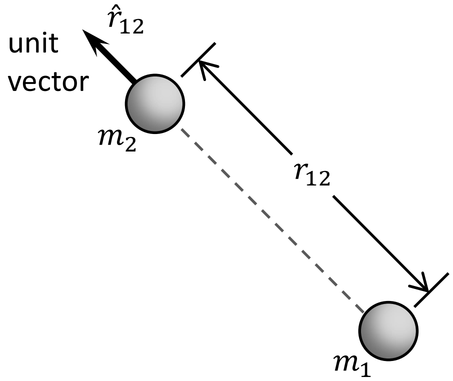


$$\vec{F}_{1\text{ on }2} = k_e \frac{q_1 q_2}{r^2} \hat{r}_{12}$$

$$k_e = 8.99 \times 10^9 \text{ N m}^2/\text{C}^2$$

Charles Coulomb did his experimental investigations of this force in the 18th century by exploring the forces between two small charged spheres. Much later, in the 20th century, Coulomb's law enabled scientists to design cyclotrons and other types of accelerators for moving charged particles in circular orbits at high speeds.

Newton's discovery of the universal law of gravitation came the other way around. He thought about orbits first. This was back in the 17th century, long before Coulomb began his studies. A statement of Newton's universal law of gravitation describing the force experienced by mass 1 due to the presence of mass 2 is shown below in modern mathematical notation:



$$\vec{F}_{1\text{ on }2} = -G\frac{m_1 m_2}{r^2}\hat{r}_{12}$$

$$G = 6.67 \times 10^{-11}\ \mathrm{N\,m^2/kg^2}$$

About the time that Coulomb did his experiments with electrical charges in the 18th century, one of his contemporaries, Henry Cavendish, did a direct experiment to determine the nature of the gravitational force between two spherical masses in a laboratory. This confirmed Newton's gravitational force law and allowed him to determine the gravitational constant, G. A fact emerges that is quite amazing. Both types of forces, electrical and gravitational, are very similar. Essentially the same mathematics can be used to describe orbital and linear motions due to either electrical or gravitational interactions of the tiniest fundamental particles or the largest galaxies. This statement needs to be qualified a bit when electrons, protons and other fundamental particles are considered. A new field called quantum mechanics was developed in the early part of the 20th century to take into account the wave nature of matter, which we don't actually study in introductory physics. However, even in wave mechanical calculations electrical forces like those shown above are used.

Activity 2: The Electrical *vs.* the Gravitational Force

Examine the mathematical expression for the two force laws.

(a) What is the same about the two force laws?

(b) What is different? For example, is the force between two like masses attractive or repulsive? How about two like charges? What part of each equation determines whether the like charges or masses are attractive or repulsive?

(c) Do you think negative mass could exist? If there is negative mass, would two negative masses attract or repel?

Which Force is Stronger—Electrical or Gravitational?

Gravitational forces hold the planets in our solar system in orbit and account for the motions of matter in galaxies. Electrical forces serve to hold atoms and molecules together. If we consider two of the most common fundamental particles, the electron and the proton, how do their electrical and gravitational forces compare with each other?

Let's peek into the hydrogen atom and compare the gravitational force on the electron due to interaction of its mass with that of the proton to the electrical force between the two particles as a result of their charge. In order to do the calculation you'll need to use some well known constants.

Electron: $m_e = 9.1 \times 10^{-31}$ kg, $q_e = -1.6 \times 10^{-19}$ C

Proton: $m_p = 1.7 \times 10^{-27}$ kg, $q_p = +1.6 \times 10^{-19}$ C

Distance between the electron and proton: $r = 0.53 \times 10^{-10}$ m

Activity 3: The Electrical *vs.* the Gravitational Force in the Hydrogen Atom

(a) Calculate the magnitude of the electrical force on the electron. Is it attractive or repulsive?

(b) Calculate the magnitude of the gravitational force on the electron. Is it attractive or repulsive?

(c) Which is larger? By what factor (i.e. what is the ratio)?

(d) Which force are you more aware of on a daily basis? If your answer does not agree with the result of part (c), explain why, i.e. why do we usually not experience electrical forces in our everyday lives?

Activity 4: The Gravitational Force of the Earth

(a) Use Newton's law of universal gravitation to show that the magnitude of the acceleration due to gravity on an object of mass m at a height h above the surface of the earth is given by the following expression:

$$\frac{GM_e}{(R_e + h)^2}$$

Hint: Because of the spherical symmetry of the Earth you can treat the mass of the Earth as if it were all concentrated at a point at the Earth's center.

(b) Calculate the acceleration due to gravity of a mass m at the surface of the earth (h=0). The radius of the earth is $R_e \approx 6.38 \times 10^3$ km and its mass $M_e \approx 5.98 \times 10^{24}$ kg. Does the result look familiar? How is this acceleration related to the gravitational acceleration g?

(c) Use the equation you derived in part (a) to calculate the acceleration due to gravity at the ceiling of the room you are now in. How does it differ from the value at the floor? Can you measure the difference in the lab using the devices available?

(d) Suppose you travel *halfway* to the moon. What is the new value of the acceleration due to gravity (neglecting the effect of the moon's pull)? (Recall that the earth-moon distance is about 384,000 km.)

(e) Is the gravitational acceleration "constant," g, really a constant? Explain.

(f) In part (d) you showed that there is a significant gravitational attraction halfway between the earth and the moon. Why, then, do astronauts experience "weightlessness" when they are orbiting a mere 120 km above the earth?

Lab 16 Centripetal Force[1]

Name: ______________________ Lab Partner(s): ______________________

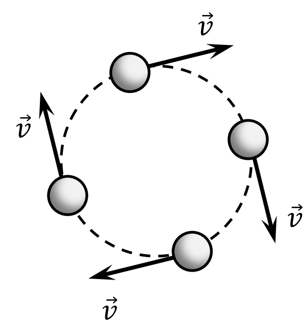
Objective

To explore the phenomenon of uniform circular motion and the accelerations and forces needed to maintain it.

Overview

In a previous unit you began the study of the application of Newton's laws to projectile motion. In this unit we are going to consider the application of Newton's laws to another phenomenon in two dimensions. Since Newton's laws can be used to predict types of motion or the conditions for no motion, their applications are useful in many endeavors including human body motion, astrophysics, and engineering.

You will explore uniform circular motion, in which an object moves at a constant speed in a circle. In particular, you will develop a mathematical description of centripetal acceleration and the force needed to keep an object moving in a circle.



Apparatus

- An airplane.
- A video analysis system (*Tracker*).
- A spring scale.
- Graphing software (*Excel*).

Moving in a Circle at a Constant Speed

When a race car speeds around a circular track, or when David twirled a stone at the end of a rope to clobber Goliath, or when a planet like Venus orbits the sun, they undergo uniform circular motion. Understanding the forces which govern orbital motion has been vital to astronomers in their quest to understand the laws of gravitation.

But we are getting ahead of ourselves, for as we have done in the case of linear and projectile motion we will begin our study by considering situations involving external applied forces that lead to circular motion in the absence of friction. We will then use our belief in Newton's laws to see how the circular motions of the planets can be used to help astronomers discover the laws of gravitation.

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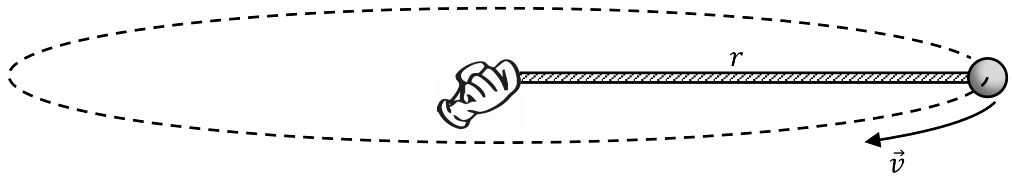


Let's begin our study with some very simple considerations. Suppose an astronaut goes into outer space, ties a ball to the end of a rope, and spins the ball so that it moves at a constant speed.

Activity 1: Uniform Circular Motion

(a) Consider the figure above. What is the speed of a ball that moves in a circle of radius $r = 2.5$ m if it takes 0.50 s to complete one revolution?

(b) The speed of the ball is constant! Would you say that this is accelerated motion?

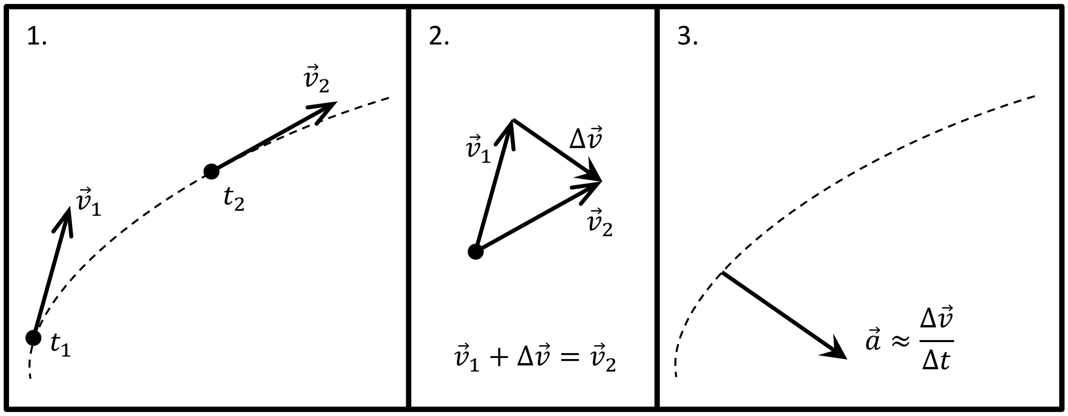
(c) What is the definition of acceleration? (Remember that acceleration is a vector!)

(d) Are velocity and speed the same thing? Is the velocity of the ball constant? (Hint: Velocity is a vector quantity!)

(e) In light of your answers to (c) and (d), would you like to change your answer to part (b)? Explain.

Using Vectors to Diagram How Velocity Changes

By now you should have concluded that since the direction of the motion of an object undergoing uniform circular motion is constantly changing, its velocity is also changing and thus it is accelerating. We would like you to figure out how to calculate the direction of the acceleration and its magnitude as a function of the speed v of an object such as a ball as it revolves and as a function of the radius of the circle in which it revolves. In order to use vectors to find the direction of velocity change in circular motion, let's review some rules for adding velocity vectors.

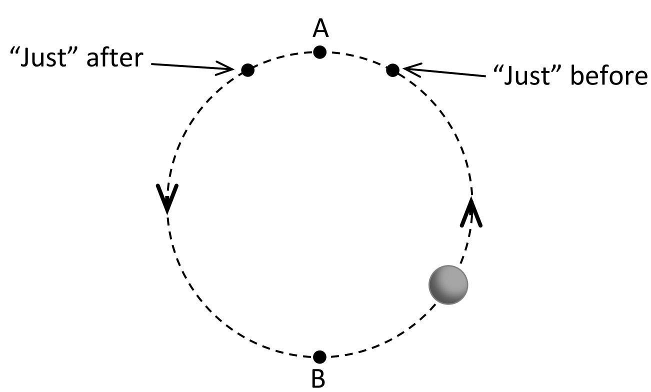


1. To Draw Velocities: Draw an arrow representing the velocity, $\vec{v}_1$, of the object at time t_1. Draw another arrow representing the velocity, $\vec{v}_2$, of the object at time t_2.
2. To Draw Velocity Change: Find the change in the velocity $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$ during the time interval described by $\Delta t = t_2 - t_1$. Start by using the rules of vector sums to rearrange the terms so that $\vec{v}_1 + \Delta\vec{v} = \vec{v}_2$. Next place the tails of the two velocity vectors together halfway between the original and final location of the object. The change in velocity is the vector that points from the head of the first velocity vector to the head of the second velocity vector.
3. To Draw Acceleration: The acceleration equals the velocity change $\Delta\vec{v}$ divided by the time interval t needed for the change. Thus, $\vec{a}$ is in the same direction as $\Delta\vec{v}$ but is a different length (unless $\Delta t = 1$). Thus, even if you do not know the time interval, you can still determine the direction of the acceleration because it points in the same direction as $\Delta\vec{v}$.

The acceleration associated with uniform circular motion is known as centripetal acceleration. You will use the vector diagram technique described above to find its direction.

Activity 2: The Direction of Centripetal Acceleration

(a) Determine the direction of motion of the ball shown below if it is moving counter-clockwise at a constant speed. Note that the direction of the ball's velocity is always tangential to the circle as it moves around. Draw an arrow representing the direction and magnitude of the ball's velocity as it passes the dot just before it reaches point A. Label this vector $\vec{v}_1$.



(b) Next, use the same diagram to draw the arrow representing the velocity of the ball when it is at the dot just after it passes point A. Label this vector $\vec{v}_2$.

(c) Find the direction and magnitude of the change in velocity as follows. In the space below, make an exact copy of both vectors, placing the tails of the two vectors together. Next, draw the vector that must be added to vector

$\vec{v}_1$ to add up to vector $\vec{v}_2$; label this vector $\Delta\vec{v}$. Be sure that vectors $\vec{v}_1$ and $\vec{v}_2$ have the same magnitude and direction in this drawing that they had in your drawing in part (a)!

(d) Now, draw an exact copy of $\Delta\vec{v}$ on your sketch in part (a). Place the tail of this copy at point A. Again, make sure that your copy has the exact magnitude and direction as the original $\Delta\vec{v}$ in part (c).

(e) Now that you know the direction of the change in velocity, what is the direction of the centripetal acceleration, $\vec{a}_c$?

(f) If you re did the analysis for point B at the opposite end of the circle, what do you think the direction of the centripetal acceleration, $\vec{a}_c$), would be now?

(g) As the ball moves on around the circle, what is the direction of its acceleration?

(h) Use Newton's second law in vector form ($\sum \vec{F} = m\vec{a}$) to describe the direction of the net force on the ball as it moves around the circle.

(i) If the ball is being twirled around on a string, what is the source of the net force needed to keep it moving in a circle?

Using Mathematics to Derive How Centripetal Acceleration Depends on Radius and Speed

You haven't done any experiments yet to see how centripetal acceleration depends on the radius of the circle and the speed of the object. You can use the rules of mathematics and the definition of acceleration to derive the relationship between speed, radius, and magnitude of centripetal acceleration.

Activity 3: How Does $\vec{a}_{\mathrm{c}}$ Depend on $\vec{v}$ and $\vec{r}$?

(a) Do you expect you would need more centripetal acceleration or less centripetal acceleration to cause an object moving at a certain speed to rotate in a smaller circle? In other words, would the magnitude, a_{c}, have to increase or decrease as r decreases if circular motion is to be maintained? Explain.

(b) Do you expect you would need more centripetal acceleration or less centripetal acceleration to cause an object to rotate at a given radius r if the speed v is increased? In other words, would the magnitude, a_{c}, have to increase or decrease as v increases if circular motion is to be maintained? Explain.

You should have guessed that it requires more acceleration to move an object of a certain speed in a circle of smaller radius and that it also takes more acceleration to move an object that has a higher speed in a circle of a given radius. Lets use the definition of acceleration in two dimensions and some accepted mathematical relationships to show that the magnitude of centripetal acceleration should actually be given by the equation

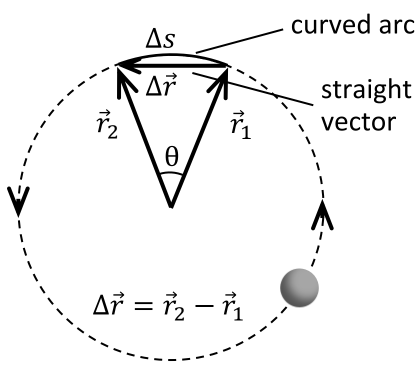
$$a_{\mathrm{c}} = \frac{v^2}{r} \qquad [Eq.\,1]$$

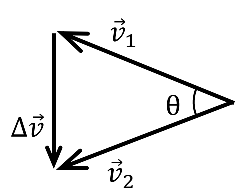
In order to do this derivation you will want to use the following definition for acceleration

$$\vec{a}_{\mathrm{avg}} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta\vec{v}}{\Delta t} \qquad [Eq.\,2]$$

Activity 4: Finding the Equation for a_{c}

(a) Refer to the diagram below. Explain why, at the two points shown on the circle, the angle between the position vectors at times t_1 and t_2 is the same as the angle between the velocity vectors at times t_1 and t_2. Hint: In circular motion, velocity vectors are always perpendicular to their position vectors.





(b) Since the angles are the same and since the magnitudes of the displacements never change (i.e., $r = r_1 = r_2$) and the magnitudes of the velocities never change (i.e., $v = v_1 = v_2$), use the properties of similar triangles to explain why $\frac{\Delta v}{v} = \frac{\Delta r}{r}$.

(c) Now use the equation in part (b) and the definition of a_{avg} to show that $(a_{\mathrm{c}})_{\mathrm{avg}} = \frac{\Delta v}{\Delta t} = \frac{(\Delta r)}{(\Delta t)}\frac{v}{r}$.

(d) The speed of the object as it rotates around the circle is given by $v = \frac{\Delta s}{\Delta t}$. Is the change in arc length, Δs, larger or smaller than the magnitude of the change in the position vector, Δr? Explain why the arc length change and the change in the position vector are approximately the same when t is very small (so that the angle θ becomes very small) i.e., why is $\Delta s \simeq \Delta r$?

(e) If $\Delta s \simeq \Delta r$, then what is the equation for the speed in terms of Δr and Δt?

(f) Using the equation in part (c), show that as $\Delta t \to 0$, the instantaneous value of the centripetal acceleration is given by Eq. 1.

(g) If the object has a mass m, what is the equation for the magnitude of the centripetal force needed to keep the object rotating in a circle (in terms of v, r, and m)? In what direction does this force point as the object rotates in its circular orbit?

Experimental Test of the Centripetal Force Equation

The theoretical considerations in the last activity should have led you to the conclusion that, whenever you see an object of mass m moving in a circle of radius r at a constant speed v, it must at all times be experiencing a net centripetal force directed toward the center of the circle that has a magnitude of

$$F_{\mathrm{c}} = ma_{\mathrm{c}} = m\frac{v^2}{r} \qquad [Eq.\,3]$$

Let's check this out. Does this rather odd equation really work for an external force?

To test the validity of the derivation we must compare it to experience. We will use a "toy" airplane suspended from a string and flying in a circular path. We will use the video analysis system to measure the properties of the motion and determine the horizontal and vertical components of the force exerted on the airplane using Equation 3. We will compare that result with a direct measurement of the tension in the string.

Activity 5: Verifying the F_c Equation

(a) Measure the mass of the airplane and the length L of the portion of the string that hangs below the horizontal post with the hole in it. Record the values here:

(b) The airplane is suspended from a spring scale. The string should pass straight down from the scale through the small hole in the horizontal post. The camera should be placed 1 to 2 meters above the airplane looking down. Turn on the camera and center the spring scale in the frame. In order to turn on the camera, you will have to open "**Camera**" (see **Appendix B: Video Analysis Using Tracker**). Check that the reading of the spring scale is consistent with the mass of the airplane. Launch the plane into uniform circular motion. When the motion is steady record a movie of at least one complete revolution and record the reading of the spring scale here:

$F_{\text{scale}} =$

(c) To analyze the movie, use *Tracker* (see **Appendix B: Video Analysis Using Tracker**). Set the origin of coordinate axes at the axis of revolution of the plane. After your plane has made one complete revolution, make a graph of x vs t. Double click on the graph to expand it, print the graph and include with this unit. Read the period of revolution for the complete cycle and record it here:

$t_{\text{rev}} =$

(d) Next, make a graph of y vs x. The result should be a circle. Double click on the graph to expand it. Determine the diameter of the circle by measuring it at several places on the graph and averaging them. Record the average radius of the path here:

$r =$

Print the graph and include with this unit.

(e) To test the validity of the expression for F_c we must know the speed. Use the measurements of the radius of the airplane's trajectory and the time for one complete revolution to calculate the average speed.

$v_{\text{ave}} =$

(f) Use your results for the mass, the average velocity of the airplane, and the radius of the circular motion to predict the centripetal force exerted by the string.

$F_c =$

(g) We will now determine the net force exerted by the string on the airplane by using a little trigonometry. Recall that we know r, the radius of the airplane's circular path, and L, the total length of the string that is actually rotating.

1. Using these two distances (r and L), calculate the angle the string makes with the horizontal. You may want to draw a diagram to do this.

   $\theta =$

2. We determined the horizontal component of the force on the airplane F_c in part (f). Knowing the angle θ we can determine an expression for the net force exerted by the string on the airplane, F_{plane}. Draw a vector force diagram including F_c and F_{plane}, derive an expression for F_{plane}, calculate it, and record it here:

   $F_{\text{plane}} =$

(h) Compare your result for F_{plane} with the measurement of the spring scale F_{scale}. Should they be different? Should they be the same? Explain.

(i) Calculate the percent difference between F_{plane} and F_{scale} ($F_{\text{plane}} - F_{\text{scale}}$)and record it below. Go around to the other lab groups and get their results for this quantity. Make a histogram of the results you collect and calculate the average and standard deviation. For information on making histograms, see **Appendix D**. For information on calculating the average and standard deviation, see **Appendix E**. Record the average and standard deviation here. Attach the histogram to this unit.

(j) What is your expectation for the difference $F_{\text{plane}} - F_{\text{scale}}$? Do the data from the class support this expectation? Use the average and standard deviation for the class to quantitatively answer this question.

(k) What does the histogram of the class data tell you?

(l) Discuss the major sources of uncertainty in this experiment.

Lab 17 Work and Power[1]

Name: ______________________ Lab Partner(s): ______________________

Objectives

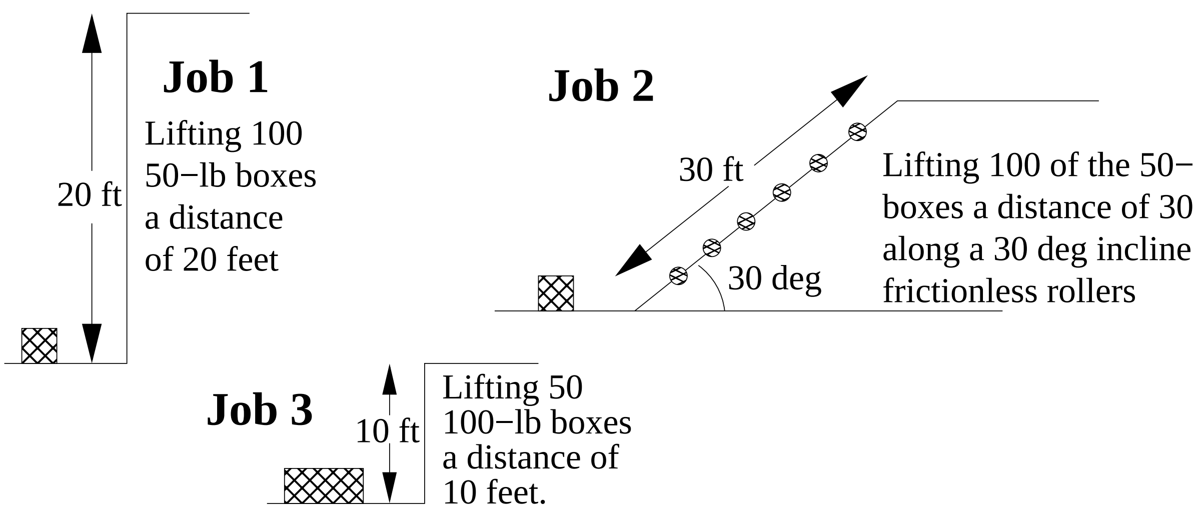
- To extend the intuitive notion of work as physical effort to a formal mathematical definition of work as a function of force and distance.
- To understand the concept of power and its relationship to work.

Apparatus

- 5.0 newton spring scale
- Wooden block with hook
- Variety of masses
- meter stick
- stop watch

The Concept of Physical Work

Suppose you are president of the Richmond Load 'n' Go Co. A local college has three jobs available and will allow you to choose which one you want before offering the other two jobs to rival companies. All three jobs pay the same amount of money. Which one would you choose for your crew?



Activity 1: Choosing Your Job

Examine the descriptions of the jobs shown in figure above. Which one would you be most likely to choose? Least likely to choose? Explain the reasons for your answer.

You obviously want to do the least amount of work for the most money. Before you reconsider your answers later

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in this unit, you should do a series of activities to get a better feel for what physicists mean by work and how the president of Load 'n' Go can make top dollar.

In everyday language we refer to doing work whenever we expend effort. In order to get an intuitive feel for how we might define work mathematically, you should experiment with moving your textbook back and forth along a table top and a rougher surface such as a carpeted floor.

Activity 2: This is Work!

(a) Pick a distance of a meter or so. Sense how much effort it takes to push a heavy book that distance. How much more effort does it take to push it twice as far?

(b) Pile another similar book on top of the original one and sense how much effort it takes to push the two books through the distance you picked. Comment below.

(c) From your study of sliding friction, what is the relationship between the mass of a sliding object and the friction force it experiences? On the basis of your experience with sliding friction, estimate how much more force you have to apply to push two books compared to one book.

(d) If the "effort" it takes to move an object is associated with physical work, guess an equation that can be used to define work mathematically when the force on an object and its displacement (i.e., the distance it moves) lie along the same line.

In physics, work is not simply effort. In fact, the physicist's definition of work is precise and mathematical. In order to have a full understanding of how work is defined in physics, we need to consider its definition in a very simple situation and then enrich it later to include more realistic situations.

A Simple Definition of Physical Work: If an object that is moving in a straight line experiences a constant force in the direction of its motion during the time it is undergoing a displacement, the work done by the external force, F_{ext}, is defined as the product of the force and the displacement of the object,

$$W_{\text{ext}} = F_{\text{ext}} \Delta x$$

where W_{ext} represents the work done by the external force, F_{ext} is the magnitude of the force, and Δx is the displacement of the object.

What if the force of interest and the displacement are in opposite directions? For instance, what about the work done by the force of sliding friction, f_k, when a block slides on a rough surface? The work done by the friction force is dissipative; thus,

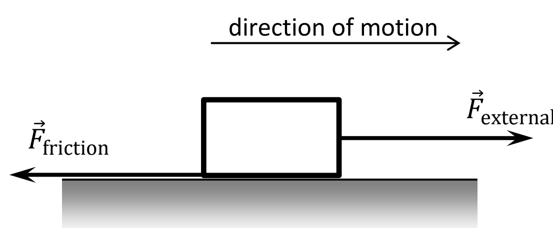
$$W_{\text{diss}} = -f_k \Delta x$$

Activity 3: Applying the Physics Definition of Work

(a) Does effort necessarily result in physical work? Suppose two guys are in an evenly matched tug of war. They are obviously expending effort to pull on the rope, but according to the definition of physical work, are they doing any physical work? Explain.

(b) A wooden block with a mass of .30 kg is pushed along a sheet of ice that has no friction with a constant external force of 10 N which acts in a horizontal direction. After it moves a distance of 0.40 m how much work has been done on the block by the external force?

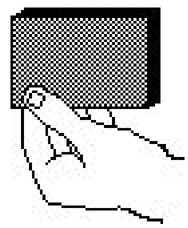
(c) The same wooden block with a mass of .30 kg is pushed along a table with a constant external force of 10 N which acts in a horizontal direction. It moves a distance of 0.40 m. However, there is a friction force opposing its motion. Assume that the coefficient of sliding friction, μ_k, is 0.20.



1. According to the definition of work done by a force, what is the work associated with the external force? Is the work positive or negative? Show your calculation. Note: This is the same as part (b) above.

2. According to our discussion above of the work done by a friction force, what is the work associated with the friction force? Is the work positive or negative? Show your calculation. (See the equation above, just before Activity 3.)

(d) Suppose you lift a 0.3 kg object through a vertical distance of 1.0 m at a constant velocity.

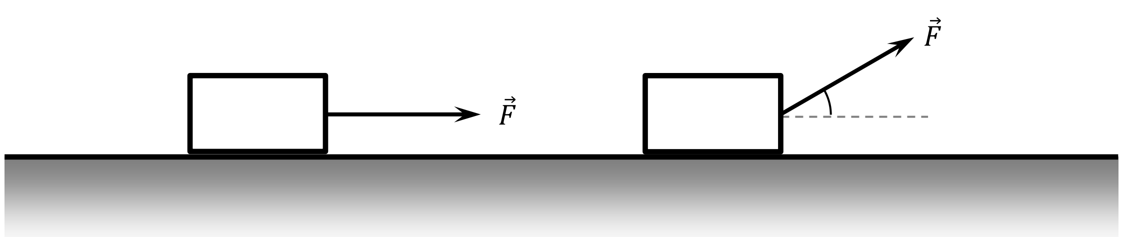


1. What is the work associated with the force that the earth exerts on the object? Is the work positive or negative? Show your calculation.

2. What is the work associated with the external force you apply to the object? Is the work positive or negative? Show your calculation.

Pulling at an Angle: What Happens When the Force and the Displacement Are Not Along the Same Line?

Let's be more quantitative about measuring force and distance and calculating the work. How should work be calculated when the external force and the displacement of an object are not in the same direction?



To investigate this, you will use a spring scale to measure the force necessary to slide a block along the table at a constant speed. Before you make your simple force measurements, put some weights on your block so that it slides along a smooth surface at a constant velocity even when it is being pulled with a force that is 30 or 60 degrees from the horizontal.

Activity 4: Calculating Work

(a) Hold a spring scale horizontal to the table and use it to pull the block (with 500 grams on it) a distance of 0.5 meters along the horizontal surface in such a way that the block moves at a constant speed. Record the force in newtons and the distance in meters in the space below and calculate the work done on the block in joules. (Note that there is a special unit for work, the joule, or J for short. One joule is equal to one newton times one meter, i.e., $\mathrm{J} = \mathrm{N\,m}$.)

(b) Repeat the measurement, only this time pull on the block at a 30° angle with respect to the horizontal. Pull the block at about the same speed for the same distance as in part (a). Is the force needed larger or smaller than you measured in part (a)? Write down the force in newtons:

(c) Repeat the measurement once more, this time pulling the block at a 60° angle with respect to the horizontal. Pull the block at about the same speed and the same distance as before. Write down the force in newtons:

(d) The distances the block moved in parts (a), (b), and (c) were all the same. Should the work done in the three cases be the same? Use your data to postulate a mathematical equation that relates the physical work, W, to the magnitude of the applied force, F, the magnitude of the displacement, Δs, and the angle, θ, between F and Δs. Calculate the the work done in the three cases using your equation. Hint: sin 30° = .500, sin 60° = .866, cos 30° = .866, cos 60° = .500.

Are the three results the same? If not, can you think of a reason why not?

Work as a Dot Product

Review the definition of dot (or scalar) product as a special product of two vectors in your textbook, and convince yourself that the dot product can be used to define physical work in general cases when the force is constant but not necessarily in the direction of the displacement resulting from it.

$$W = \vec{F} \cdot \Delta\vec{s}$$

Activity 5: How Much Work Goes with Each Job?

(a) Re-examine the descriptions of the jobs shown in the figure on the first page of this experiment. What is the minimum physical work done in job 1?

(b) What is the minimum physical work done in job 2? Get the angle right. Remember that even though you are going up an inclined plane, the force you exert to counteract gravity is straight upward.

(c) What is the minimum physical work done in job 3?

(d) Was your original intuition about which job to take correct? Which job should Richmond Load 'n' Go try to land?

The Concept of Power

People are interested in more than physical work. They are also interested in the rate at which physical work can be done. Average power, $\langle P \rangle$, is defined as the ratio of the amount of work done, ΔW, to the time interval, Δt, it takes to do the work, so that

$$P_{\text{avg}} = \frac{\Delta W}{\Delta t}.$$

Instantaneous power is given by the derivative of work with respect to time, or

$$P = \frac{dW}{dt}.$$

If work is measured in joules and time in seconds then the fundamental unit of power is in joules/second where 1 joule/second equals one watt. However, a more traditional unit of power is the horsepower, which represents the rate at which a typical work horse can do physical work. It turns out that *1 horsepower (or hp) = 746 watts = 746 joules/second.*

Those of you who are car buffs know that horsepower is a big deal in rating high performance cars. The hp in a souped-up car is in the hundreds. How does your stair climbing ability stack up? Let's see how long it takes you to climb the two stories of stairs in the science center.

Activity 6: Rate the Horsepower in Your Legs

(a) Determine the time it takes you to climb the two flights of stairs in the science center. Then measure the height of the climb and compute the work done against the force of gravity.

(b) Compute the average power, P_{avg}, you expended in hp. How does this compare to the horsepower of your favorite automobile? If you're not into cars, how do you stack up against a horse?

Lab 18 Work and Kinetic Energy[1]

Name: ______________________ Lab Partner(s): ______________________

Objectives

- To discover Hooke's law.
- To understand the concept of kinetic energy and its relationship to work as embodied in the work-energy theorem.
- To develop an understanding of the physical significance of mathematical integration.

The Force Exerted on a Mass by an Extended Spring

So far we have pushed and pulled on an object with a constant force and calculated the work needed to displace that object. In most real situations the force on an object can change as it moves.

What happens to the average force needed to stretch a spring from 0 to 1 cm compared to the average force needed to extend the same spring from 10 to 11 cm? How does the applied force on a spring affect the amount by which it stretches, i.e., its displacement?

Apparatus

- A large spring
- A support rod to hang the spring
- A 2-meter stick
- A variety of masses

Activity 1: Are Spring Forces Constant?

Hang the spring from a support rod with the large diameter coils in the downward position. Extend the spring from 0 to 1 cm. Feel the force needed to extend the spring. Extend the spring from 10 to 11 cm. Feel the force needed to extend the spring again. How do the two forces compare? Are they the same?

The Force and Work Needed to Stretch a Spring

Now we would like to be able to quantify the force and work needed to extend a spring as a function of its displacement from an equilibrium position (i.e., when it is "unstretched").

Activity 2: Force *vs.* Displacement for a Spring

(a) Measure the distance from the floor to the lower end of the spring and record this distance as s_0 below. Start filling in the first four columns of the data table on the next page. (Alternatively, this data table can be created directly in *Excel*. If you use *Excel*, print the data table and attach it to this lab.) Hang different masses on the spring in 0.100 kg increments up to 1.000 kg and calculate the external gravitational force F_{ext} applied to the spring. For each mass, measure the distance s from the floor to the lower end of the spring, and also calculate the stretch of the spring x ($= s_0 - s$).

$s_0 =$

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m (kg)	s (m)	x (m)	Δx (m)	F_{ext} (N)	$\langle F_{\text{ext}} \rangle$ (N)	ΔW (J)	W_{total} (J)
0.0		0.0	-	0.0	-	-	0.0
0.1							
0.2							
0.3							
0.4							
0.5							
0.6							
0.7							
0.8							
0.9							
1.0							

(b) Using *Excel*, create a graph of F_{ext} (vertical axis) *vs.* x (horizontal axis). Is the graph linear? If the force, F_{ext}, increases with the displacement in a proportional way, fit the data with a linear function including a trendline with equation. Print the graph and include a copy with this unit. Use the LINEST function in *Excel* (see **Appendix D: Excel**) to determine the slope of the line and its uncertainty. Use the symbol k to represent the slope of the line. Record your value of k and its uncertainty here. What are its units? Note: k is known as the spring constant.

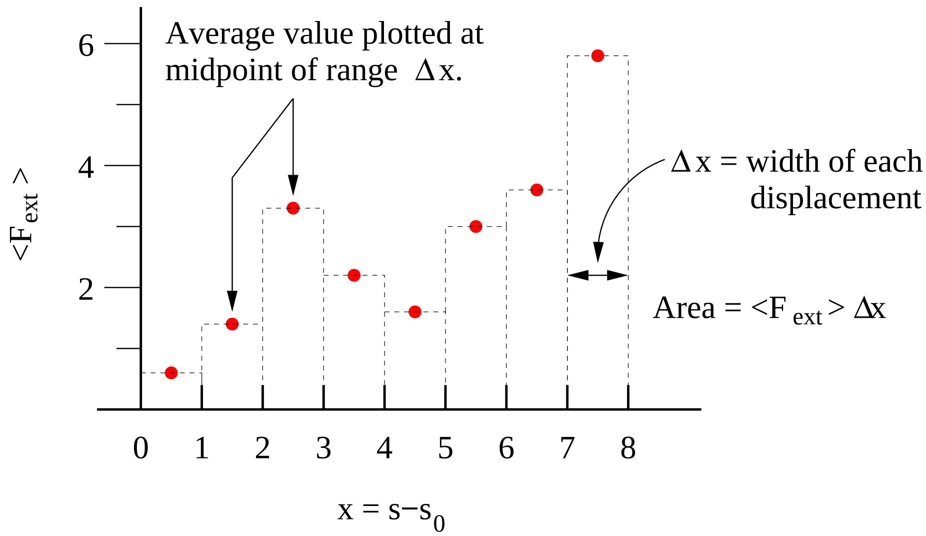
(c) Write the equation describing the relationship between the external force, F_{ext}, and the total displacement, x, of the spring from its equilibrium using the symbols F_{ext}, k, and x.

Note: Any restoring force on an object which is proportional to its displacement is known as a Hooke's Law Force. There was an erratic, contentious genius named Robert Hooke who was born in 1635. He played with springs and argued with Newton.

Calculating Work when the Force is not Constant

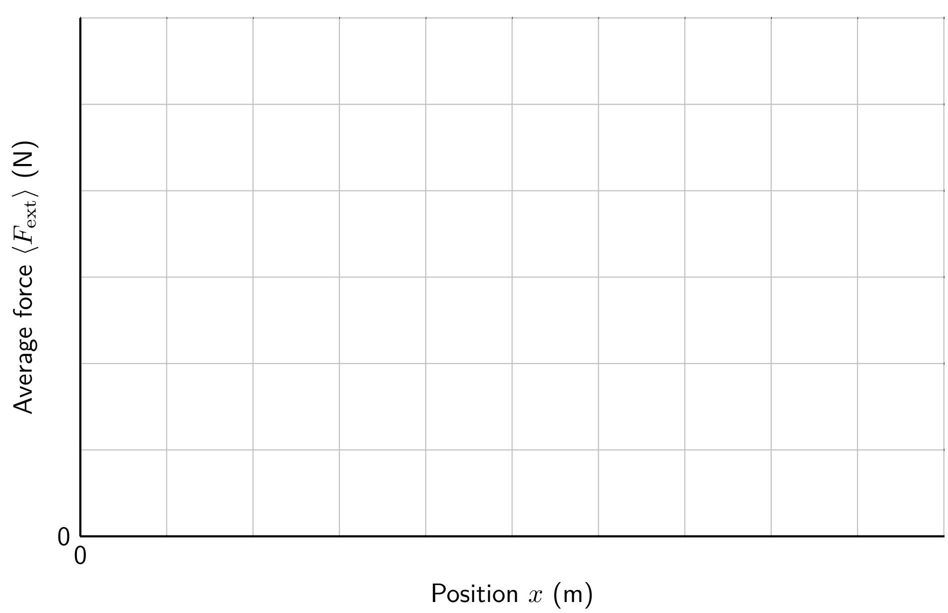
We would like to expand the definition of work so it can be used to calculate the work associated with stretching a spring and the work associated with other forces that are not constant. A helpful approach is to plot the average force needed to move an object for each successive displacement Δx as a bar graph like that shown in the figure below. The figure shows a graph representing the average applied force causing each unit of displacement of an object. This graph represents force that is not constant but not the force *vs.* displacement of a typical spring.

Note: The bar graph below is intended to illustrate mathematical concepts. Any similarity between the values of the forces in the bar graph and any real set of forces is purely coincidental. In general, the force causing work to be done on an object is not constant.



Activity 3: Force *vs.* Distance in a Bar Graph

(a) Using your data from Activity 2, calculate the width of each displacement, Δx, and the average external force, $\langle F_{\text{ext}} \rangle$ for each displacement, and record the values in the table above. This means for each mass take an average of the force value and the previous force value. You now have columns 5 and 6 filled in. Plot $\langle F_{\text{ext}} \rangle$ *vs.* x as a bar graph on the grid below. (Choose appropriate scales for the axes before making the graph.) Alternatively, you can plot the bar graph in *Excel*.



How can we calculate the work done in stretching the spring? We can use several equivalent techniques: (1) adding up little pieces of $\langle F_{\text{ext}} \rangle \Delta x$ from the above bar graph, (2) finding the area under the "curve" you created in Activity 2, or (3) using mathematical integration.

All three methods should yield about the same result. If you have not yet encountered integrals in a calculus course, you can compare the results of using the first two methods. If you have studied integrals in calculus you may want to consult your instructor or the textbook about how to set up the appropriate definite integral to calculate the work needed to stretch the spring.

Activity 4: Calculation of Work

(a) Calculate the work needed to stretch the spring by calculating small increments of $\langle F_{\text{ext}} \rangle \Delta x$ (this is ΔW, or column 7 in your table). Record the running sum in the table (this is W_{total}, or column 8) for each mass. (The "running sum" is the sum of all the values of ΔW up to that point.) Indicate the final value of W_{total} below. Don't forget to specify units.

$W_{\text{total}} =$

(b) Calculate the work needed to stretch the spring by computing the area under the curve in the graph of F_{ext} *vs.* x that you created in Activity 2.

(c) Calculate the work needed to stretch the spring by computing the definite integral of the function you found in Activity 2(c), evaluated from the initial x value to the final x value.

(d) Do the three values of the work done agree with one another? How would you account for any discrepancy?

Defining Kinetic Energy and Its Relationship to Work

What happens when you apply an external force to an object that is free to move and has no friction forces on it? Obviously it should experience an acceleration and end up being in a different state of motion. Can we relate the change in motion of the object to the amount of work that is done on it?

Let's consider a fairly simple situation. Suppose an object is lifted through a distance s near the surface of the earth and then allowed to fall. During the time it is falling it will experience a constant force as a result of the attraction between the object and the earth glibly called the force of gravity. You can use the theory we have already developed for the gravitational force to compare the velocity of the object to the work done on it by the gravitational field as it falls through a distance y. This should lead naturally to the definition of a new quantity called kinetic energy, which is a measure of the amount of "motion" gained as a result of the work done on the mass.

Activity 5: Equations for Falling v *vs.* y

(a) An object of mass m is dropped near the surface of the earth. What are the magnitude and direction of its acceleration g?

(b) If the object has no initial velocity and is allowed to fall for a time t under the influence of the gravitational force, what kinematic equation describes the relationship between the distance the object falls, y, and its time of fall, t? Assume $y_0 = 0$ and take positive down.

(c) Do you expect the magnitude of the velocity to increase, decrease or remain the same as the distance increases? Note: This is an obvious question!!

(d) Differentiate the equation you wrote down in part (b) (i.e. find $v = dy/dt$) to find a relationship between v, the acceleration g, and time t.

(e) Eliminate t from the equations you obtained in parts (b) and (d) to get an expression that describes how the velocity, v, of the falling object depends on the distance, y, through which it has fallen.

You can use the kinematic equations to derive the functional relationship you hopefully discovered experimentally in the last activity. If we define the kinetic energy (K) of a moving object as the quantity $K = \frac{1}{2}mv^2$, then we can relate the change in kinetic energy as an object falls to the work done on it. Note that for an object initially at rest the initial kinetic energy is $K_i = 0$, so the change in kinetic energy is given by the difference between the initial and final kinetic energies. $\Delta K = K_f - K_i = \frac{1}{2}mv^2$.

Activity 6: Computing Work and Kinetic Energy of a Falling Mass

(a) Suppose the mass of your falling object is 0.35 kg. What is the value of the work done by the gravitational force when the mass is dropped through a distance of $y = 1.2$ m?

(b) Use the kinematic equation you derived in Activity 5(e) that relates v and y to find the velocity of the falling object after it has fallen 1.2 m.

(c) What is the kinetic energy of the object before it is dropped? After it has fallen 1.2 m? What is the change in kinetic energy, ΔK, as a result of the fall?

(d) How does the work done by the gravitational force compare to the kinetic energy change, ΔK, of the object?

Activity 7: The Mathematical Relationship between Work and Kinetic Energy Change During a Fall

(a) Since our simplified case involves a constant acceleration, write down the equation you derived in Activity 5(e) to describe the speed, v, of a falling object as a function of the distance y which it fell.

(b) Using the definition of work, show that $W = mgy$ when the object is dropped through a distance y.

(c) By combining the equations in parts (a) and (b) above, show that in theory the work done on a mass falling under the influence of the gravitational attraction exerted on it by the earth is given by the equation $W = \Delta K$.

You have just proven an example of the work-kinetic energy theorem which states that the change in kinetic energy of an object is equal to the net work done by all the forces acting on it.

$$W = \Delta K \qquad \text{[Work-Kinetic Energy Theorem]}$$

Although you have only shown the work-kinetic energy theorem for a special case where no friction is present, it can be applied to any situation in which the net force can be calculated. For example, the net force on an object might be calculated as a combination of applied, spring, gravitational, and friction forces.

Lab 19 Conservation of Mechanical Energy[1]

Name: ______________________ Lab Partner(s): ______________________

Objectives

- To understand the concept of potential energy.
- To investigate the conditions under which mechanical energy is conserved.

Overview

The last unit on work and energy culminated with a mathematical proof of the work-kinetic energy theorem for a mass falling under the influence of the force of gravity. We found that when a mass starts from rest and falls a distance y, its final velocity can be related to y by the familiar kinematic equation

$$v_f^2 = v_i^2 + 2gy \qquad \text{or} \qquad gy = \frac{1}{2}\left(v_f^2 - v_i^2\right) \qquad [Eq.\,1]$$

where v_f is the final velocity and v_i is the initial velocity of the mass.

We believe this equation is valid because: (1) you have derived the kinematic equations mathematically using the definitions of velocity and constant acceleration, and (2) you have verified experimentally that masses fall at a constant acceleration. We then asked whether the transformation of the mass from a speed v_i to a speed v_f is related to the work done on the mass by the force of gravity as it falls.

The answer is mathematically simple. Since $F_{\mathrm{G}} = mg$, the work done on the falling object by the force of gravity is given by

$$W_{\mathrm{G}} = F_{\mathrm{G}}y = mgy \qquad [Eq.\,2]$$

But according to Equation 1, $gy = \frac{1}{2}v_f^2 - \frac{1}{2}v_i^2$, so we can re-write Equation 2 as

$$W_{\mathrm{G}} = mgy = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \qquad [Eq.\,3]$$

The $\frac{1}{2}mv_f^2$ is a measure of the motion resulting from the fall. If we define it as the energy of motion, or, more succinctly, the kinetic energy, we can define a work-kinetic energy theorem for falling objects:

$$W = \Delta K \qquad [Eq.\,4]$$

or, the work done on a falling object by the earth is equal to the change in its kinetic energy as calculated by the difference between the final and initial kinetic energies.

If external work is done on the mass to raise it through a height y (a fancy phrase meaning "if some one picks up the mass"), it now has the potential to fall back through the distance y, gaining kinetic energy as it falls. Aha! Suppose we define *potential energy* to be *the amount of external work, W_{ext}, needed to move a mass at constant velocity through a distance y against the force of gravity.* Since this amount of work is positive while the work done by the gravitational force has the same magnitude but is negative, this definition can be expressed mathematically as

$$U = W_{\mathrm{ext}} = mgy \qquad [Eq.\,5]$$

Note that when the potential energy is a maximum, the falling mass has no kinetic energy but it has a maximum potential energy. As it falls, the potential energy becomes smaller and smaller as the kinetic energy increases. The kinetic and potential energy are considered to be two different forms of mechanical energy. What about the total mechanical energy, consisting of the sum of these two energies? Is the total mechanical energy constant

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during the time the object falls? If it is, we might be able to hypothesize a law of conservation of mechanical energy as follows: *In some systems, the sum, E, of the kinetic and potential energy is a constant.* This hypothesis can be summarized mathematically by the following statement.

$$E = K + U = \text{constant} \qquad [Eq.\,6]$$

The idea of mechanical energy conservation raises a number of questions. Does it hold quantitatively for falling masses? How about for masses experiencing other forces, like those exerted by a spring? Can we develop an equivalent definition of potential energy for the mass-spring system and other systems and re-introduce the hypothesis of conservation of mechanical energy for those systems? Is mechanical energy conserved for masses experiencing frictional forces, like those encountered in sliding?

In this unit, you will explore whether or not the mechanical energy conservation hypothesis is valid for a falling mass.

Activity 1: Mechanical Energy for a Falling Mass

Suppose a ball of mass m is dropped from a height h above the ground.

(a) Where is U a maximum? A minimum?

(b) Where is K a maximum? A minimum?

(c) If mechanical energy is conserved what can you say about the sum of $K + U$ for any point along the path of a falling mass?

Mechanical Energy Conservation

How do people in different reference frames near the surface of the earth view the same event with regard to mechanical energy associated with a mass and its conservation? Suppose the president of your college drops a 2.0-kg water balloon from the second floor of the administration building (10.0 meters above the ground). The president takes the origin of his or her vertical axis to be even with the level of the second floor. A student standing on the ground below considers the origin of his coordinate system to be at ground level. Have a discussion with your classmates and try your hand at answering the questions below.

Activity 2: Mechanical Energy and Coordinate Systems

(a) What is the value of the potential energy of the balloon before and after it is dropped according to the president? According to the student? Show your calculations and don't forget to include units!

The president's perspective is that $y = 0.0$ m at $t = 0$ s and that $y = -10.0$ m when the balloon hits the student):

$U_i =$

$U_f =$

The student's perspective is that $y = 10.0$ m at $t = 0$ s and that $y = 0.0$ m when the balloon hits:

$U_i =$

$U_f =$

Note: If you get the same potential energy value for the student and the president, you are on the wrong track!

(b) What is the value of the kinetic energy of the balloon before and after it is dropped according to the president? According to the student? Show your calculations. Hint: Use a kinematic equation to find the velocity of the balloon at ground level.

President's perspective:

$K_i =$

$K_f =$

Student's perspective:

$K_i =$

$K_f =$

Note: If you get the same values for both the student and the president for values of the kinetic energies you are on the right track!

(c) What is the value of the total mechanical energy of the balloon before and after it is dropped according to the president? According to the student? Show your calculations. Note: If you get the same values for both the student and the president for the total energies you are on the wrong track!!!!!

President's perspective:

$E_i =$

$E_f =$

Student's perspective:

$E_i =$

$E_f =$

(d) Why don't the two observers calculate the same values for the mechanical energy of the water balloon?

(e) Why do the two observers agree that mechanical energy is conserved?

Activity 3: Energy Analysis for a Falling Mass

(a) Perform a video analysis (*i.e.* get t, x, and y) of the movie entitled *conserveME7.mp4* in the *Phys131* folder to obtain the vertical position of the the ball as a function of time. Make sure you scale the graph. See Appendix B for details on video analysis.

(b) Calculate the vertical distance the ball moves during the time intervals between adjacent frames. Suppose, for example, your time data are in column `A` and your vertical position data are in column `D` and the first entry is in row 14. Click in row 14 of an empty column like column `F`. In the formula box above the spreadsheet table enter '`=D15-D14`' (don't include the quotation marks) and hit `Return`. This will calculate the distance the ball covered in going from the first entry in row 14 to the second entry in row 15. Next, click on the new entry you just made. You should see a small box in the lower right corner of the cell. Grab it and drag it straight down to the second-to-last row of your data and release. You should now see the distance covered between adjacent entries in all the rows above. If you don't see this consult your instructor. Label the column 'dy (m)'. Verify the results for one or two cells. Why did you drag down to the second-to-last cell and not the last one?

(c) Calculate the average time interval for each step. Recall our example above. If you calculated the difference in vertical position between adjacent frames in column `F` starting in row 14, then click in column `G` row 14. In the formula box above the spreadsheet table enter '`=A15-A14`' and hit `Return`. Click on the new entry you just made. You should see a small box in the lower right corner of the cell. Grab it and drag it straight down to the second-to-last row of your data and release. You should now see the time difference between adjacent frames in all the rows above. If you don't see this consult your instructor. Label the column 'dt (s)'. Verify the results for one or two cells.

(d) If you calculated the difference in vertical position between adjacent frames in column `F` starting in row 14 and the difference in time in column `G`, then click in column `H` row 14. In the formula box above the spreadsheet table enter '`=F14/G14`' and hit `Return`. What is this quantity?

(e) The step above is the calculation of the average velocity going from cell `H14` to cell `H15`. We will use this as an approximate measure of the instantaneous velocity. Last, click on the new entry you just made. You should see a small box in the lower right corner of the cell. Grab it and drag it straight down to the second-to-last row of your data and release. You should now see the velocity in all the rows above. If you don't see this consult your instructor. Label the column 'v (m/s)'. Verify the results for one or two cells.

(f) Calculate the kinetic energy for each time interval (the mass of the ball is $m = 0.058\ kg$). To do this, recall our example above. If you calculated the velocity in column `H` starting in row 14, then click in column `I` row 14. In the formula box above the spreadsheet table enter '`=0.5*0.058*H14*H14`' and hit `Return`. This will calculate the kinetic energy for cell `I14`. Last, click on the new entry you just made. You should see a small box in the lower right corner of the cell. Grab it and drag it straight down to the second-to-last row of your data and release.

You should now see the kinetic energy in all the rows above. If you don't see this consult your instructor. Label the column $KE(J)$. Verify the results for one or two cells.

(g) Calculate the potential energy PE for each frame using the same techniques you used above. Label the column $PE(J)$. Verify the results for one or two cells.

(h) Calculate the total energy $ME = KE + PE$ for each frame. Label the column $ME(J)$. Verify the results for one or two cells.

(i) Create a graph of KE, PE, and ME vs. time. Put all three on the same graph. Print the graph and attach it to this unit.

(j) Does mechanical energy appear to be conserved within experimental uncertainties? How would you quantitatively estimate the value of the experimental uncertainty? Once you establish the method apply it to your data and record the results here.

Lab 20 Conservative and Non-Conservative Forces[1]

Name: ______________________ Lab Partner(s): ______________________

Objectives

- To investigate the conditions under which mechanical energy is conserved.
- To relate conservative and non-conservative forces to the net work done by a force when an object moves in a closed loop.

Apparatus

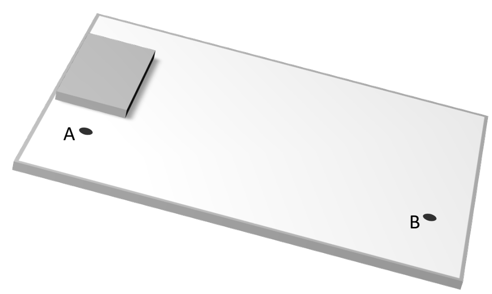
- *Capstone* software (*Force_Sensor.cap* experiment file)
- CS2000 compact scale
- Meter stick
- Wireless force sensor with hook
- Wooden block with hook
- Wooden board to use as an incline
- Stop watch
- Variety of masses

Introduction

You already know that mechanical energy is conserved for a freely falling object. Let's investigate whether mechanical energy is conserved when an object slides down an inclined plane in the presence of a friction force.

Activity 1: Is Mechanical Energy Conserved for a Sliding Block?

(a) Raise the incline until the block slides at a constant velocity once it is pushed gently to overcome the static friction force. What is the potential energy of the block at point A relative to the bottom of the ramp? Show your data and calculation.



(b) What is the potential energy change ΔU of the block when it reaches point B? Explain.

(c) Did the velocity of the block appear reasonably constant as it slid down the ramp?

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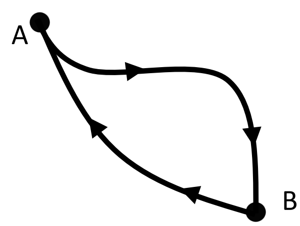
(d) Assuming the initial kinetic energy (just after your starter shove) is the same as the final kinetic energy, what is the kinetic energy change, ΔK, of the block when it reaches point B?

(e) Is mechanical energy conserved as a result of the sliding? Cite the evidence for your answer.

As you examine the activities you just completed you should conclude that the conservation of mechanical energy will probably only hold in situations where there are no friction forces present.

Conservative and Non-Conservative Forces

Physicists have discovered that certain conservative forces such as gravitational forces and spring forces do no total work on an object when it moves in a closed loop. Other forces involving friction are not conservative and hence the total work these forces do on an object moving in a closed loop is not zero. In this next activity you will try to determine the validity of this assertion.

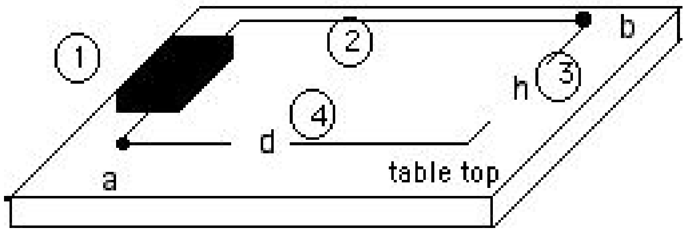


It is not hard to see that a gravitational force does no net work when an object moves at a constant speed through a complete round trip. In the above figure, it takes negative external work to lower a mass from point A to point B, as the force of gravity takes care of the work. On the other hand, raising the mass from point B to point A requires positive external work to be done against the force of gravity, and thus the net work done by the gravitational force for the complete trip is zero. When a frictional force is present it always does net work on an object as it undergoes a round trip. For example, when a block slides from point a to point b on a horizontal surface, it takes work to overcome the frictional force that opposes the motion. When the block slides back from point b to point a, the frictional force still opposes the motion of the block so that net work is done. Let's make this idea more concrete by sliding a block around a horizontal loop on your lab table in the presence of a frictional force and computing the work it does. Then you can raise and lower the block around a similar vertical loop and calculate the work the gravitational force does.

Activity 2: Are Frictional Forces Conservative According to the Loop Rule?

(a) Open the *Force_Sensor.cap* file in the *Phys131* folder. Turn on the force sensor at your station and connect it to the computer via Bluetooth. The measurements from the force sensor may not be zero when the force is actually zero. To tare the sensor, select the desired sensor (the **Wireless Force Sensor**) in the **Controls** palette and then click **Zero Sensor Now**.

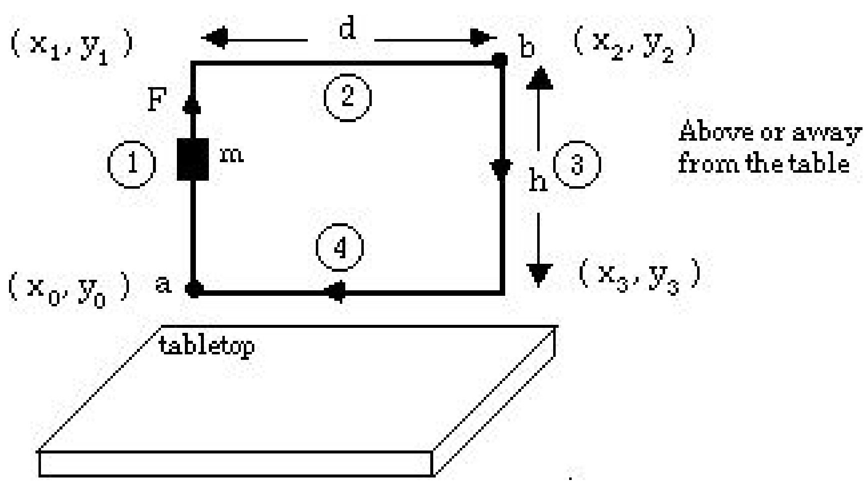
(b) Put masses on the wooden block and attach the force sensor to the wooden block. Click the **Record** button and use the force sensor to pull the block around a rectangular path on your table top at constant velocity. Draw arrows along the path for the direction of motion and the direction of the friction force exerted on the block. At constant velocity, the friction force will have the same magnitude as the force you measure on the sensor, but in the opposite direction. List the measured forces and distances for each of the four segments of the path, and calculate the work done by friction around the closed path (from a to b to a again, as shown the figure that follows). Remember how we calculated the work done by friction in Activity 2 of Experiment 17.



Work (done by friction) around closed loop = ____________

(c) Is the frictional force conservative or non-conservative (i.e., is the total work done by the friction force zero or non-zero) according to the loop rule? Explain.

Let's return once again to our old friend the gravitational force and apply an external force to move the same block (without the masses) in a vertical loop above the table. Be very, very careful to pay attention to the direction of the gravitational force relative to the direction of the motion. Remember that work is the dot product of this force and the displacement in each case so that the work done by the gravitational force when the block moves up and when it moves down are not the same. What happens to the work when the gravitational force is perpendicular to the direction of motion, as is the case in moving from left to right and then later from right to left?



Activity 3: Are Gravitational Forces Conservative?

(a) Use the force sensor to raise and lower the wooden block around a rectangular path above your tabletop at a constant speed. Draw arrows along the path for the direction of motion and the direction of the gravitational force exerted on the block. Use your measured distances, the measurement of the mass of the block and the dot product notation to calculate the work done by the gravitational force on the block over each of the four segments of the path, and over the entire path a to b to a as shown. Don't forget to specify the units! Hints: (1) In path 1 $\Delta x = x_1 - x_0$, $\Delta y = y_1 - y_0$, etc. (2) Keep track of the signs. For which paths is the work negative? positive?

Work (done by gravitational force) around closed loop = ________________

(b) Is the gravitational force conservative or non-conservative according to the loop rule? Explain.

The "Missing" Energy

We have seen in the case of the sliding block (Activity 1) that the Law of Conservation of Mechanical Energy does not seem to hold for forces involving friction. The question is, where does the "missing" energy ΔE go when frictional forces are present? The energy in the system might not all be potential energy or kinetic energy. If we are clever and keep adding new kinds of energy to our collection, we might be able to salvage a Law of Conservation of Energy. If we can, this law has the potential to be much more general and powerful than the Law of Conservation of Mechanical Energy.

Activity 4: What Happens to the Missing Energy?

(a) Rub the sliding block back and forth vigorously against your hand? What sensation do you feel?

(b) How might this sensation account for the missing energy?

Physicists call the energy which is lost by a system as a result of work done against frictional forces thermal energy. This thermal energy may lead to an increase in the system's internal energy. Using the symbol E_{th} to represent the internal energy of a system that experiences frictional forces allows us to express the Law of Conservation of Total Energy mathematically with the expression

$$E_{\text{sys}} = U + K + E_{\text{th}} = \text{constant}$$

An alternative way to express energy conservation is to note that when energy exchanges take place, the total system energy does not change, so that

$$\Delta E_{\text{sys}} = \Delta E_{\text{mech}} + \Delta E_{\text{th}} = 0$$

We are not prepared in this part of the course to consider the nature of internal energy or its actual measurement, so the Law of Conservation of Energy will for now remain an untested hypothesis. However, we will re-consider the concept of internal energy next semester when we deal with heat and temperature.

Lab 21 Momentum and Momentum Change[1]

Name: ______________________ Lab Partner(s): ______________________

Objectives

- To understand the definition of momentum and its vector nature as it applies to one- dimensional collisions.
- To reformulate Newton's second law in terms of change in momentum, using the fact that Newton's "motion" is what we refer to as momentum.
- To develop the concept of impulse to explain how forces act over time when an object undergoes a collision.
- To use Newton's second law to develop a mathematical equation relating impulse and momentum change for any object experiencing a force.

Overview

In the next few units we will explore the forces of interaction between two or more objects and study the changes in motion that result from these interactions. We are especially interested in studying collisions and explosions in which interactions take place in fractions of a second or less. Early investigators spent a considerable amount of time trying to observe collisions and explosions, but they encountered difficulties. This is not surprising, since the observation of the details of such phenomena requires the use of instrumentation that was not yet invented (such as the high speed camera). However, the principles of the outcomes of collisions were well understood by the late seventeenth century, when several leading European scientists (including Sir Isaac Newton) developed the concept of "quantity of motion" to describe both elastic collisions (in which objects bounce off each other) and inelastic collisions (in which objects stick together). These days we use the word momentum rather than motion in describing collisions and explosions.

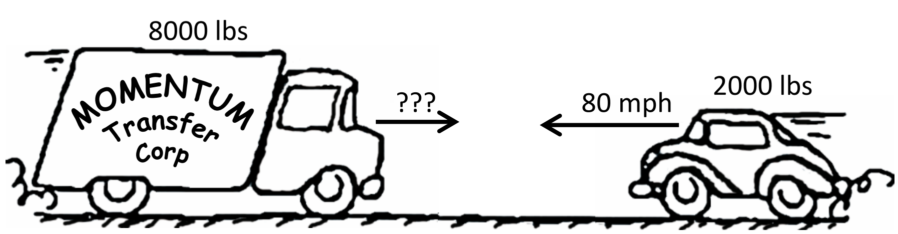
We will begin our study of collisions by exploring the relationship between the forces experienced by an object and its momentum change. It can be shown mathematically from Newton's laws and experimentally from our own observations that the integral of force experienced by an object over time is equal to its change in momentum. This time-integral of force is defined as a special quantity called impulse, and the statement of equality between impulse and momentum change is known as the impulse-momentum theorem.

Apparatus

- One dynamics cart, one collision cart and track
- Set of weights

Defining Momentum

In this session we are going to develop the concept of momentum to predict the outcome of collisions. But you don't officially know what momentum is because we haven't defined it yet. Lets start by predicting what will happen as a result of a simple one-dimensional collision. This should help you figure out how to define momentum to enable you to describe collisions in mathematical terms.



[1]1990-93 Dept. of Physics and Astronomy, Dickinson College. Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Portions of this material may have been modified locally and may not have been classroom tested at Dickinson College.

It's early fall and you are driving along a two lane highway in a rented moving van. It is full of all of your possessions so you and the loaded truck were weighed in at 8000 lbs. You have just slowed down to 15 MPH because you're in a school zone. It's a good thing you thought to do that because a group of first graders is just starting to cross the road. Just as you pass the children you see a 2000 lb sports car in the oncoming lane heading straight for the children at about 80 MPH. What a fool the driver is! A desperate thought crosses your mind. You figure that you just have time to swing into the oncoming lane and speed up a bit before making a head-on collision with the sports car. You want your truck and the sports car to crumple into a heap that sticks together and doesn't move. Can you save the children or is this just a suicidal act? For simulated observations of this situation you can use two carts of different masses set up to stick together in trial collisions.

Activity 1: Can You Stop the Car?

(a) Predict how fast you would have to be going to completely stop the sports car. Explain the reasons for your prediction.

(b) Try some head on collisions with the carts of different masses to simulate the event. Describe some of your observations. What happens when the less massive cart is moving much faster than the more massive cart? Much slower? At about the same speed?

(c) Based on your intuitive answers in parts (a) and (b) and your observations, what mathematical definition might you use to describe the momentum (or motion) of a moving vehicle traveling with a known mass and velocity?

Just to double check your reasoning, you should have come to the conclusion that momentum is defined by the vector equation

$$\vec{p} = m\vec{v}.$$

Expressing Newton's Second Law Using Momentum

Originally Newton did not use the concept of acceleration or velocity in his laws. Instead he used the term "motion," which he defined as the product of mass and velocity (a quantity we now call momentum). Let's examine a translation from Latin of Newton's first two laws (with some parenthetical changes for clarity).

Newton's First Two Laws of Motion

1. *Every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed on it.*
2. *The (rate of) change of motion is proportional to the motive force impressed: and is made in the direction in which that force is impressed.*

The more familiar contemporary statement of the second law is that the net force on an object is the product of its mass and its acceleration where the direction of the force and of the resulting acceleration are the same. Newton's statement of the law and the more modern statement are mathematically equivalent, as you will show.

Activity 2: Re-expressing Newton's Second Law

(a) Write down the contemporary mathematical expression for Newton's second law relating net force to mass and acceleration. Please use vector signs and a summation sign where appropriate.

(b) Write down the definition of instantaneous acceleration in terms of the rate of change of velocity. Again, use vector signs.

(c) If an object has a changing velocity and a constant mass then $m\frac{d\vec{v}}{dt} = \frac{d\,(m\vec{v})}{dt}$. Explain why.

(d) Show that $\sum \vec{F} = m\vec{a} = \frac{d\vec{p}}{dt}$.

Momentum Change and Collision Forces

What's Your Intuition?

You are sleeping in your sister's room while she is away at college. Your house is on fire and smoke is pouring into the partially open bedroom door. The room is so messy that you cannot get to the door. The only way to close the door is to throw either a blob of clay or a super ball at the door — there's not enough time to throw both.

Activity 3: What Packs the Biggest Wallop-A Clay Blob or a Super ball?

Assuming that the clay blob and the super ball have the same mass, which would you throw to close the door: the clay blob (which will stick to the door) or the super ball (which will bounce back with almost the same velocity it had before it collided with the door)? Give reasons for your choice, using any notions you already have or any new concepts developed in physics such as force, momentum, Newton's laws, etc. Remember, your life depends on it!

Momentum Changes

It would be nice to be able to use Newton's formulation of the second law of motion to find collision forces, but it is difficult to measure the rate of change of momentum during a rapid collision without special instruments. However, measuring the momenta of objects just before and just after a collision is usually not too difficult. This led scientists in the seventeenth and eighteenth centuries to concentrate on the overall changes in momentum that resulted from collisions. They then tried to relate changes in momentum to the forces experienced by an object during a collision. In the next activity you are going to explore the mathematics of calculating momentum changes.

Activity 4: Predicting Momentum Changes

Which object undergoes the most momentum change during the collision with a door: the clay blob or the super ball? Explain your reasoning carefully.

Let's check your reasoning with some formal calculations of the momentum changes for both inelastic and elastic collisions. This is a good review of the properties of one-dimensional vectors. Recall that momentum is defined as a vector quantity that has both magnitude and direction. Mathematically, momentum *change* is given by the equation

$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i$$

where $\vec{p}_i$ is the initial momentum of the object just before and $\vec{p}_f$ is its final momentum just after a collision.

Activity 5: Calculating 1D Momentum Changes

(a) Suppose a dead ball (or clay blob) is dropped on a table and "sticks" in such a way that it has an initial momentum just before it hits of $\vec{p}_i = -p_{iy}\,\widehat{j}$ where $\widehat{j}$ is a unit vector pointing along the positive y axis. What is the final momentum of the dead ball (after it stops on the table)?

(b) Calculate the change in momentum of the dead ball as a result of its collision with the table, using the equation above for $\Delta\vec{p}$ and the $\vec{p}_i$ and $\vec{p}_f$ from part (a). Use the same type of unit vector notation to express your answer. Show the calculation.

(c) Suppose that a live ball (or a super ball) is dropped on a table and "bounces" on the table in an elastic collision so that its speed just before and just after the bounce are the same. Also suppose that just before it bounces it has an initial momentum $\vec{p}_i = -p_{iy}\,\widehat{j}$, where $\widehat{j}$ is a unit vector pointing along the positive y-axis. What is the final momentum of the ball in the same vector notation? Hint: Does the final $\vec{p}$ vector point along the $+y$ or $-y$ axis?

(d) Calculate the change in momentum of the ball as a result of the collision, using the equation above for $\Delta\vec{p}$ and the $\vec{p}_i$ and $\vec{p}_f$ from part (c). Use the same type of unit vector notation to express your result. Show the calculation.

(e) The answer is not zero. Why? How does this result compare with your prediction?

(f) Now we will do the same thing with numbers. Suppose the mass of each ball is 0.2 kg and that they are dropped from 1 m above the table. Using this value for the mass of each ball and a calculated value for the velocity of each ball just before it hits the table, calculate the momentum just before the collision $\vec{p}_i$ for each of the balls. Also calculate the momentum of each ball just after the collision $\vec{p}_f$ and the change in momentum $\Delta\vec{p}$ for each ball. Show your calculations in the space below.

Applying Newton's Second Law to the Collision Process (The Egg Toss)

Suppose somebody tosses you a raw egg and you catch it. In physics jargon, one would say (in a very official tone of voice) that "the egg and the hand have undergone an inelastic collision." What is the relationship between the force you have to exert on the egg to stop it, the time it takes you to stop it, and the momentum change that the egg experiences? You ought to have some intuition about this matter. In more ordinary language, would you catch an egg slowly or fast?

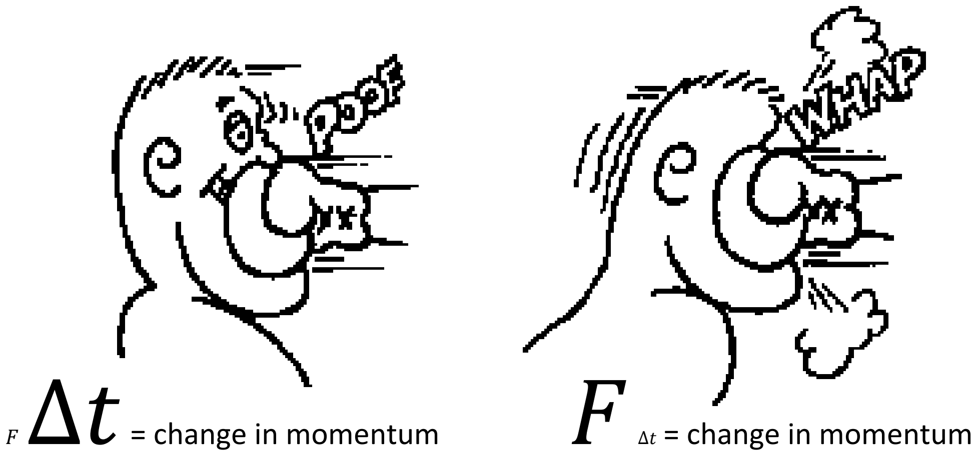
Activity 6: Momentum Changes and Average Forces on an Egg: What's Your Intuition?

(a) If you catch an egg of mass m that is heading toward your hand at speed v, what is the magnitude of the momentum change that it undergoes?

(b) Does the total momentum change differ if you catch the egg more slowly or is it the same?

(c) Suppose the time you take to bring the egg to a stop is Δt. Would you rather catch the egg in such a way that Δt is small or large? Why?

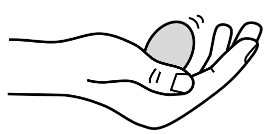
(d) What do you suspect might happen to the average force you exert on the egg while catching it when Δt is small?



You can use Newton's second law to derive a mathematical relationship between momentum change, force, and collision times for objects. This derivation leads to the impulse-momentum theorem. Let's apply Newton's second law to the egg catching scenario.

Activity 7: Force and Momentum Change

(a) Sketch an arrow representing the magnitude and direction of the force exerted by your hand on the egg as you catch it.



(b) Write the mathematical expression for Newton's second law in terms of the net force and the time rate of change of momentum. (See Activity 2(d) for details.)

(c) Show that for a constant force $\vec{F}$ the change in momentum is given by $\Delta\vec{p} = \vec{F}\,\Delta t$. Hint: Begin with expression you wrote in part (b).

Lab 22 Impulse, Momentum, and Interactions[1]

Name: ____________________ Lab Partner(s): ____________________

Objectives

- To verify the relationship between impulse and momentum experimentally.
- To study the forces between objects that undergo collisions and other types of interactions in a short time period.

Apparatus

- *Capstone* software (*Impulse-Momentum.cap* experiment file)
- CS2000 compact scale
- Dynamics track with track ends
- Wireless smart cart with spring

The Impulse-Momentum Theorem

Real collisions, like those between eggs and hands, a tennis ball and a wall, or a falling ball and a table top are tricky to study because Δt is so small and the collision forces are not really constant over the time the colliding objects are in contact. Thus, we cannot calculate the impulse as $F\,\Delta t$. Before we study more realistic collision processes, let's redo the theory for a variable force. In a collision, according to Newton's second law, the force exerted on a falling ball by the table top at any infinitesimally small instant in time is given by

$$\vec{F} = \frac{d\vec{p}}{dt} \tag{1}$$

To describe a general collision that takes place between an initial time t_i and a final time t_f , we must take the integral of both sides of the equation with respect to time. This gives

$$\int_{t_i}^{t_f} \vec{F} dt = \int_{t_i}^{t_f} \frac{d\vec{p}}{dt} dt = (\vec{p}_f - \vec{p}_i) = \Delta\vec{p}. \tag{2}$$

Impulse $\vec{J}$ is a vector quantity defined by the equation

$$\vec{J} = \int_{t_i}^{t_f} \vec{F}\,dt, \tag{3}$$

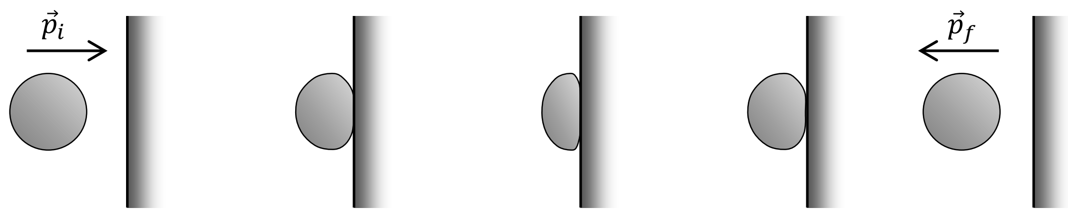
which represents the area under the F *vs.* t curve. By combining equations (2) and (3) we can formulate the impulse-momentum theorem in which

$$\vec{J} = \Delta\vec{p}. \tag{4}$$

[1]1990-93 Dept. of Physics and Astronomy, Dickinson College. Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Portions of this material may have been modified locally and may not have been classroom tested at Dickinson College.

Activity 1: Predicting Collision Forces That Change

Let's see qualitatively what an impulse curve might look like in a real collision in which the forces change over time during the collision. In particular, let's consider the collision of a tennis ball with a wall as shown below.

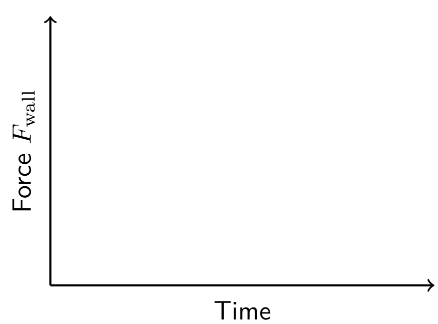


(a) Suppose a tennis ball is barreling toward a wall and collides with it. If gravity and air friction are neglected, what is the net force exerted on the object just before it starts to collide?

(b) When will the magnitude of the force on the ball be a maximum?

(c) Roughly how long does the collision process take? Half a second? Less? Several seconds?

(d) Attempt a rough sketch of the shape of the force the wall exerts on a moving object during a collision.

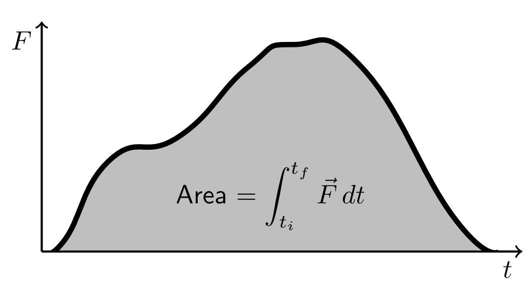


Activity 2: Verification of the Impulse-Momentum Theorem

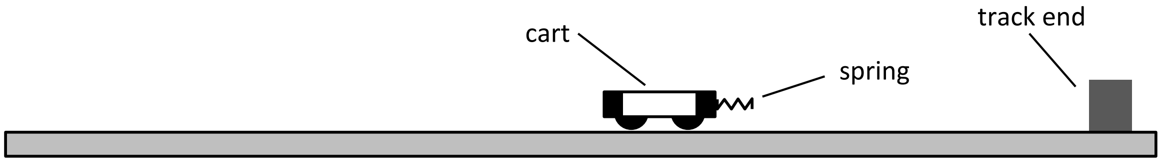
To verify the impulse-momentum theorem experimentally we will show that for an actual collision involving a single force on an object, the equation

$$\int_{t_i}^{t_f} \vec{F}\, dt = \Delta \vec{p}$$

holds, where the impulse integral can be calculated by finding the area under the curve of a graph of F *vs.* t.



The experimental setup is shown in the figure below. Open the file *Impulse-Momentum.cap* in the *Phys131* folder. Turn on the cart at your station and connect it to the computer via Bluetooth. To tare the cart's built-in sensors, select the desired sensor (either the **Smart Cart Acceleration Sensor** or the **Smart Cart Force Sensor**) in the **Controls** palette and then click **Zero Sensor Now**. When taring the sensors, the cart should be at rest and there should be no applied force on the spring.



(a) Measure the mass m of the cart + spring, using the compact scale, and record it here:

(b) With the track horizontal, give the cart a gentle shove towards the track end. The spring attached to the cart's built-in force sensor will measure the force as a function of time during the cart's collision with the track end. Record data in the table below for several trials, using the initial and final velocities of the cart to calculate its change in momentum Δp. To calculate the impulse, $\vec{J}$, you will measure the area under the force curve. (See **Appendix A: Capstone** for instructions on how to measure the area.)

If your instructor requests it, print the graphs for one of your trials and include it with this report. Also, you can use *Excel* to make your table instead if you prefer, but be sure to print it and attach it at the end of this unit.

v_i (m/s)	v_f (m/s)	Δp (kg·m/s)	J (N·s)	difference	% difference

(c) For each trial above, calculate the difference between J and Δp, expressing it as a percentage difference as well.

(d) Do your results verify the impulse-momentum theorem? Explain, *quantitatively*.

(e) What do you expect for the values in the last column of your table (Percent diff)? Make a histogram of your results in the % difference column and calculate the average and standard deviation. For information on making histograms, see **Appendix D**. For information on calculating the average and standard deviation, see **Appendix E** (can also be done in *Excel*). Record the average and standard deviation here. Attach the histogram to this unit. Is your data consistent with your expectation?

(f) What does the histogram of your data tell you?

(g) Is there any indication of a systematic uncertainty? What are the possible sources of error?

Lab 23 Newton's Laws and Momentum Conservation[1]

Name: ______________________ Lab Partner(s): ______________________

Objectives

- To study the forces between objects that undergo collisions and other types of interactions in a short time period.
- To formulate the Law of Conservation of Momentum as a theoretical consequence of Newton's laws and to test it experimentally.

Apparatus

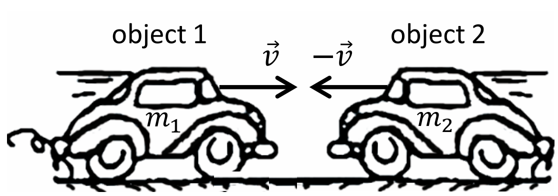
- Pasco 550 Interface
- *Capstone* software (*Two_Force_Probes.cap* experiment file)
- Force probes (x2)
- Rubber bumpers, hooks and springs for the force probes
- Pasco collision carts (x2)
- A track for the carts
- A video analysis system (*Tracker*)
- A movie scaling ruler
- Graphing and curve fitting software (*Excel*)
- Circular bubble level

Predicting Interaction Forces Between Objects

We recently focused our attention on the change in momentum that an object undergoes when it experiences a force that is extended over time (even if that time is very short!). Since interactions like collisions and explosions never involve just one object, we would like to turn our attention to the mutual forces of interaction between two or more objects. As usual, you will be asked to make some predictions about interaction forces and then be given the opportunity to test these predictions.

Activity 1: Predicting Interaction Forces

(a) Suppose that two identical objects are moving toward each other at the same speed so that $m_1 = m_2$ and $\vec{v}_1 = -\vec{v}_2$.



Predict the relative magnitudes of the forces between the objects. Place a check next to your prediction.

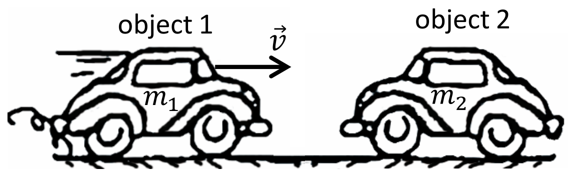
________ Object 1 exerts more force on object 2.

________ The objects exert the same force on each other.

________ Object 2 exerts more force on object 1.

[1]1990-93 Dept. of Physics and Astronomy, Dickinson College. Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Portions of this material may have been modified locally and may not have been classroom tested at Dickinson College.

(b) Suppose the masses of two objects are the same and that object 1 is moving toward object 2, but object 2 is at rest, so that $m_1 = m_2$ and $\vec{v}_1 > \vec{v}_2$.



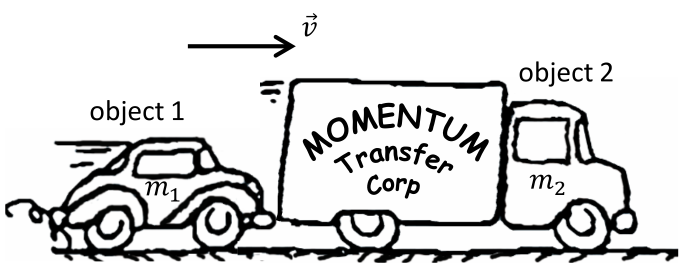
Predict the relative magnitudes of the forces between the objects. Place a check next to your prediction.

________ Object 1 exerts more force on object 2.

________ The objects exert the same force on each other.

________ Object 2 exerts more force on object 1.

(c) Suppose the mass of object 1 is much less than that of object 2 and that it is pushing object 2, which has a dead motor. Both objects move in the same direction at speed v, so that $m_1 \ll m_2$ and $\vec{v}_1 = \vec{v}_2$.



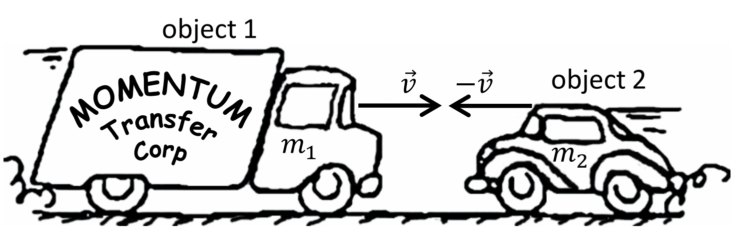
Predict the relative magnitudes of the forces between the objects. Place a check next to your prediction.

________ Object 1 exerts more force on object 2.

________ The objects exert the same force on each other.

________ Object 2 exerts more force on object 1.

(d) Suppose the mass of object 1 is greater than that of object 2 and that the objects are moving toward each other at the same speed so that $m_1 > m_2$ and $\vec{v}_1 = -\vec{v}_2$.



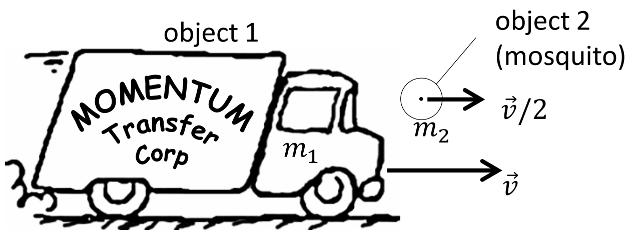
Predict the relative magnitudes of the forces between the objects. Place a check next to your prediction.

________ Object 1 exerts more force on object 2.

________ The objects exert the same force on each other.

________ Object 2 exerts more force on object 1.

(e) Suppose the mass of object 1 is greater than that of object 2 and that object 2 is moving in the same direction as object 1 but not quite as fast, so that $m_1 > m_2$ and $\vec{v}_1 > \vec{v}_2$.



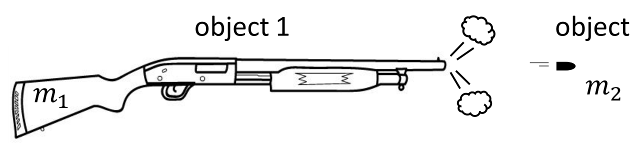
Predict the relative magnitudes of the forces between the objects. Place a check next to your prediction.

________ Object 1 exerts more force on object 2.

________ The objects exert the same force on each other.

________ Object 2 exerts more force on object 1.

(f) Suppose the mass of object 1 is greater than that of object 2 and that both objects are at rest until an explosion occurs, so that $m_1 > m_2$ and $\vec{v}_1 < \vec{v}_2$.



Predict the relative magnitudes of the forces between object 1 and object 2. Place a check next to your prediction.

________ Object 1 exerts more force on object 2.

________ The objects exert the same force on each other.

________ Object 2 exerts more force on object 1.

(g) Provide a summary of your predictions. What are the circumstances under which you predict that one object will exert more force on another object?

Measuring Mutual Forces of Interaction

In order to test the predictions you made in the last activity you can study gentle collisions between two force probes attached to carts. First, open the file *Two_Force_Probes.cap* in the *Phys131* folder. To make the interpretation of your data easier you should flip the sign of one of the force probes. To do this first click on `Hardware Setup` on the left side of the `CapStone` window, then click the gear symbol on `Forces Sensor Ch A`, and check the box labeled `Change sign`. To get back out click `OK` and then `Hardware Setup`. You can strap additional masses to one of the carts to increase its total mass so it has significantly more mass than the other. If a compression spring is available you can set up an "explosion" between the two carts by compressing the spring between the force probes on each cart and letting it go. You can make and display the force measurements with the Two Force Probes file already opened. You can also determine the areas under the force *vs.* time graphs to find the impulses experienced by the carts during the collisions. See **Appendix A: Capstone** for instructions on finding the area under a curve.

Activity 2: Measuring Slow Forces

(a) Play a gentle tug-of-war in which you pull the ends of the two force probes away from each other for about 10 seconds with your partner using the hooks on the force probes hooked together. What do you observe about the mutual forces?

(b) Remove the hooks and attach rubber bumpers to the two force probes. Play a gentle tug-of-war in which you push the ends of two force probes against each other for about 10 seconds with your partner. What do you observe about the mutual forces?

(c) Now that you're warmed up to this two force measurement technique go ahead and try some different types of gentle collisions between two carts of different masses and initial velocities. To do this, remove the rubber bumpers and attach springs to the two force probes.

Activity 3: Measuring Collision Forces

(a) Use the two carts to explore various situations that correspond to the predictions you made about mutual forces. Your goal is to find out under what circumstances one object exerts more force on another object. Describe what you did in the space below and attach a printout of at least one of your graphs of force 1 *vs.* time and force 2 *vs.* time.

(b) What can you conclude about forces of interactions during collisions? Are there any circumstances under which one object experiences a different magnitude of force than another during a collision? How do the magnitudes and directions of the forces compare on a moment by moment basis in each case? Compare your answers here with your predictions.

(c) Do your conclusions have anything to do with Newton's third law?

(d) How does the vector impulse due to object 1 acting on object 2 compare to the impulse of object 2 acting on object 1 in each case? Are they the same in magnitude or different? Do they have the same sign or a different sign? Remember $\vec{J} = \int_{t_i}^{t_f} \vec{F}\, dt$.

Newton's Laws and Momentum Conservation

In your investigations of interaction forces, you should have found that the forces between two objects are equal in magnitude and opposite in sign on a moment by moment basis for all the interactions you studied. This is of course a testimonial to the seemingly universal applicability of Newton's third law to interactions between ordinary masses. You can combine the findings of the impulse-momentum theorem (which is really another form of Newton's second law since we derived it mathematically from the second law) to derive the Law of Conservation of Momentum shown below.

Law of Conservation of Momentum

$$\sum \vec{p} = \vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f} = \text{constant in time}$$

where 1 refers to object 1 and 2 refers to object 2 and i refers to the initial momenta and f to the final momenta.

Activity 4: Deriving Momentum Conservation

(a) What did you conclude in the last activity about the magnitude and sign of the impulse on object 1 due to object 2 and vice verse when two objects interact? In other words, how does $\vec{J}_1$ compare to $\vec{J}_2$?

(b) Since you have already verified experimentally that the impulse-momentum theorem holds, what can you conclude about how the change in momentum of object 1, $\Delta\vec{p}_1$, as a result of the interaction compares to the change in momentum of object 2, $\Delta\vec{p}_2$, as a result of the interaction? Remember $\vec{J} = \Delta\vec{p}$.

(c) Use the result of part (b) to show that the Law of Conservation of Momentum holds for a collision, i.e. $\sum \vec{p} = \vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f} = \text{constant in time}$.

In the next few units you will continue to study one- and two-dimensional collisions using momentum conservation. Right now you will attempt to test the Law of Conservation of Momentum for a simple situation by using video analysis. To do this you will make and analyze a video movie in which two carts of DIFFERENT masses undergo a one-dimensional elastic collision. You may not be able to finish this in class, but you can complete the project for homework.

Testing Momentum Conservation

You just used theoretical grounds to derive momentum conservation. This idea still must be tested against experiment. You will make this test by colliding two carts (of different masses) on a track and recording and analyzing their motion before and after they hit. Measure the masses of the carts and record the total mass of each cart here:

Activity 5: Colliding Carts

(a) Make a movie of two carts colliding by following these steps.

1. Open **Camera** and turn on the camera as explained in **Appendix B: Video Analysis Using Tracker**. Center the track in the field of view of the camera which should be about 1 m above the center of the track where one cart (the target) will sit. The target cart should be located straight down below the camera. Align the track so that it is parallel to the border of the movie image. Make sure the track is level by using the small level available at each station. Place a ruler somewhere in the field of view where it won't interfere with the collision. This ruler will be used later to determine the scale.

2. Make a movie of one cart (the projectile) rolling into the other, stationary cart (the target). See **Appendix B: Video Analysis Using Tracker** for details on making the movie. When you save the movie file give it the name *Collision*.

(b) Determine the position of both carts (the target and the projectile) during the motion using *Tracker*. In *Tracker*, align one of the axes along the path of the incoming projectile. Track one cart, then the other as functions of time. For each cart you will have a table showing values of time and x position for each cart.

(c) Within *Tracker*, create graphs of position versus time for both carts. Print the graphs and include them with this unit. You can determine the initial and final velocities of each cart from the slopes of the appropriate segments of the graphs.

(d) Use your data to calculate the momenta of carts 1 and 2 before the collision.

(e) Use the data to calculate the momenta of carts 1 and 2 after the collision.

(f) Using the results of parts (d) and (e), calculate the total momentum before and after the collision. Also calculate the difference between the total momentum before and after the collision ($p_f - p_i$) and the percent difference $(p_f - p_i)/p_{\text{ave}}$ (where p_{ave} is the average of p_i and p_f), and record them below. Go around to the other lab groups and get their results for the difference $p_f - p_i$. Make a histogram of the results you collect and calculate the average and standard deviation. For information on making histograms, see **Appendix D**. For information on calculating the average and standard deviation, see **Appendix E**. Record the average and standard deviation here. Attach the histogram to this unit.

(g) What is your expectation for the difference between the initial and final momentum? Do the data from the class support this expectation? Use the average and standard deviation for the class to quantitatively answer this question.

(h) What does the histogram of the class data tell you?

(i) Within the limits of experimental uncertainty, is momentum conserved (i.e., is the total momentum of the two cart system the same before and after the collision)?

Lab 24   Momentum Conservation and Center of Mass[1]

Name: ______________________   Lab Partner(s): ______________________

Objectives

To explore the applicability of conservation of momentum to the mutual interactions among objects that experience no external forces (so that the system of objects is isolated). You will calculate momentum changes for an isolated system consisting of two very unequal masses and to observe momentum changes for a system consisting of two equal masses.

Apparatus

- Two Pasco dynamics carts with equal masses
- Two Pasco collision carts
- A track for the carts
- A video analysis system (*Tracker*)
- Variety of masses
- Circular bubble level
- Movie scaling ruler

Overview

You have now tested Newton's third law under different conditions and it always seems to hold. The implications of that are profound, because whenever an object experiences a force, another entity must also be experiencing a force of the same magnitude. A single force is only half of an interaction. Whenever there are interactions between two or more objects, it is often possible to draw a boundary around a system of objects and say there is no net external force on it. A closed system with no external forces on it is known as an isolated system. Some examples of isolated systems are shown in the figure below.

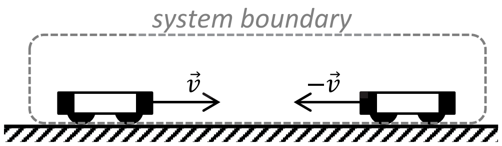
As a consequence of Newton's laws, momentum is believed to be conserved in isolated systems. This means that, no matter how many internal interactions occur, the total momentum of each of the systems pictured below should remain constant. When one of the objects gains some momentum another part of the system must lose the same amount of momentum. If momentum doesn't seem to be conserved then we believe that there is an outside force acting on the system. Thus, by extending the boundary of the system to include the source of that force we can save our Law of Momentum Conservation. The ultimate isolated system is the whole universe. Most astrophysicists believe that momentum is conserved in the universe!

You will begin this unit by examining a situation in which it appears that momentum is not conserved and then seeing how the Law of Conservation of Momentum can hold when the whole isolated system is considered. In the next activity you will make qualitative observations using two carts of equal mass moving toward each other at the same speed. You will observe momentum changes for several types of interactions, including an elastic and inelastic collision and an explosion.

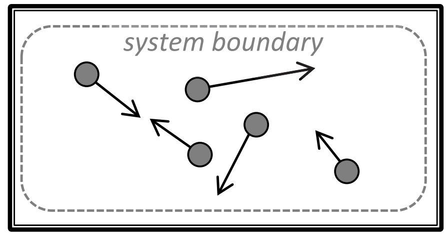
Next, a new quantity, called the center of mass of a system, will be introduced as an alternate way to keep track of the momentum associated with a system or an extended body. In the next unit, you will use this concept to demonstrate that the Law of Conservation of Momentum holds for both one-dimensional and two-dimensional interactions in isolated systems. Several other attributes of the center of mass of a system will be studied.

Examples of isolated systems in which the influence of outside forces is negligible are shown below.

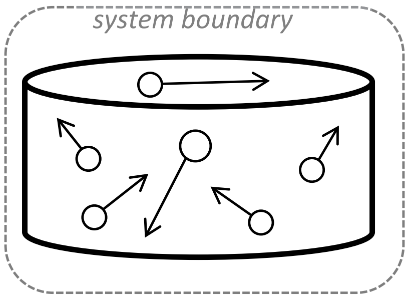
[1]1990-93 Dept. of Physics and Astronomy, Dickinson College. Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Portions of this material may have been modified locally and may not have been classroom tested at Dickinson College.



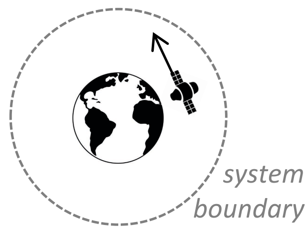
Carts with almost frictionless bearings interact but experience only negligible forces from the track



Pucks riding on a cushion of air on an air table interact with each other before hitting the walls of the air table. (Friction forces with the surface of the table are negligible.)



Gas molecules interact with each other and with the walls of their container. (Other forces, such as those of the table holding up the container and the gravitational force, are considered to have a negligible effect on the motions of the molecules and container.



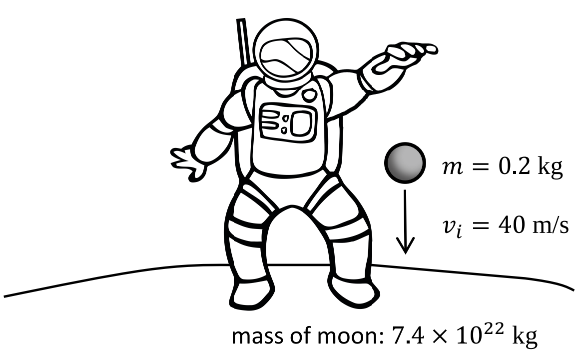
An orbiting satellite and the Earth interact. (Forces between these objects and others such as the sun and moon are considered to have a negligible effect on their motion.)

When an Irresistible Force Meets an Immovable Object

Let's assume that a superball and the moon (with an astronaut on it) are the objects in a closed system. (The pull of the earth doesn't affect the falling ball, the astronaut, or the moon nearly as much as they affect each other.) Suppose that the astronaut drops the superball and it falls toward the moon so that it rebounds at the same speed it had just before it hit. If momentum is conserved in the interaction between the ball and the moon, can we notice the moon recoil?

Activity 1: Whapping the Moon with a Superball

(a) Suppose a ball of mass 0.20 kg is dropped and falls toward the surface of the moon so that it hits the ground with a speed of 40 m/s and rebounds with the same speed. According to the Law of Conservation of Momentum, what is the velocity of recoil of the moon?



(b) Will the astronaut notice the jerk as the moon recoils from him? Why or why not?

(c) Consider the ball and the moon as an interacting system with no other outside forces. Why might the astronaut (who hasn't taken physics yet!) have the illusion that momentum isn't conserved in the interaction between the ball and the moon?

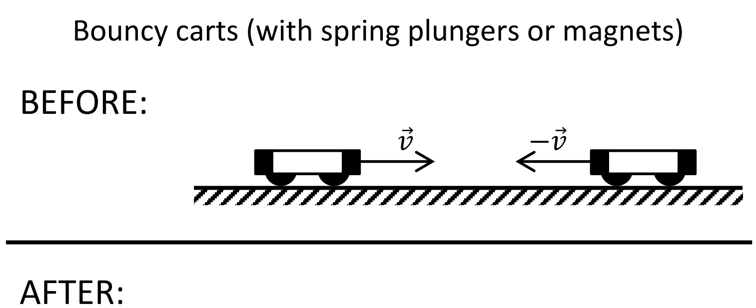
(d) Why might an introductory physics student here on earth have the impression when throwing a ball against the floor or a wall that momentum isn't conserved?

Collisions with Equal Masses: What Do You Know?

Let's use momentum conservation to predict the results of some simple collisions. The diagrams below show objects of equal mass moving toward each other. If the track exerts negligible friction on them then the two cart system is isolated. Assume that the carts have opposite velocities so that $\vec{v}_{1i} = -\vec{v}_{2i}$. To observe what actually happens, you can use relatively frictionless carts with springs, magnets, and Velcro.

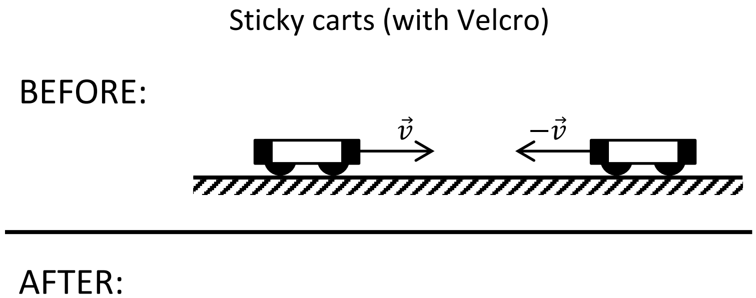
Activity 2: Predictions of the Outcome of Collisions

(a) Sketch a predicted result of the interaction between two carts that bounce off each other so their speeds remain unchanged as a result of the collision. Use arrows to indicate the direction and magnitude of the velocity of each object after the collision.



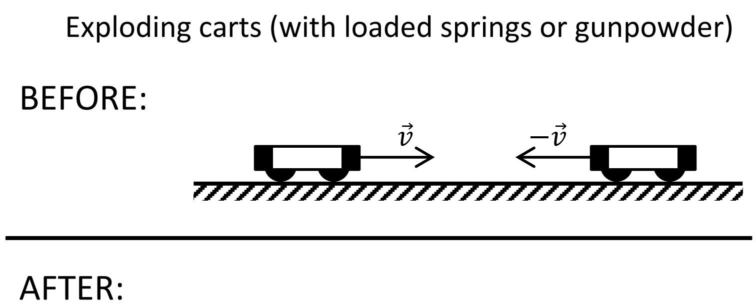
(b) Observe a bouncy collision (also known as an elastic collision) using two collision carts. Discuss whether or not the outcome was what you predicted it to be. If not, draw a new sketch with arrows indicating the magnitudes and directions of the velocities. What is the apparent relationship between the final velocities $\vec{v}_{1f}$ and $\vec{v}_{2f}$? How do their magnitudes compare to those of the initial velocities?

(c) Sketch the predicted result of the interaction between two objects that stick to each other. Use arrows to indicate the direction and magnitude of the velocity of each object after the collision.



(d) Observe a sticky collision (also known as an inelastic collision) using one collision cart and one dynamics cart with the plunger pushed all the way in. (The velcro pieces on the carts must be facing one another so that the carts will stick together on impact.) Discuss whether or not the outcome was what you predicted it to be. If not, draw a new sketch with arrows indicating the magnitudes and directions of the velocities. What is the apparent relationship between the final velocities $\vec{v}_{1f}$ and $\vec{v}_{2f}$? How do their magnitudes compare to those of the initial velocities?

(e) Sketch a predicted result of the interaction between two objects that collide in a "superelastic" way, where energy is added in the collision due to an explosion. Use arrows to indicate the direction and magnitude of the velocity of each object after the collision.



(f) Observe an exploding or "superelastic" collision using two dynamics carts with the plungers pushed part way in and facing one another. Discuss whether or not the outcome was what you predicted it to be. If not, draw a new sketch with arrows indicating the magnitudes and directions of the velocities. What is the apparent relationship between the final velocities $\vec{v}_{1f}$ and $\vec{v}_{2f}$? How do their magnitudes compare to those of the initial velocities?

(g) What is the total momentum (i.e., the vector sum of the initial momenta) before the collision or explosion in all three situations?

(h) Does momentum appear to be conserved in each case? Is the final total momentum the same as the initial total momentum of the two cart system?

Defining a Center for a Two Particle System

What happens to the average position of a system in which two moving carts having the same mass interact with each other? That is, what happens to $\langle x \rangle = (x_1 + x_2)/2$ as time goes by? What might the motion of the average position have to do with the total momentum of the system? To study this situation you will need a video movie-making and analysis system. In making these observations you'll need to look at the pattern of data points that you place over the frames. You will not need to create graphs or work with numbers.

Activity 3: Motion of the Average Position

(a) Imagine interactions between identical carts moving toward each other at the same speed as described in Activity 2. Do you expect the average position of the carts to move before, during, or after the collision or explosion in each case? Might this have anything to do with the fact that the total momentum of such a system is zero? (Do not make a movie for this case.)

(b) Let's use video analysis to study a real situation in which the total momentum of the system is NOT zero. Do the following:

1. Turn the video camera on and center the track in the field of view. The camera should be about 1 m above the center of the track. Align the track so that it is parallel to the border of the movie image. Make sure the track is flat by using the small bubble level available at each station. Place a ruler somewhere in the field of view where it won't interfere with the collision and parallel to one side of the field of view. This ruler will be used later to determine the scale.

2. Use two collision carts that have small magnets built in. Make a movie of the collision of two equal mass carts moving in the same direction with different speeds. Orient the carts so they collide on the sides that hold the magnets. See **Appendix B: Video analysis Using Tracker** for details on making the movie. When you save the movie file give it the name *Collision*.

(c) Determine the motion of the average position of both carts during the collision process using *Tracker*. In *Tracker*, align one of the axes along the path of the carts. Track the average position (the point half way between the centers of the two carts) as a function of time.

How does the position average (the point halfway between the centers of the two carts) appear to move? Does it move at constant velocity? Might this motion have anything to do with the fact that the total momentum of the system is directed in one direction and is constant?

(d) You should have found that if the momentum of the carts is constant then the average position moves at a constant rate also. Suppose the masses of the carts are unequal? How does the average position of the two objects move then? Lets have a look at a collision between unequal masses. Make and analyze a new movie as you did before, but add a significant amount of mass to one of the carts. Once again track the motion of the average position by clicking halfway between the centers of the two carts. Is the motion of this average position uniform? (i.e. does it move at constant velocity?)

(e) You should have found that the average position of a system of two unequal masses does not move at a constant velocity. We need to define a new quantity called the *center of mass* that is halfway between two equal masses but somewhere else when one of the masses is larger. Take a guess as to where the center of mass is located for the two unequal masses. Make and analyze another movie as you did before, but this time click on each frame at the place where you think the center of mass is located. Describe in the space below what you did and whether your "assumed" center of mass position moved at constant velocity.

Lab 25   Introduction to Rotation[1]

Name: ______________________   Lab Partner(s): ______________________

Objectives

To understand the definitions of angular velocity and angular acceleration and the kinematic equations for rotational motion on the basis of observations. We will discover the relationship between linear velocity and angular velocity and between linear acceleration and angular acceleration.

Apparatus

- A rotator consisting of an axle, a metal disk, and a fixture to hold the disk.
- A stopwatch.
- A meter stick, drawing compass, flexible ruler, protractor, and some string.

Overview

Earlier in the course, we studied centripetal force and acceleration, which characterize circular motion. In general, however, we have focused on studying motion along a straight line as well as the motion of projectiles. We have defined several measurable quantities to help us describe linear and parabolic motion, including position, velocity, acceleration, force, and mass. In the real world, many objects undergo circular motion and/or rotate while they move. The electron orbiting a proton in a hydrogen atom, an ice skater spinning, and a hammer that tumbles about while its center of mass moves along a parabolic path are just three of many rotating objects.

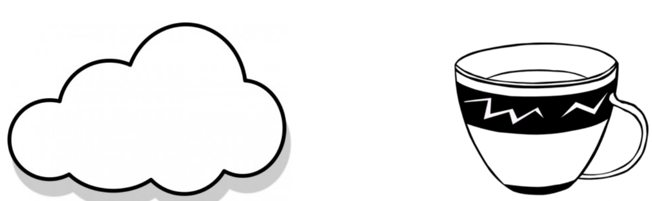
Since many objects undergo rotational motion it is useful to be able to describe their motions mathematically. The study of rotational motion is also very useful in obtaining a deeper understanding of the nature of linear and parabolic motion.

We are going to try to define several new quantities and relationships to help us describe the rotational motion of rigid objects, i.e., objects that do not change shape. These quantities will include angular velocity, angular acceleration, rotational inertia and torque. We will then use these new concepts to develop an extension of Newton's second law to describe rotational motion for masses more or less concentrated at a single point in space (e.g., the electron in the hydrogen atom) and for extended objects (like the tumbling hammer).

Rigid *vs.* Non-rigid Objects

We will begin our study of rotational motion with a consideration of some characteristics of the rotation of rigid objects about a fixed axis of rotation. The motions of objects, such as clouds, that change size and shape as time passes are hard to analyze mathematically. In this unit we will focus primarily on the study of the rotation of particles and rigid objects around an axis that is not moving. A rigid object is defined as an object that can move along a line or can rotate without the relative distances between its parts changing.

Shown in the figure below are examples of a non-rigid object in the form of a cloud that can change shape and of a rigid object in the form of an empty coffee cup that does not change shape.



[1]1990-93 Dept. of Physics and Astronomy, Dickinson College. Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Portions of this material may have been modified locally and may not have been classroom tested at Dickinson College.

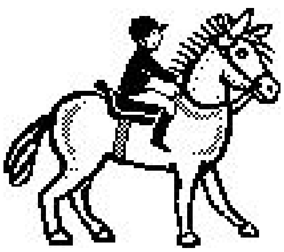
A hammer tossed end over end and an empty coffee cup are examples of rigid objects. A ball of clay that deforms permanently in a collision and a cloud that grows are examples of non-rigid objects.

A Puzzler

Use your imagination to solve the rotational puzzler outlined below. It's one that might stump someone who hasn't taken physics.

Activity 1: Horses of a Different Speed

You are on a white horse, riding off at sunset with your beau on a chestnut mare riding at your side. Your horse has a speed of 4.0 m/s and your beau's horse has a speed of 3.5 m/s, yet he/she constantly remains at your side. Where are your horses? Make a sketch to explain your answer.

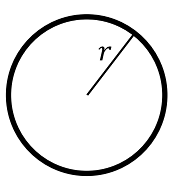


Review of the Geometry of Circles

Remember way back before you came to college when you studied equations for the circumference and the area of a circle? Let's review those equations now, since you'll need them a lot from here on in.

Activity 2: Circular Geometry

(a) What is the equation for the circumference C of a circle of radius r? What are the units of C?

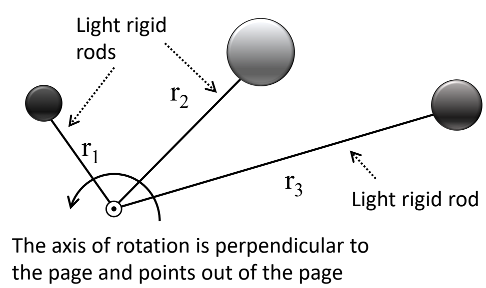


(b) What is the equation for the area A of a circle of radius r? What are the units of A?

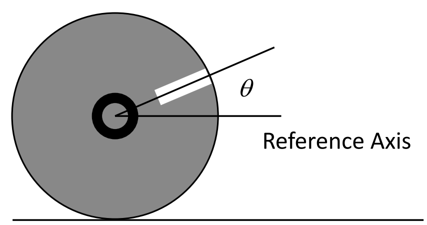
(c) If someone told you that the area of a circle was $A = r$, how could you refute them immediately? What's wrong with the idea of area being proportional to r?

Distance from an Axis of Rotation and Speed

Let's begin our study by examining the rotation of objects about a common axis that is fixed. What happens to the speeds of different parts of a rigid object that rotates about a common axis? How does the speed of the object depend on its distance from an axis? You should be able to answer this question by observing the rotational speed of the rotator at each experimental station.



Place the disk in the fixture and slowly rotate it a constant speed. The figure below shows the rotator and the definition of angular displacement.



Activity 3: Spinning the Rotator: Speed *vs.* Radius

(a) Measure how long it takes the white marker to sweep through an entire circle. Record the time and the angle in the space below.

(b) Using the equation for circumference calculate the distance of the paths traced out by the outer edge of the white marker and the inner edge as it rotated through a complete circle you just recorded. (Note: What do you need to measure to perform this calculation?) Record your data below.

(c) Calculate the average speed of the outer edge of the white marker and the average speed of the inner edge of the marker. How do they compare?

(d) How are the speeds related to the distances of inner and outer edges of the white marker from the axis of rotation? Describe the relationship mathematically (in other words write the equation for v as a function of r that you used in your calculations above).

(e) Consider your equation above, if we chose to record data for motion through half a circle how would the equation change? Can you write the equation in terms of Δt and $\Delta\theta$?

(f) As the disk rotates, does the distance from the axis of rotation to the outer edge of the white marker change?

(g) As the disk rotates, does the distance from the axis of rotation to the inner edge of the white marker change?

(h) At any given time during your rotation, is the angle between the reference axis and the inner edge of the white marker the same as the angle between the axis and the outer edge of the white marker, or do the angles differ?

(i) At any given time during the rotation, is the rate of change of the angle between the reference axis and the inner edge of the white marker the same as the rate of change of the angle between the axis and the outer edge, or do the rates differ?

(j) What happens to the linear velocity, $\vec{v}$, of the outer edge of the marker as it rotates at a constant rate? Hint: What happens to the magnitude of the velocity, i.e., its speed? What happens to its direction?

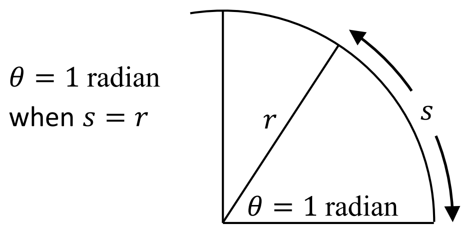
(k) Is the outer edge of the white marker accelerating? Why or why not?

Radians, Radii, and Arc Lengths

An understanding of the relationship between angles in radians, angles in degrees, and arc lengths is critical in the study of rotational motion. There are two common units used to measure angles: degrees and radians.

1. A degree is defined as 1/360th of a rotation in a complete circle.

2. A radian is defined as the angle for which the arc along the circle is equal to its radius as shown in the figure below.



So, $s = \theta r$ when θ is in radians.
In the next series of activities you will be relating angles, arc lengths, and radii for a circle.

Activity 4: Relating Arcs, Radii, and Angles

(a) Let's warm up with a review of some very basic mathematics. What should the constant of proportionality be between the circumference of a circle and its radius? Write the appropriate equation.

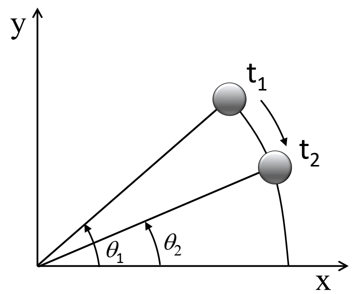
(b) Approximately how many degrees are in one radian? Let's do this experimentally. Using the compass draw a circle and measure its radius. Then, use the flexible ruler to trace out a length of arc s that has the same length as the radius. Next measure the angle in degrees that is subtended by the arc.

(c) Theoretically, how many degrees are in one radian? Please calculate your result to three significant figures. Figure out a general relationship to convert between degrees and radians. **Hint:** How many degrees are in a circle? How many radians are in a circle?

(d) If an object moves 30 degrees on the circumference of a circle of radius 1.5 m, what is the length of its path?

(e) If an object moves 0.42 radians on the circumference of a circle of radius 1.5 m, what is the length of its path?

(f) Remembering the relationship between the speed of the outer edge of the rotator and the distance, r, from the rotator's axis the outer edge, what equations would you use to define the magnitude of the average "angular" velocity, $\langle\omega\rangle$ in terms of v and r? **Hint:** In words, $\langle\omega\rangle$ is defined as the amount of angle swept out by the object per unit time.



(g) How many radians are there in a full circle consisting of 360 degrees?

(h) When an object moves in a complete circle in a fixed amount of time, what quantity (other than time) remains unchanged for circles of several different radii?

Relating Linear and Angular Quantities

It's very useful to know the relationship between the variables s, v, and a, which describe linear motion and the corresponding variables θ, ω, and α, which describe rotational motion. You now know enough to define these relationships.

Activity 5: Linear and Angular Variables

(a) Using the definition of the radian, what is the general relationship between a length of arc, s, on a circle and the variables r and θ in radians.

(b) Assume that an object is moving in a circle of constant radius, r. Using the relationship you found in part (a) above, take the derivative of s with respect to time to find the velocity of the object. Show that the magnitude of the linear velocity, v, is related to the magnitude of the angular velocity, ω, by the equation $v = \omega r$.

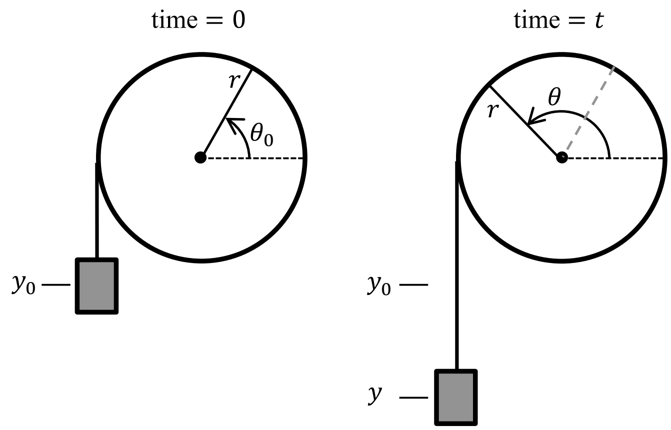
(c) Assume that an object is accelerating in a circle of constant radius, r. Using the relationship you found in part (b) above, take the derivative of v with respect to time to find the tangential acceleration of the object. Show that the linear acceleration, a_t, tangent to the circle is related to the angular acceleration, α, by the equation $a_t = \alpha r$.

The Rotational Kinematic Equations for Constant α

The set of definitions of angular variables are the basis of the physicist's description of rotational motion. We can use them to derive a set of kinematic equations for rotational motion with constant angular acceleration that are similar to the equations for linear motion.

Activity 6: The Rotational Kinematic Equations

The figure below shows a massless string wound around a spool of radius r. The mass falls with a constant acceleration, a. Refer to this figure and the results of Activity 5 to answer the following questions. NOTE: The distance y that the mass falls is equal to the arc length s moved by a point on the edge of the spool.



(a) What is the equation for y in terms of θ and r?

(b) What is the equation for v in terms of ω and r?

(c) What is the equation for a in terms of α and r?

(d) Consider the falling mass in the figure above. Note that the positive y axis is pointing down. Using the relationships between the linear and angular variables in parts (a), (b), and (c), derive the rotational kinematic equation for constant acceleration for each linear kinematic equation listed below. **Warning:** Don't just write the analogous equations! Show the substitutions needed to derive the equations on the right from those on the left.

1. $v = v_0 + at$ $\qquad\qquad \omega =$

2. $y = y_0 + v_0 t + \frac{1}{2}at^2$ $\qquad\qquad \theta =$

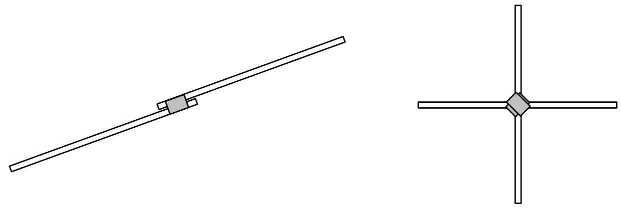
3. $v^2 = v_0^2 + 2ay$ $\qquad\qquad \omega^2 =$

Lab 26 Getting a Feel for Moment of Inertia

Name: ______________________ Lab Partner(s): ______________________

Apparatus

- four shish-kebab sticks[1]
- masking tape



Setup

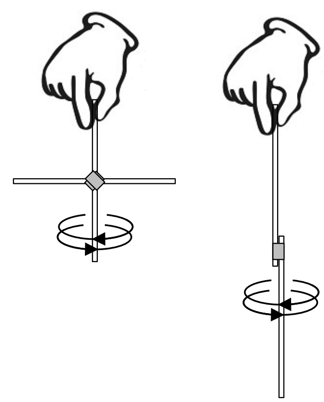
If it hasn't been done already, tape two pairs of shish-kebob sticks together, as shown above. (One pair taped end-to-end, the other pair taped like a cross.)

Activity

1. Each pair of sticks should have about the same mass (two sticks plus some tape). Hold each pair of sticks in your hand, and shake it quickly side to side. They should feel about the same, and really different from, say, shaking a heavy textbook. Do they?

2. Next, you'll spin each pair of sticks back and forth between two fingers as shown—but don't try it yet, until you make a prediction: will both pairs of sticks be equally easy to spin back and forth, or will one feel harder to start spinning and stop spinning than the other?

Prediction:



Now try it. Does one feel harder to start spinning? (If yes, which one?)

Experiment:

3. Which pair of sticks "feels heavier" when you spin it?

The effect you feel is that each pair of sticks has a different *moment of inertia*, which tells you how hard it is to make an object spin.

- An object's moment of inertia depends on how its mass is distributed. It is larger when the mass is further away from the axis of rotation. That's why the cross is harder to spin, even though both pairs of sticks have the same mass.
- The moment of inertia *also depends* on mass: you would feel the difference if you tried to spin two things of the same shape but different masses. That's why the crossed sticks "feel heavier" when you spin them.

4. For a quick, qualitative explanation of what *mass* means, you might say something like:

"Mass tells you how much an object resists accelerating when you apply a force."

Write a sentence analogous to the one above, using the terms *moment of inertia*, *angular acceleration*, and *torque*.

[1] In Turkish, *şiş* means roughly "stick" and *kebap* means something like "grilled meat." So, strictly speaking, this lab only uses the *shish* part of the shish-kebab. (Also, strictly speaking, the term "veggie kebabs" should really be something like "shish veggies," but that's another story.) *Fun facts to know and share!*

Lab 27 Newton's Second Law for Rotation[1]

Name: ______________________ Lab Partner(s): ______________________

Objectives

To understand torque and its relation to angular acceleration and moment of inertia on the basis of both observations and theory.

Apparatus

- A Rotating Disk System
- A hanging mass of 200 g (for applying torque)
- String and pulley
- A movie scaling ruler and a simple cm ruler
- Vernier caliper
- Small water bubble level
- A video analysis system (*Tracker*).

Overview

We have used the definition of moment of inertia, I, to determine a theoretical equation for the moment of inertia of a uniform disk. This equation was given by

$$I = \frac{1}{2}MR^2.$$

Does this equation adequately describe the moment of inertia of a rotating disk system? If so, then we should find that, if we apply a known torque, τ, to the disk system, its resulting angular acceleration, α, is actually related to the system's moment of inertia, I, by the equation

$$\tau = I\alpha \quad \text{or} \quad \alpha = \tau/I$$

The purpose of this experiment is to determine if, within the limits of experimental uncertainty, the measured angular acceleration α_{exp} of a rotating disk system is the same as its theoretical value α_{th}. The theoretical value of angular acceleration can be calculated using theoretically determined values for the torque on the system and its moment of inertia.

Activity 1: Theoretical Calculations

In this section you'll take some basic measurements on the rotating disk system to determine the theoretical value for angular acceleration.

(a) Calculate the theoretical value of the moment of inertia of the metal disk using basic measurements of its radius and mass. Ignore the small hole in the middle in your calculation (i.e. assume the disk is uniform). Be sure to state units!

$R_d =$ $M_d =$

$I =$

[1]1990-93 Dept. of Physics and Astronomy, Dickinson College. Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Portions of this material may have been modified locally and may not have been classroom tested at Dickinson College.

(b) In preparation for calculating the torque on your system, summarize the measurements for the falling mass, m, and the radius of the spool that has the string wrapped around it in the space below. The diameter of the spool can be measured using the vernier caliper. Don't forget the units!

$m =$ $r_s =$

(c) Calculate the theoretical value for the torque τ on the rotating system as a function of the magnitude of the hanging mass and the radius, r_s, of the spool, assuming the tension in the string is equal to the weight of the falling mass (this introduces an error of less than 1 percent if $m = 200$ g). Remember that torque equals force (tension in the string) times moment arm (radius of spool). Be sure to include units.

$\tau =$

(d) Based on the values of torque and moment of inertia that you just calculated, what is the theoretical value of the angular acceleration α_{th} of the disk? (Refer to equations in **Overview**). Be sure to include units.

$\alpha_{\text{th}} =$

Activity 2: Experimental Measurement of Angular Acceleration

(a) Place the video camera about 1 m above the rotator, and align the camera with the center of the rotator. Open **Camera** and turn on the camera as explained in **Appendix B: Video analysis Using Tracker**. Center the rotator in the field of view of the camera. Use the small level to ensure that the surface of the rotator is level. Place a ruler of known length in the field of view of the camera and parallel to one side of the frame.

(b) Place the rotator so the string will pass smoothly over the pulley and put 200 g of mass on the end of the string. Release the rotator and use the video camera to record the motion of the disk for at least two full turns. See **Appendix B: Video analysis Using Tracker** for details on making the movie.

(c) Determine the angular displacement of the rotator as a function of time. **Important:** Be careful to place the origin of your coordinate system on the axle of the rotator BEFORE recording data, so the angular displacement you measure will be the desired one. Using *Tracker*, track the outer edge of the white marker on the rotating disk for at least two full rotations. The resulting file should contain three columns with the values of time, x-position, and y-position for two complete rotations of the disk.

(d) What is the expression for the angular displacement of the disk (relative to the x axis) in terms of the x- and y-positions of the marker that you recorded above? (Note that these positions will be relative to an origin that you placed on the axle of the rotator in part (c).)

$\theta =$

(e) We want to graph the angular displacement of the disk as a function of time. To do this:

1. Click on the word *Table* immediately above your data in Tracker and select θ in the Visible Table Columns window that pops up. A new column of data labeled θ should appear.
2. Next **right** click on the θ column label in the data table and select Number and Units. At the bottom of the window that pops up select radians.
3. Use this new data of the angular displacement to plot the angular displacement as a function of time.

(f) We now want to extract the angular acceleration α_{exp} from the data.

1. To describe the time dependence of the angular displacement what type of function should we use to fit the data? (Refer to Activity 6 part (d) of the lab Introduction to Rotation.) Which coefficient of the polynomial is related to the angular acceleration?

2. Add a trendline to the data with a polynomial fit of order 2 and display the resulting equation for the time dependence on the graph. (See Appendix B for help with doing this in Tracker or export your data to Excel.) Write the equation for the angular position as a function of time in the space below. Be sure to include the proper units with the coefficients. Determine the experimental value for the angular acceleration α_{exp} from the fit and record it below.

Activity 3: Comparing Theory with Experiment

(a) Summarize the theoretical and experimental values of angular acceleration. Be sure to include the proper units.

$\alpha_{\text{th}} =$

$\alpha_{\text{exp}} =$

(b) Do theory and experiment agree within the limits of experimental uncertainty? Calculate the percent difference.

Lab 28 Rotational Kinetic Energy

Name: ______________________ Lab Partner(s): ______________________

Objective

To determine the change in kinetic energy of a rotating disk and falling mass system and compare with the change in potential energy of the system.

Apparatus

- Rotating Disk System and string
- Hanging mass of 100 g
- Table clamp and rotary motion sensor
- *Capstone* software (*rotational_KE.cap* experiment file)
- meter stick
- scale
- calipers

Overview

Conservation of energy states that the change in energy of an isolated system equals zero. Therefore, for our isolated system any change in potential energy should be equal to the change in kinetic and thermal energy. By minimizing friction we will observer the conversion of potential energy to kinetic energy. We will compare a system with purely translational motion to one with both translational and rotational motion.

Since many objects undergo rotational motion it is useful to be able to describe their motions mathematically. The study of rotational motion is also very useful in obtaining a deeper understanding of the nature of linear and parabolic motion.

In this experiment kinetic energy arises from allowing a mass to fall while attached to a disk, which is free to spin. If the mass falls a distance h, the mass loses gravitational potential energy and gains kinetic energy.

Kinetic Energy of the system

(a) What objects begin to move once the weight begins to fall?

(b) What type of kinetic energy does each object have? Write the equation for the kinetic energy for each object.

Moment of Inertia

We have used the definition of moment of inertia I to determine a theoretical equation for the moment of inertia of a uniform disk. The equation was given by

$$I = \frac{1}{2}MR^2.$$

Does this equation adequately describe the moment of inertia of the rotating disks?

(a) Calculate the moment of inertia of the disk on the rotational motion apparatus. (Which radius do you use?) Ignore the small hole in the middle in your calculations; assume the disk is uniform.

(b) Estimate the moment of inertia for the disk on the front of the rotary motions sensor. We will ignore the motion of the rotary motion sensor in this lab. Is it reasonable to assume the rotational kinetic energy is dominated by the disk on the rotational motion apparatus and ignore the motion of the of the rotary motion sensor? Why or why not?

Activity 1: Theoretical Calculations

(a) Determine the height h through which your mass will fall.

(b) Calculate the final velocity of a 100 g mass dropped from from this height (assuming a simple system with no rotation).

(c) Using conservation of energy, write an equation (in terms of variables such as I, ω, R, r, m...) for the change in potential and kinetic energy of our experimental system.

(d) Rewrite the equation for energy substituing in the linear velocity of the string and mass for the angular velocity of the disk. Below the equation indicate which radius each r or R in the equation is refering to, and which mass each m or M is refering to.

(e) We will measure the angular velocity of the rotary motion sensor. How does the angular velocity of the rotary motion sensor ω_{ms} relate to the linear velocity of the string v_s? What radius do you use?

$v_s =$

(f) Record the radius of the rotary motion sensor.

$r_{\text{ms}} =$

(g) Calculate the final velocity of the mass in your system, assuming a 100 g mass falling through the height you recorded above.

Activity 2: Measuring motion

(a) Open the file *rotational_KE.cap*.

(b) Place the 100 g mass on the string and place the string over the middle (medium) pulley grove on the rotary motion sensor.

(c) Record the starting position of the mass.

h_i=

(d) Click run and release the mass.

(e) Analyze your data. You may need to repeat parts (c) and (d) multiple times to obtain good interpretable data. What was the maximum angular velocity of the rotatory motion sensor?

(f) What is the linear velocity of the string and mass v_s (see part e above)?

(g) How does your theoretical velocity from Activity 1, part (g) compare with your experimental velocity? Suggest some reasons for any differences.

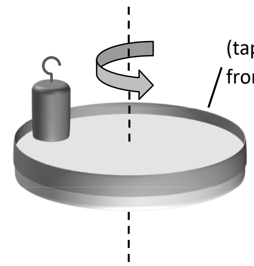
(h) Repeat two additional times, to check that your results are reproducible.

Lab 29 Conservation of Angular Momentum

Name: ______________________ Lab Partner(s): ______________________

Apparatus

- Rotating disk system, with paper tabs
- Photogate and Pasco interface
- Masses of 500 g, 1 kg
- Simple cm ruler
- Vernier caliper
- Small water bubble level
- *Capstone* software (*conserve_ang_mom.cap* experiment file)
- Masking tape



Overview

As a consequence of Newton's laws, angular momentum like linear momentum is believed to be conserved in isolated systems. This means that, no matter how many internal interactions occur, the total angular momentum of any system should remain constant if there are no external torques. When one of the objects gains some angular momentum another part of the system must lose the same amount. If angular momentum isn't conserved, then we believe that there is some outside torque acting on the system.

In this unit you will test the notion of the conservation of angular momentum. As in the test of the conservation of linear momentum, we will investigate what happens when two bodies undergo a "rotational" collision. You will drop a large weight onto a rotating disk and determine the angular momentum of the rotator-disk-weight system before and after this perfectly inelastic collision.

Activity 1: The Moment of Inertia Before and After the Collision

(a) Calculate the theoretical value of the moment of inertia of the disk using basic measurements of its radius and mass. Be sure to include units and show the expression you used!

$R_d =$ $M_d =$

$I_d =$

After dropping the weight on the rotating disk, the system will have a new moment of inertia.

(b) First, record the mass m_w and radius r_w of the weight, as well as the distance d that the center of weight will be from the center of the disk once it slides all the way to the masking tape. You can use the vernier calipers to measure the weight's diameter.

$m_w =$ $r_w =$ $d =$

(c) Next, write down a formula for the moment of inertia I_w of the weight as it revolves with the disk, and calculate its value based on your measurements above. (If you know how to use the parallel axis theorem, use it! If not, you can treat the weight as just a point mass a distance d from the center of the disk, which turns out to be a reasonable approximation here.)

$I_w =$

(c) What is the moment of inertia I_{final} of the whole system after the mass has been dropped on the disk?

$I_{\text{final}} =$

Activity 2: Measurement of Angular Velocity and Angular Momentum

Open the experiment file *conserve_ang_mom.cap* in the *Phys131* folder. You will be using a photogate to measure the angular speed ω of the rotating disk. The photogate simply measures the elapsed time whenever its beam is broken. The experiment file has already been set up to calculate ω from these measured times. (You're welcome!)

Set up your photogate so that its beam is periodically broken by the paper sticking out from between the disks. When you are ready, give the disk a moderate spin ($\approx$ 10 rad/sec should be good), and hit record in the program. Wait a few seconds to get a stable measurment of the initial angular speed, then drop the weight gently on the spinning disk, continuing to record the speed for a few seconds. Print out your graph of ω vs. time if your instructor requests it.

(a) From your graph, determine the angular speeds ω_{initial} and ω_{final} of the system both before and after the collision. Note that there's probably a little bit of friction in the system, which means the speed won't ever be exactly constant. You'll need to think about how to handle this, and use this to estimate and record the uncertainty $\pm\delta\omega$ for each of your measurements.

$\omega_{\text{initial}} =$

$\omega_{\text{final}} =$

(b) How did you estimate your uncertainties $\pm\delta\omega$, above?

(c) Calculate the angular momentum L before and after the collision (including units!). Be sure to include the uncertainty $\pm\delta L$ for each.

$L_{\text{initial}} =$

$L_{\text{final}} =$

(d) Was the angular momentum of the system conserved during this collision, to within the precision of your measurement? If not, can you think of any reasons why it might not have been?

(e) Go around to the other lab groups and get their results for the difference $\Delta L = L_{\text{final}} - L_{\text{initial}}$ between the angular momenta before and after the collision. Make a histogram of the results you collect and calculate the average and standard deviation. For information on making histograms, see **Appendix D**. For information on calculating the average and standard deviation, see **Appendix E**. Record the average and standard deviation here. Attach the histogram to this unit.

(f) What is your expectation for the difference between the initial and final angular momentum? Do the data from the class support this expectation? Use the average and standard deviation for the class to quantitatively answer this question.

(g) What does the histogram of the class data tell you? Be quantitative in your answer.

(h) Does the class data indicate a systematic error? What do you suppose causes this?

(i) Would the procedure you followed above change if the weight was moving horizontally at a constant velocity when you dropped it? If it changed, what would be different?

Lab 30 Hooke's Law

Name: ______________________ Lab Partner(s): ______________________

Apparatus

- Two springs and supports
- Collection of masses
- Two-meter stick

Introduction

There is no such thing as a perfectly rigid body. The stiffest of metal bars can be twisted, bent, stretched, and compressed. Delicate measurements show that even small forces cause these distortions. Under certain circumstances (typically, when the forces are not too large), a body deformed by forces acting upon it will return to its original size and shape when the forces are removed, a capacity known as elasticity. Permanent distortion from large forces is referred to as plastic deformation. In this lab, you will stay within the elastic limit.

Activity

(a) Suspend one of the springs from the support. Using the meter stick, observe the position of the lower end of the spring and record the value in the table below.

(b) Hang 50 grams from the lower end of the spring and again record the position of this end.

(c) Repeat part (b) with loads of 100, 200, 300, and 400 grams hung from the spring.

(d) Repeat parts (b) and (c) with the second spring.

Mass on spring 1	Force on spring 1 (N)	Position reading (m)	Elongation (m)	Mass on spring 2	Position reading (m)	Elongation (m)
0	0		0	0		0
50				50		
100				100		
200				200		
300				300		
400				400		

(e) Determine the elongation produced by each load.

(f) Plot a curve using the values of the elongation as the abscissas (x values) and the forces due to the corresponding loads as ordinates (y values) for each spring. Make sure you use compatible units. Write the equation for each curve in the space below.

Questions

(a) What do your curves show about the dependence of each spring's elongation upon the applied force?

(b) List the proportionality constant (including proper units) for each spring in the space below.

(c) The slope of each line (the proportionality constant) is known as the force constant, k. Use the LINEST function in *Excel* (see Appendix D: Excel) to determine the slope of the line and the uncertainty in the slope. Write the force constant as $k = slope \pm \Delta slope$. Be sure to include the proper units.

Lab 31 Periodic Motion

Name: ______________________ Lab Partner(s): ______________________

Objectives

- To learn about some of the characteristics of periodic motion, namely period, frequency, and amplitude.
- To investigate the relationships between position, velocity, acceleration, and force in periodic motion.
- To investigate energy in simple harmonic motion.

Apparatus

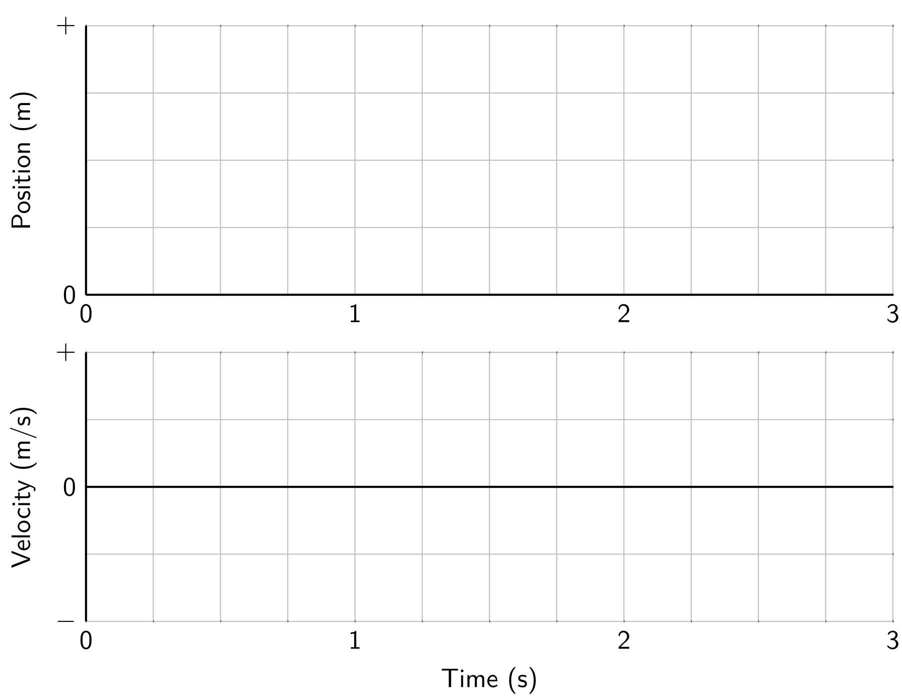
- *Capstone* software (*P_V_Graphs.cap* and *SHM.cap* experiment files)
- Spring
- Support with cross bar
- Variety of masses
- Wireless force sensor
- Wireless motion sensor

Introduction

Periodic motion is motion that repeats itself. You can see the repetition in graphs of position, velocity, or acceleration vs. time. The length of time to go through one complete cycle is called the *period*. The number of cycles in each second is called the *frequency*. The unit of frequency, cycles per second, is called *Hertz*.

Activity 1: Periodic Motion of a Mass-Spring System

(a) Open the file *P_V_Graphs.cap* in the *Phys131* folder. Turn on the motion sensor at your station and connect it to the computer via Bluetooth. Hang the large spring from the force sensor's hook with the large diameter coils down and hang a 200-g mass from the spring. Place the motion sensor facing up directly below the spring. Lift the mass straight upward about 10 cm, and let go. You may need to adjust the height of the support for the sensor to see the mass well. Record data for a few seconds to display position-time and velocity-time graphs of the motion. Sketch the graphs on the following axes.



Note that when an object returns to the same position, it does not necessarily mean that a cycle is ending. It must return to the same position, and the velocity and acceleration must also return to the same values in both magnitude and direction for this to be the start of a new cycle.

(b) Do the position and velocity graphs appear to have the same period? Do their peaks occur at the same times? If not, how are the peaks related in time, i.e. what fraction of a period is their phase difference?

(c) Use the **Delta Tool** to measure the period of the motion. (For better accuracy, measure the total time over as many cycles as possible and divide by the number of cycles.)

(d) Using the **Statistics** function, determine and record the maximum and minimum displacement. Determine the amplitude of the motion from $A = (x_{\max} - x_{\min})/2$.

(e) Save the graph. You will be using it again in Activity 5.

Activity 2: Predictions: What Factors Determine the Period of the Mass-Spring System?

The motion of a mass hanging from a spring that you looked at in Activity 1 is a close approximation to a kind of periodic motion called simple harmonic motion (abbreviated SHM).

(a) What can you do to change the period of the SHM of the mass-spring system? What will happen to the period if you increase the amplitude? Increase the mass? Increase the spring constant (use a stiffer spring)?

(b) Repeat the procedure of Act 1, but with a different starting position (other than 10 cm). (Warning: Do not make the amplitude so large that the mass comes closer than 15 cm from the motion sensor.) When you have good graphs, find and record the period and the amplitude using the methods described in Activity 1.

(c) Is there evidence that the period depends on amplitude? (Did the change in amplitude result in a comparable change in period?) How does this compare with your prediction?

(d) Carefully measure the period for four other masses. Record the masses and the measured periods in a table in the space below along with the mass and period from Activity 1. **Note:** Add one third of the mass of the spring to each mass value. This accounts for the fact that the theory of SHM neglects the mass of the spring.

(e) Does the period depend on the mass? Does it increase or decrease as mass is increased?

(f) Determine the mathematical relationship between the period T and the mass m by finding a function that fits the data. Do this by using *Excel* to plot a graph of T vs. m (use the total mass, including one third of the mass of the spring) and fitting the data with a POWER function. Write the equation that provides the best fit to the data in the space below. What is the power of m for your best fit? Print the graph and include with this unit.

You should have found that T is independent of amplitude and proportional to $\sqrt{m}$. The actual expression for the period is

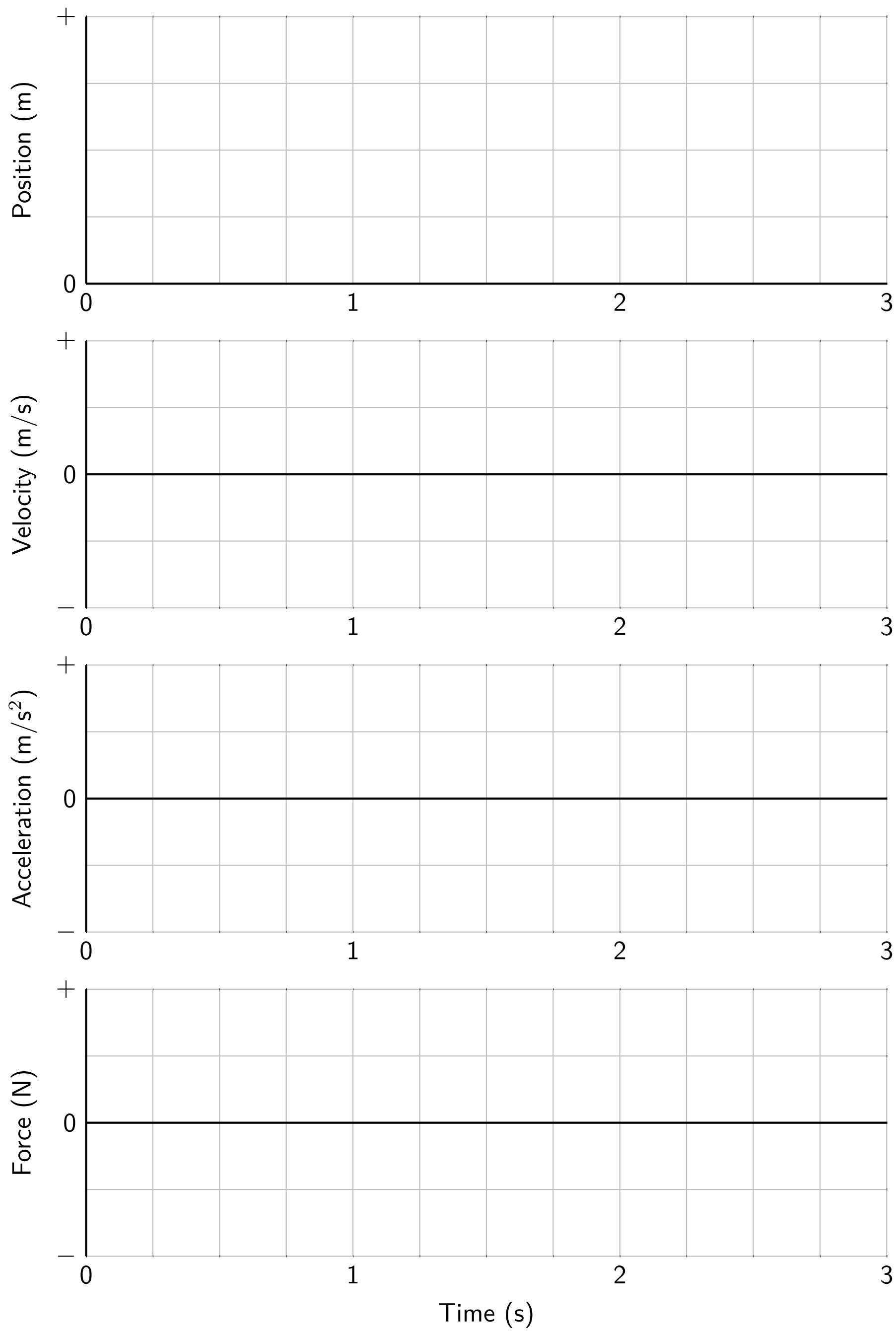
$$T = 2\pi\sqrt{\frac{m}{k}}.$$

Activity 3: Determination of the Spring Constant

Measure the distance the spring stretches for four different masses and use these data to determine the spring constant, k, as you did in either the Work and Kinetic Energy experiment or the Hooke's Law experiment. Record your data and the result for k in the space below.

Activity 4: Velocity, Acceleration, and Force for SHM

(a) Consider the motion you looked at in Activity 1 when the mass was 200 g and the initial position was 10 cm. Sketch the position and velocity graphs that you observed on the axes below using dashed lines.



(b) Based on what you know about the relationships between velocity, acceleration, and force, use dashed lines to sketch your predictions for the acceleration and force graphs.

(c) Open the file *SHM.cap* in the *Phys131* folder. Turn on the motion and force sensors at your station and connect them to the computer via Bluetooth. Tare the force sensor with noe applied force on the sensor's hook. Hang the large spring from the force sensor's hook with the large diameter coils down and hang a 200-g mass from the spring. Place the motion sensor facing up directly below the spring. Start the mass oscillating with an amplitude of 10 cm and record data for a few seconds. When you have obtained good graphs, sketch the results on the above axes using solid lines. Print your graph and attach to this unit.

(d) When the mass is at its maximum distance from the motion sensor, is the absolute value of the velocity maximum, minimum or some other value according to your graphs? Does this agree with your predictions? Does this agree with your observations of the oscillating mass? Explain.

(e) When the mass has its maximum positive velocity, is its distance from the motion sensor maximum, minimum, the equilibrium value or some other value according to your graphs? What about when it reaches maximum negative velocity? Does this agree with your predictions? Does this agree with your observations of the oscillating mass? Explain.

(f) According to your graphs, for what distances from the motion sensor is the acceleration maximum? For what distances is the acceleration zero? What is the velocity in each of these cases?

(g) Compare the force and acceleration-time graphs. Describe any similarities. Does the force graph agree with your prediction?

(h) From your graphs, what would you say is the relationship between force and acceleration?

(i) Compare the force and distance(position)-time graphs. What would you say is the relationship between force and position?

Activity 5: Energy of a Mass Undergoing SHM

Now use the graphs from Activity 1 to examine the energy relationships in simple harmonic motion. To do this you will need to print the graphs and draw a horizontal line through the equilibrium position on the position vs. time graph. The x values must be determined relative to this equilibrium position (which represents a displacement of zero).

(a) At what points is the kinetic energy of the mass zero? Label these points on your distance and velocity graphs above with "$K = 0$".

(b) Calculate the elastic potential energy due to the spring at one of these points. Label the point you use on your velocity and distance graphs with a 1. Use $U = \frac{1}{2}kx^2$, where x is the distance from the equilibrium position and k is the force constant of the spring, which you have already measured. Use the **Delta Tool** in the menubar (see Appendix A) to measure position, then subtract the equilibrium position to determine x. Show your data and calculations in the space below.

(c) At what points is the potential energy zero? Label these points with a "$U = 0$" on your distance and velocity graphs.

(d) If you measured the kinetic energy at one of these points, what would you expect its value to be? Explain.

(e) Check your prediction. Calculate the kinetic energy at one of these points. Label the point you use on your velocity graph with a 2. Use $K = \frac{1}{2}mv^2$, where m is the total mass including half the mass of the spring. Use the *Delta Tool* to determine v. Show your data and calculations in the space below.

(f) Did your calculated kinetic energy agree with your prediction?

(g) If you calculated the potential and kinetic energies at a point where neither of these was zero, what would you expect the total energy to be? Explain.

(h) Check your prediction. Make a table below with headings for v (m/s), position (m), x (m) where x is the difference between the position of the mass and the equilibrium position, K, U, and the total energy. Pick 5 or 6 points where the mass has both kinetic and potential energy, calculate them both (K and U), and then calculate the total energy for each point ($K + U$). Label these points on your distance and velocity graphs with numbers. Make a histogram of your results for the total energy and calculate the average and standard deviation. For information on making histograms, see Appendix D. For information on calculating the average and standard deviation, see Appendix E. Record the average and standard deviation here. Attach the histogram to this unit.

(i) What does the histogram of the total energy tell you? Be quantitative in your answer.

(j) Is the total energy conserved? Be quantitative in your answer.

Lab 32 The Derivatives of the Sine and Cosine Function

Name: ________________________ Lab Partner(s): __________________________________

Objectives

- Develop mathematical tools to study the harmonic oscillator.
- To apply an oft-used approximation to the sine and cosine functions function to calculate their derivatives.
- To compare the previous results to analytical methods.

Overview

Periodic motion is common in nature and the simple harmonic oscillator (*i.e.* a spring) is a common example. The motion of a system undergoing such motion is usually described with sine and/or cosine functions like the following

$$x(t) = A\cos(\omega t + \phi) \tag{1}$$

where A is the amplitude of the oscillation, ω is the angular frequency, ϕ is the phase angle, and t is time. Inserting this expression for $x(t)$ into Newton's Second Law for an oscillator with spring constant k

$$F_s = -kx = ma = m\frac{d^2x}{dt^2} \tag{2}$$

which shows we need to take derivatives of the sine and cosine to study the harmonic oscillator.

Activity 1: The Derivative of the Sine

The definition of the derivative of a function $f(t)$ is

$$\frac{d\,f(t)}{dt} = \lim_{\Delta t \to 0} \frac{f(t+\Delta t) - f(t)}{\Delta t} \tag{3}$$

which can be approximated as

$$\frac{d\,f(t)}{dt} \approx \frac{f(t+\Delta t) - f(t)}{\Delta t} \tag{4}$$

if Δt is small enough. We will use this approximation to calculate derivatives and compare the results with the analytical form of the derivatives.

(a) First make a plot of the sine function using *Excel*. There are instructions in Appendix E of *Physics For Doing!* Use the first column to hold the angles for the points along the curve. Go to the top row of the first column and enter 'Angle' (cell A1). Go to the cell below (cell A2) and enter '=(ROW()-2)*0.1' where the 'ROW()' function returns the value of the row for that cell. We subtract two from the row number so the sine curve will start at zero, and then multiply by a scale factor 0.1 which corresponds to a time step of 0.1 s.

(b) Click in the A2 cell and then click and drag down on the small square in the lower, right corner of the cell. Drag down to row 65 of so. *Excel* will fill the cells with the angle data. Enter 'Sine' in cell B1 at the top of the second column. Click in cell B2, enter '=SIN(A2)'. Click and drag down on the square in the lower, right corner of cell B2 to the same bottom row you used before.

(c) Now make a scatterplot the data in columns A and B. Highlight both columns of data and go to `Insert` and select scatterplot in the `Charts` menu. You should see a plot of the sine.

(d) Now go to cell C1 and enter 'dx/dt'. In cell C2 calculate the slope of the sine function between the time for this row and the next one (row 3). Click and drag the small square in the lower, right corner of cell C2 to the second to last row of your angle calculation. Why shouldn't you go all the way to the last row?

(e) Add the data in column C to your plot. Right-click in your plot and then left-click on 'Select data'. You should see the 'Select Data Source' window. Click `Add`. You should now get the 'Edit Series' window. Make the appropriate entries for the series name, x values, and y values. When you're done making the plot, click on the large plus sign at upper, right and check the boxes for 'Legend' and 'Axis Titles'.

(f) What familiar function does the approximate derivative curve look like? If you're having trouble with this, consult your instructor.

(g) Your curve for dx/dt should closely resemble a cosine which is equal to the derivative of the sine function. To check the accuracy of your approximation add a cosine curve to your plot using the same techniques described in steps (b) and (e). How does the cosine curve compare with your approximate derivative of the sine?

You should have found your approximate derivative of the sine function closely resembles the cosine. This is the type of approximation routinely used in physics, for example, in the labs where you measure velocity and acceleration extracted from position measurements. Keep your spreadsheet and plot for Activity 2. Print your plot and attach it to this unit.

Activity 2: The Derivative of the Cosine

Since we will also need the derivative of the cosine function in our study of the harmonic oscillator, we now apply the same approximate derivative formula (Eq. 4) to the cosine.

(a) You should already have the data for a cosine curve in your spreadsheet. Use that data and the same procedures used in Activities 1.(d)-(e) to calculate the derivative of the cosine and add that data to your spreadsheet.

(b) Add the data for the approximate derivative of the cosine to your plot. What familiar function does the approximate derivative curve resemble? If you're having trouble with this, consult your instructor.

(c) How does the sine curve on your plot from Activity 1 compare with your approximate derivative of the cosine?

You should have found your approximate derivative of the cosine function closely resembles the negative of the sine. We will use both of these derivatives demonstrated here frequently when investigating the simple harmonic oscillator.

Lab 33 Oscillators with Damping and Driving Forces

Name: ______________________ Lab Partner(s): ______________________

Apparatus:

- *Capstone* software (*Damping.cap* experiment file)
- Collection of masses
- Lab stands, to support spring, vibrator, and motion sensor
- Light duty spring
- PASCO 550 Universal Interface
- Masking tape
- String vibrator with two banana cables
- Two index cards
- Wireless motion sensor

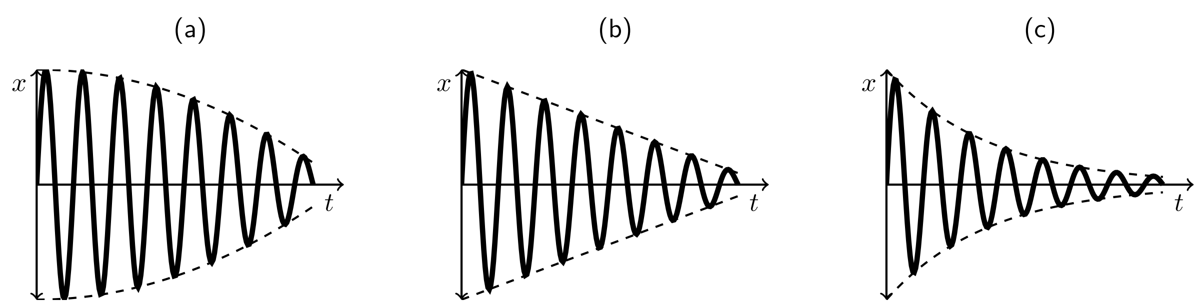
Activity 1: An Introduction to Damping

In your study of oscillations so far, you have seen that a mass and spring system with a "restoring force" of $F = -kx$ leads to simple harmonic motion that can be described by

$$x(t) = A\cos(\omega t) \tag{1}$$

where $\omega = \sqrt{k/m}$ is the system's angular frequency. But Equation (1) implies that the system keeps vibrating forever, and your experience tells you that's not true. You know that if you start a mass vibrating, the amplitude will gradually decrease over time; it certainly won't still be going if you come back to check on it an hour later. Something is missing from our description of simple harmonic motion so far.

(a) Make a prediction: if you hang a mass from a spring and start it oscillating up and down, which graph below looks most like the motion you would see? Why do you think so?



Well, let's try it! Hang a 50 g mass on the spring, over the edge of the table. Move the "string vibrator" box out of the way for now; you'll use that in Activity 4. Adjust the height of the spring so the bottom of the mass is about 60 cm above the floor. Use masking tape to stick an index card on the bottom of your mass. Open the file *Damping.cap* in the *Phys131* folder. Turn on the sensor at your station and connect it to the computer via Bluetooth. It will be most convenient if you set the equilibrium position of your mass to be at $x = 0$. To do this, let your mass hang at equilibrium over the motion sensor, click on *Hardware Setup* on the left sidebar, click on the gear icon to the right of *Wireless Motion Sensor*, and click on the *Zero Sensor Now* button.

(b) Now take some data: lift your mass up 10 cm above its equilibrium position, let it go, and hit *Record*. You'll want to take data for about one minute. Was your prediction correct? If not, which graph does your data most resemble?

The piece that was missing from our description of simple harmonic motion so far is an additional "damping force" that slows the system down over time. The damping force could be a frictional force, $F_{\text{fric}} = \mu_k F_{\text{normal}}$, from two things rubbing together, or it could be a drag force, $F_{\text{drag}} = \frac{1}{2} C_d A \rho v^2$, from an object moving through the air. (Here A is the object's cross-sectional area, ρ is the density of the air, and C_d is a dimensionless "drag coefficient" that depends on the object's shape.) But both of those are mathematically difficult to analyze. Instead, we'll use a slightly different model for drag force that assumes F_{drag} is proportional to v, not v^2:

$$F_{\text{drag}} = -bv, \tag{2}$$

where the proportionality constant b, also called the "damping coefficient," presumably depends on things like the size and shape of the object, and the density and viscosity of the medium the object moves in.

(c) What does the negative sign in Equation (2) indicate?

When we include the damping force as well as the restoring force, then Newton's second law $F_{\text{net}} = ma$ for our object becomes

$$-kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2}. \tag{3}$$

Actually *deriving* the solution for the function $x(t)$ that satisfies Equation (3) would require more advanced math than fits in this course. Instead, we'll simply present the solution here without any proof:

$$x(t) = \underbrace{A_0 e^{-bt/2m}}_{A(t)} \cos(\omega t) \tag{4}$$

You can see that while the amplitude in Equation (1) was just a constant A, the amplitude above is now a function $A(t)$ that decays exponentially with time, according to $A(t) = A_0 e^{-bt/2m}$.

(d) What is the initial amplitude A_0 in your data?

(e) Equation (4) says that $A(t)$ decays to about one third of its initial value (actually $\dfrac{1}{e}A_0$, or $\approx 0.37A_0$) at time $t = 2m/b$. At what time t does that happen in your data?

(f) From your answer above, what is the value of the damping coefficient b for your system? What are the units of b?

(*g*) Make a prediction: if you reduced the surface area of the index card under your weight to 1/2 of its original size? Would the damping coefficient b *increase*, *decrease*, or *stay the same*? Why do you think so?

(*h*) Test your prediction by folding your index card in half, and securing it with a small piece of masking tape. (Folding is better than cutting, because it doesn't change the mass.) *Note: do not delete your previous data. You'll refer to it again later.* Start your mass by lifting it up by the same 10 cm as before and letting it go. What is the value of b for your system with a half-size card? Was your prediction correct?

Activity 2: Damping and Frequency

It turns out that the damping coefficient b also affects the frequency of an oscillator. There probably wasn't a noticeable change in the two trials you did in Activity 1, because the damping coefficients here are just too small. But you can probably imagine what would happen if you increased the damping coefficient b by *a lot*, for instance by putting your system under water, or even inside a big vat of molasses.

(*a*) Would doing this experiment under water make the frequency of your system *increase*, *decrease*, or *stay the same*?

Again, the derivation is more than we can do here, but the actual angular frequency of a damped oscillator is given by

$$\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}. \tag{5}$$

You can see from the equation above that any increase in b reduces the angular frequency ω of the system from its undamped value of $\omega_0 = \sqrt{k/m}$.

(*b*) Remove the index card from the bottom of your mass, and record a few cycles of the mass oscillating up and down. Make a careful measurement of the angular frequency by measuring the time of ten complete cycles. Record your values of the period T, the angular frequency ω_0, the frequency f_0, and the spring constant k below.

(*c*) Use the value of b that you calculated in Activity 1, part (*f*), to calculate what the angular frequency ω should be with the full index card stuck to the bottom. How big a difference in frequency do you expect?

Activity 3: Driving Forces (the Theory Part)

In addition to a damping force, we can also add a periodic driving force, $F_{\text{drive}} = F_0 \sin(\omega_d t)$ to our system. This is an external force that keeps the oscillations going—just as you would be doing if you pumped your legs back and forth on a swing at a playground. Such a system eventually reaches a steady state where the effect of the driving force (increasing the oscillations) and the damping force (decreasing the oscillations) exactly balance each other. The steady state motion is described by

$$x = A\cos(\omega_d t + \phi), \tag{6}$$

where it's important to note that the resulting angular frequency of the oscillator is the *driving frequency* ω_d, not the natural frequency $\omega_0 = \sqrt{k/m}$ of the mass and spring. The amplitude A of the motion is given by the (admittedly intimidating) expression

$$A = \frac{F_0/m}{\sqrt{\left(\omega_d^2 - \omega_0^2\right)^2 + \left(\frac{b\omega_d}{m}\right)^2}}. \tag{7}$$

There's a lot packed into this last equation, so let's step through it piece by piece.

(a) If you keep everything constant, but increase *only* the magnitude of the sinusoidal[1] driving force F_{drive}, what happens to the amplitude A? Does it *increase*, *decrease*, or *stay the same*?

(b) If you keep everything constant, but increase *only* the damping coefficient b, does the amplitude A *increase*, *decrease*, or *stay the same*?

(c) If the damping force is negligibly small, what value of the driving frequency ω_d would you choose to make the resulting amplitude A as large as possible?

(d) If the driving frequency is *much, much bigger* than the natural frequency, do you expect the resulting amplitude to be very large, or very small?

(e) If the driving frequency is *much, much smaller* than the natural frequency, do you expect the resulting amplitude to be very large, or very small?

[1] Shaped like a sine or cosine function.

Activity 4: Driving Forces (the Experiment Part)

You should have found in the previous activity that the limit of light damping, amplitude A of a damped, driven oscillator is maximized when the driving angular frequency ω_d exactly matches the natural frequency ω_0 of the oscillator.[2] This phenomenon is called *resonance*, and it occurs at the *resonance frequency.*

To observe this effect, you will add a small sinusoidal driving force to your system, and measure its resulting steady-state amplitude. The "string vibrator" should be clamped onto the table near your spring. Move it over slightly, and insert the end of its metal arm in between two coils of the spring, about 3 or 4 coils from the top. You may need to readjust the height of the vibrator so that the metal arm is roughly in the middle of its travel range.

The vibrator should be plugged into the red and black jacks on the right side of the Pasco 550 interface, which will provide a sinusoidally varying voltage to power the vibrator. To turn on this voltage, click the *Signal Generator* button on the left toolbar. The *Waveform* should be set to "Sine." In the *Frequency* box, enter "1.00 Hz". You may also need to click the little curved arrow buttons just to right of that to increase the number of digits shown. Finally, set both the *Amplitude* and the *Voltage Limit* to 8 Volts, and click the *On* button to start it.

(a) With the half index card taped to the bottom of your mass and the vibrator set to a frequency of $f_d = 1.00$ Hz, allow the amplitude of the mass to reach a steady state. Record the resulting amplitude in the table below. If you find that the amplitude never stabilizes, but goes through regular cycles of smaller and larger amplitudes, then record the largest amplitude that you observe.

Response curve data for damping with half index card, $b =$ (see Activity 1, Part (h))		
Frequency f_d (Hz)	Amplitude A (m)	Notes (was amplitude stable, etc.)

(b) Continue to take data for several additional values of f_d, initially adjusting the frequency in steps of 0.1 Hz. Record the steady state amplitudes you observe for driving frequencies ω_d both above and below ω_0. Remember that the output of the signal generator is given as an f, in Hz, so you will want to compare that to the natural frequency f_0 in Hz rather than to ω_0 in rad/s. (See Activity 2, Part (b)). What is the maximum amplitude A that you can achieve, and what frequency f_d do you observe it? (That frequency is called the *resonance frequency.*)

[2]For systems with significant damping, the resonant angular frequency is slightly lower than ω_0, and is given by

$$\omega_d = \sqrt{\frac{k}{m} - 2\left(\frac{b}{2m}\right)^2}.$$

Note that this is also slightly smaller than the frequency of the damped oscillator of Equation (5), in that the expression includes an extra factor of 2.

(c) The values of A for each f in the table above is called the *response curve* of the system. Make a prediction: how would the response curve change if you used a full index card instead of a half index card?

(d) Replace the half index card with a full index card. Take data again to see if there is any effect on the response curve.

Response curve data for damping with full index card, $b =$ (see Activity 1, Part (f))		
Frequency f (Hz)	Amplitude A (m)	Notes

(e) Plot your two response curves in Excel, and print out a copy. Be sure to annotate your graphs (by hand is fine) labeling the two response curves with the values of the damping coefficient b for the full and half index cards, and showing the resonance frequency f_0.

Lab 34 Galilean Relativity

Name: ______________________ Lab Partner(s): ______________________

Objective

To investigate the relationship between different inertial reference frames and develop the equations of Galilean relativity.

Overview

Before we begin our study of Einstein's special theory of relativity we will first investigate the effects of observing a phenomenon in two different inertial frames of reference. A spring-launched projectile will be "fired" from a moving platform and we will discover how the description of this phenomenon changes (or doesn't change) in a frame that moves along with the platform. You will use the video analysis software and mathematical modeling tools to find the equations that describe the horizontal motion (x *vs.* t) and the vertical motion (y *vs.* t) of the projectile in a stationary frame and then in an inertial frame moving at constant speed with the launcher. The set of relationships between the inertial reference frames forms Galilean relativity. These relationships provide an intuitive, "common sense" picture of the world that works well at low velocities, but fails in many surprising ways at high velocities. At high velocities we must resort to Einstein's special theory of relativity, which we will discuss later. To do the activities in this unit you will need:

- A video analysis system (*Tracker*).
- The video *Galilean_Transformation.mov*
- Graphing and curve fitting software (*Excel*).

Activity 1: Observing Projectile Motion From a Moving Launcher

(a) Use *Tracker* to analyze the film *Galilean_Transformation.mov* and determine the position of the projectile in each frame with a fixed origin. To do this task follow the instructions of **Appendix B: Video Analysis** for "Analyzing the Movie." When analyzing the movie, place the fixed origin at the dark spot that appears in front the cart the launcher rides. This makes the comparison with the activities below easier. Change the position of origin in the frame by performing the following steps.

1. Click on the axes icon on the menu bar. The axes will appear over the movie.
2. Click on the origin (where the axes cross each other) and drag it to the desired location.
3. Click on the *Create* icon located on the menu bar and then select "Point Mass."

(b) Collect the data for analysis by following the instructions in **Appendix B**. The data table should contain three columns with the values of time, x-position, and y-position. Copy this data into an Excel spreadsheet for analysis.

(c) Determine the position of the launcher at the first and last frames of the movie. Using these results, what is the horizontal and vertical speed of the launcher during the movie?

$x_0 =$ $x_1 =$ $\Delta t =$ $(v_{\text{launcher}})_x =$

$y_0 =$ $y_1 =$ $(v_{\text{launcher}})_y =$

(d) Plot and fit the position versus time data for the horizontal and vertical positions of the projectile. See **Appendix D: Introduction to *Excel*** for details. Print each plot and attach it to this unit. Record the equation of each fit here. Be sure to properly label the units of each coefficient.

$x(t) =$

$y(t) =$

What is the horizontal speed of the projectile? How did you determine this?

Activity 2: Changing Reference Frames

(a) We now want to consider how the phenomenon we just observed would appear to an observer that was riding along on the launcher at a constant speed. Assume the moving observer places her origin at the same place you put your origin on the first frame of the movie. Predict how each graph will change for the moving observer.

Horizontal position versus time:

Vertical position versus time:

(b) Use *Tracker* to analyze the film *Galilean_Transformation.mov* again and determine the position of the projectile in each frame. This time, though, use a moving origin that is placed at the same point on the cart on each frame. Use the same point on the launcher that you used to define the origin in the first frame during the previous activity. To change the origin from frame to frame follow these instructions.

1. Open the movie again in Tracker and click on the Axes icon to add your coordinate system (note that the origin will again appear over the movie). Next click the *Coordinate System* drop down menu and unselect the "Fixed Origin" option. This will allow you to move the position of the origin as you advance frames.

2. Click on the origin (where the axes cross each other) and move the origin to the dark spot that appears in front the cart the launcher rides.

3. Click on the *Create* icon located on the menu bar and then select "Point Mass." To begin collecting data you will need to click on *mass A* in the *Track Control* box each time before marking the position of the projectile. Additionally, you will need to move the location of the origin to the dark spot that appears in front the cart the launcher rides each time after the position of the projectile is marked. The film will advance as usual. Repeat this procedure to accumulate the x and y positions relative to the origin you've defined in each frame.

(c) Collect the data for analysis. The data table should contain four columns with the values of time, x-position, and y-position and the position of the origin. Print the data table and attach it to this unit.

(d) Plot and fit the position versus time data for the horizontal and vertical positions. See **Appendix D: Introduction to *Excel*** for details. When you have found a good fit to the data, record your result below, print the graph, and attach a copy to this unit. Be sure to properly label the units of each coefficient in your equation.

$x(t) =$

$y(t) =$

What is the horizontal speed of the projectile? How did you determine this?

Activity 3: Relating Different Reference Frames

(a) Compare the two plots for the vertical position as a function of time. How do they differ in appearance? Are the coefficients of the fit for each set of data different? Do these results agree with your predictions above? If not, record a corrected "prediction" here.

(b) Compare the two plots for the horizontal position as a function of time. How do they differ in appearance? Are the coefficients of the fit for each set of data different? Do these results agree with your predictions above? If not, record a corrected "prediction" here.

(c) What is the difference between the horizontal velocities in the two reference frames? How does this difference compare with the horizontal velocity of the launcher? How are the horizontal velocities of the projectile in each inertial reference frame and the velocity of the launcher that you determined above related to one another? Does this relationship make sense? Why or why not?

(d) Consider a point $\vec{r} = x\,\hat{i} + y\,\hat{j}$ on the ball's trajectory in the stationary observer's reference frame. If the moving observer's time frame is moving at the speed $(v_{\text{launcher}})_x$ then what would the moving observer measure for x? Call this horizontal position of the moving observer x'.

(e) What would the moving observer measure for y? Call this vertical position of the moving observer y'.

(f) The relationships you found above are from Galilean relativity. You should have obtained the following results.

$$x' = x - (v_{\text{launcher}})_x t$$

$$y' = y$$

$$v_x = v'_x + (v_{\text{launcher}})_x$$

The primes refer to measurements made in the moving frame of reference in this case. If you did not get these expressions consult your instructor.

Activity 4: Testing Galilean Relativity

You can test your mathematical relationships with *Excel*. You will use your data for the stationary observer and the relationship you derived to calculate what the observer moving with the launcher would measure.

In your data table for the stationary observer, you have columns giving x, the horizontal position, as a function of time, t. Using an *Excel* formula, apply the Galilean transformation above to these numbers to determine x', the horizontal position as measured by the moving observer. Make a plot showing x' as a function of t.

Your "transformed" data for the stationary observer should closely resemble the results for the moving observer. Is this what you observe? If not, consult your instructor. Print and attach a copy of your plot to this unit.

Lab 35 The Twins Paradox

Name: ______________________ Lab Partner(s): ______________________

Objective

To investigate some of the unusual implications of Einstein's special theory of relativity.

Overview

Einstein's theory of special relativity leads to a variety of apparent paradoxes that depart radically from our everyday expectations. One of the most celebrated is the twins paradox, in which an identical twin makes a long interstellar journey while the other twin remains on the (roughly) stationary Earth. When the space-faring twin returns she finds her partner has aged considerably more than she has. In this unit you will explore the quantitative aspects of the paradox and some of the surprising consequences.

Activity 1: Setting Things Up

Problems in special relativity are often very counterintuitive, so it is instructive to consider the situation non-relativistically first. Investigate this problem without applying any of the new ideas you have learned about the theory of special relativity.

One member of a pair of identical twins has decided to embark on a long space voyage. The two twins have lived their lives in close proximity to one another and are very similar in appearance. The adventurous twin boards a fast spacecraft and leaves the Earth behind at a speed of 0.99c or 99% of the speed of light. The space-faring twin's itinerary is rather monotonous, and she simply travels at this constant speed for a time, turns around, and returns to the Earth at the same speed. She measures the time of her trip to be Δt_0. In the meantime the Earth-bound twin has seen twenty years pass by. We will refer to this time as Δt.

(a) In mathematical terms, what is the relationship between the times Δt_0 and Δt?

(b) Which time is associated with which twin?

(c) When the twins are reunited will their appearances differ?

Activity 2: Applying Special Relativity

(a) Now we want to apply the lessons of special relativity. Time dilation implies that moving clocks run more slowly when observed by someone in a different inertial frame. For the twins paradox what does this imply about the time interval the space-faring twin measures during her trip? Will it be less than, equal to, or greater than the interval measured by the Earth-bound twin? Will the space-faring twin age more, less, or the same amount as the Earth-bound twin?

(b) What is the mathematical relationship between Δt_0 and Δt according to the special theory of relativity?

(c) How much time has passed on the Earth-bound twin's clock?

(d) How much time has passed on the space-faring twin's clock?

(e) If this result is inconsistent with your prediction above how should you resolve the contradiction?

(f) How will the two twins' appearances differ, if at all? Is the difference only in the measurement of the time intervals or are there real physiological differences between the twins after the trip?

(g) If the average speed of the space-faring twin was more like the typical orbital speed of the space shuttle (about 7.4 km/s) what would the time difference between the twins' clocks be?

Activity 3: Graphical Analysis

(a) Find a mathematical relationship for the ratio of the time interval measured by the space-faring twin to the time interval measured by the Earth-bound twin. Show your work and record your result here.

(b) You will now use *Excel* to make a plot showing how this ratio behaves as a function of β (the speed expressed as a fraction of the speed of light). To do this, start up *Excel*, and create a column headed "beta." This column should contain the series of numbers 0,0.05,0.10,0.15,. . .,1. To create such a column of numbers, enter the first two rows and highlight them. Then grab the lower-right corner of the second cell with the mouse and drag down (in the same way as if you were dragging down a formula).

After you have created the β column, create a second column containing the ratio of time intervals (i.e., the relationship you found in part (a)). Use an *Excel* formula.

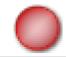
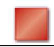
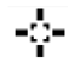
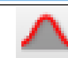
(c) Make a plot of the ratio of the time interval measured by the space-faring twin to the time interval measured by the Earth-bound twin. At what speed does the effect of time dilation become significant? Is there a limit to the ratio? Is there any reason to restrict the range of β to 0-1? Clearly state your reasoning. Print your plot and attach a copy to this unit.

(d) Consider the following scenario. As the space-faring twin's craft recedes from the Earth it is moving at a constant speed. Since no inertial frame can be considered "better" than any other there is nothing physically inconsistent with the view that the space-faring twin is observing the Earth recede from her at a constant velocity. Hence, the space-faring twin will observe clocks on the Earth to move slowly and the Earth-bound twin will age at a slower rate than the space-faring one. Is this reasoning flawed? How?

(e) If the scenario is not flawed how can it be that the space-faring twin was found to have aged less in the original problem?

Appendix A: Introduction to Capstone

Quick Reference Guide

What You Want To Do	How You Do It	Button
Start Recording Data	Click the *Record* button	
Stop Recording Data	Click the *Stop* button	
Re-scale the data so it fills the Graph display window	Click the *Scale to Fit* button	
Read exact x and y coordinates from a graph	Click the *Coordinates/Delta Tool* button. See instructions below.	
Select a section of data	Click the *Highlight* button. See instructions below.	
Select from the Statistics menu	Click the *Statistics* button, use menu. See instructions below.	
Find area under a curve	Click the *Display Area* button after selecting data of interest	

Reading Exact Coordinates

1. After clicking the *Coordinates/Delta Tool* button, select "Add Coordinates/Delta Tool."

2. Click on the gray box that appears and move it to the data point you want to read.

3. For more significant digits, right-click on the box around the data point, and select *Tool Properties* → *Numerical Format* → *Horizontal* (or *Vertical*) *Coordinate*. Click the box to "Override default number format" and increase the "Number of Deximal Places" to the value you want.

4. To delete Delta Tool: Hold cursor over the Delta Tool square, right click and select "Delete Tool".

Selecting a Section of Data

1. To select a data section, click the *Highlight* button. Drag the pink rectangle to the data you wish to highlight. Adjust the width and height using horizontal and vertical arrows at edges of highlighted region.

2. To unselect the data, with cursor in highlighted region, right click and select "Delete highlighter".

Finding Average and Standard Deviation of a Section of Data

1. Select the section of data of interest, as described above.

2. Click on the *Statistics* button on the Graph Toolbar. Use the menu on this button to select maximum, minimum, mean, standard deviation, or any combination.

3. To remove the results, click on the *Statistics* button again.

Finding the Area Under a Curve

1. Select the section of data that you want to integrate under, using the procedure described above.
2. Click the *Display Area* button on the Graph Toolbar. The results of the integration will be displayed on the graph.
3. To undo the integration, click on the *Display Area* button again.

Calibrating Force Sensors

1. Check to see that the force sensor is connected to the Pasco interface.
2. Open whatever experiment file you will be using.
3. Click the *Calibration* icon in the tools panel on the left side of the screen.
4. Verify that the type of measurement is set to “Force”, then click *Next*.
5. Choose “Two standards” as calibration type, then click *Next*.
6. In this step, you will apply known forces to the sensors using weights. You can either hang masses from the sensor directly, or using a string looped over a pulley. If you are hanging masses directly while holding the force sensor in your hand, try resting it against the edge of the table to keep it stable.
   (a) With no mass attached, set the standard value to 0 N for the first point, click “set current value to standard value”, then click *Next*.
   (b) Hang a mass (say 200 g) from the sensor, set the standard value to its weight (say 1.96 N), click “set current value to standard value”, then click *Next*.
7. After reviewing calibration, click *Finish*.
8. Click the *Calibration* icon again to close the dialog.
9. If you calibrated the sensor by holding it vertically but will now use it horizontally, you will need to account for the weight of the hook. Turn the sensor to horizontal, without any applied force, and press the “TARE” button on the side of the force sensor.
10. The force sensor should now be calibrated for the rest of your experiment. But it’s a good idea to try taking a tiny bit of data with a known weight to make sure your results aren’t totally nuts.

Appendix B: Video Analysis Using Tracker

Making a Movie with "Camera”

To make a movie, perform the following steps:

1. Make sure the camera is connected to a USB port on your computer. Close all windows, applications, programs, and browsers.
2. Click the **Start** button in the lower-left corner of your screen and type "camera’ in the Search programs and files box. Once the search results appear, click on **Camera** at top of window.
3. Position the camera 2-3 meters from the object you will be viewing. Adjust the camera height and orientation so that the field of view is centered on the expected region where the object will move.

4. Place a meter stick or an object of known size in the field of view where it won't interfere with the experiment. The meter stick should be the same distance away from the camera as the motion you are analyzing so the horizontal and vertical scales will be accurately determined. It should also be parallel to one of the sides of the movie frame. Make sure that the meter stick is not far away from the central region of field of view, and that it is perpendicular to the line of sight of camera.

5. To start recording click the video camera icon on the right of the window. When you are done, click the red square.

   The video will save to camera roll located in **This PC** → **Pictures** → **Camera Roll**

Analyzing the Movie

To determine the position of an object at different times during the motion, perform the following steps:

1. Start up Tracker by going to **Start** → **Programs** → **Physics Applications** → **Tracker**. When **Tracker** starts it appears as shown below. The menu icons and buttons that we will use are identified by arrows.

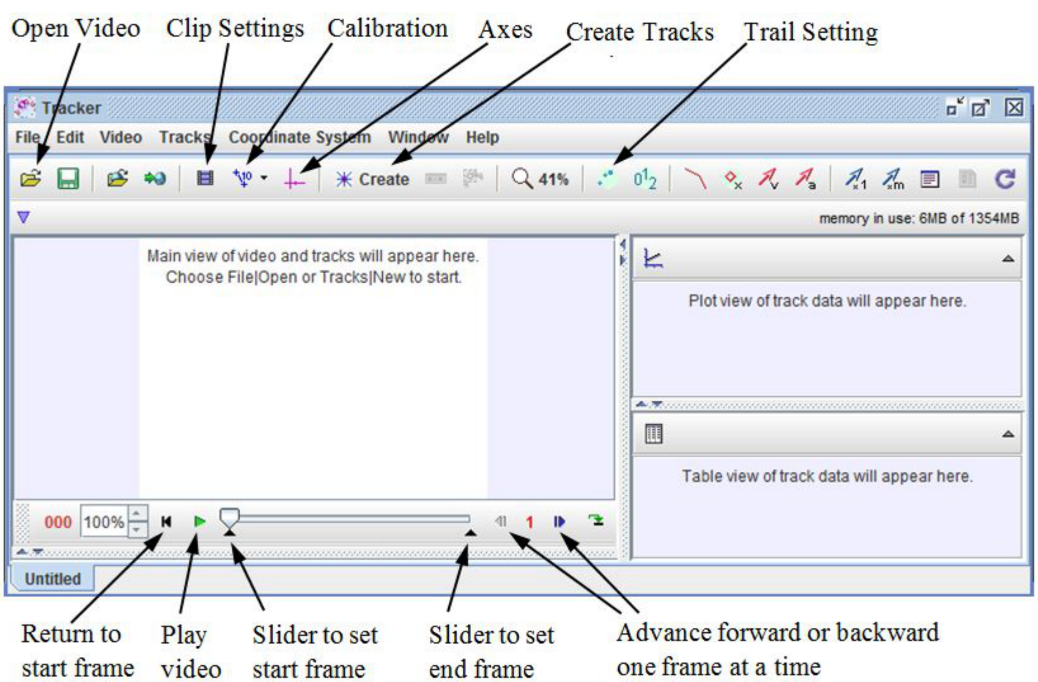


Figure 1: Initial **Tracker** window for video analysis.

2. Click the **Open Video** button on the toolbar to import your video. After your video is imported, Tracker will warn you that the video frames don't have the same time duration. This is okay since **Camera** uses a variable frame rate. Click **OK** on the warning window to ignore Tracker's recommendation.

3. Click the **Clip Settings** button to identify the frames you wish to analyze. A clip settings dialog box appears. Here, you only need to identify and set the start and end frames. Leave everything else in the dialog box unchanged. To find and set the start frame, drag the player's left slider to scan forward through the video, and stop when you get to the first frame of interest. Now, the start frame is set and the corresponding frame number should be displayed in the dialog box. If not, then click on the **Start Frame** in the dialog box, enter the number of the frame (printed in the lower left part of the Tracker window), and click outside the box. Then, click the **Play video** button to go to the last frame in the video. Next, drag the player's right slider to scan backward through the video to find the last frame of interest. Stop when

you get to the frame of interest. Now, the end frame is also set and the corresponding frame number should be displayed in the dialog box. If not, then click on the **End Frame** in the dialog box, enter the number of the frame, and click outside the box. Finally, click the **OK** button to close the dialog box, and then click the player's **Return** button to return to the start frame.

4. Click the **Calibration** button and select the **calibration stick**. A blue calibration line appears on the video frame. Drag the ends of this blue line to the ends of your calibration meter stick. Then click the readout box (the number indicating length) on the calibration line to select it. Enter the length of the meter stick in this box (without units). For example, if your calibration meter stick is 1.00 meter long, enter 1.00 in the box (if not already there) and then click outside the box to accept the value. At this point, you can right-click the video frame to zoom in for more accurate adjustment of the ends of the calibration stick. Right-click the video again to zoom out.

5. Click the **Axes** button to set the origin and orientation of the x-y coordinate axes. Drag the origin of the axes to the desired position. Sometimes this will be the initial position of the object of interest. For rotating objects it will typically be the axis of rotation. Click the video outside the origin to fix the position of the origin. To change the orientation (angle) of the axes, drag the x axis. Click the video to fix the new orientation.

6. Click the **Create** button to track the object of interest in the video. From the menu of choices select **Point Mass** for the track type. Make sure the video is at the start frame, which shows the initial position of the object of interest. Mark this position by holding down the **shift key** and clicking the mouse (crosshair cursor) on the object. As the position is marked, the video automatically advances to the next frame. Similarly, mark the position of the object on this and subsequent frames by holding down the **shift key** and clicking the mouse. Do not skip any frames.

   After marking the position on the end frame, you can adjust any one of the marked positions. Advance the video to the frame where you would like to make a fine adjustment. Right-click the video frame to zoom in and drag the marked position with the mouse.

   If you would like to track additional objects, repeat the procedure outlined here for each object.

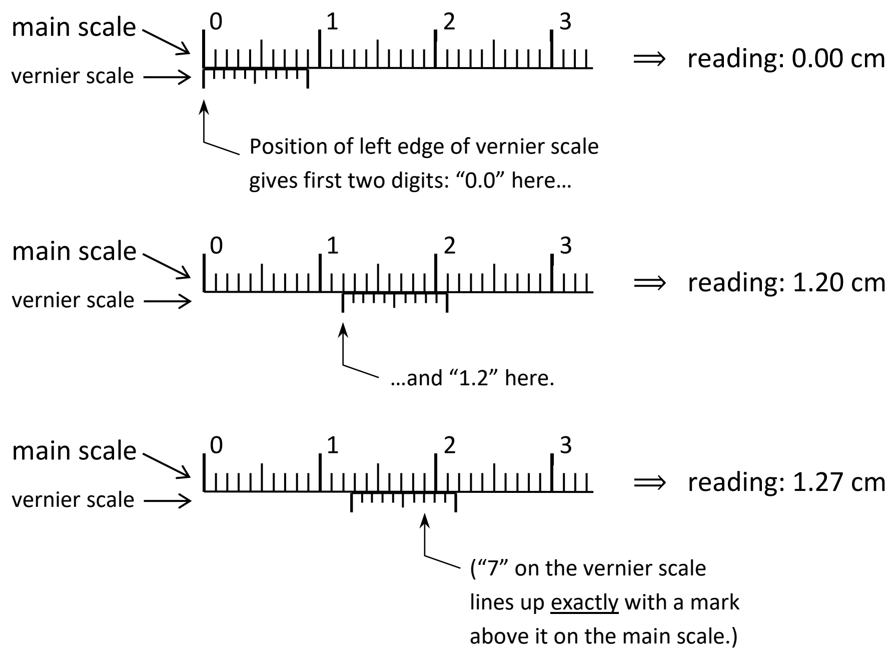
7. **Plotting and Analyzing the Tracks:** The track data (position versus time) are listed in the Table View and plotted in Plot View sections of the Tracker screen. Click the vertical axis label of the plot to change the variable plotted along that axis. To plot multiple graphs, click the **Plot** button, located above the plot, and select the desired number. Right-click on a plot to access display and analysis options in a pop-up menu. To fit your data to a line, parabola, or other functions, select the **Analyze** option. On the Data-Tool window that opens up, click the **Analyze** button and check the **Curve Fits** box from the drop-down menu. Then select the fit type from the **Fit Name** drop-down menu (lower left). Make sure the **Auto fit** box is checked. Note that the curve fitter fits the selected function to the data in the two leftmost columns of the displayed data table. The leftmost column, identified by a yellow header cell, defines the independent variable, and the second leftmost column, identified by a green header cell, defines the dependent variable. So, to fit the data in other columns, their corresponding headers must be dragged to the two leftmost columns. To print out the displayed plot and data table on the Data Tool screen, select **Print** from the **File** menu on the Data Tool screen.

8. **Printing and Exporting Data:** Track data can also be easily exported to *Excel* for further analysis by copying the data from the data table to the clipboard and pasting into *Excel.* Select the desired data (on Data Tool screen), go to **Edit** → **copy** → **selected data** (on Data Tool screen). Open *Excel* (in Microsoft Office folder), click on "HOME", and click the Paste clipboard. Data should appear in *Excel.* Graph can now be plotted in *Excel* (see **Appendix D: Excel**) and printed.

Appendix C: Vernier Calipers

A vernier is a small auxiliary scale that slides along the main scale. It allows more accurate estimates of fractional parts of the smallest division on the main scale.

On a vernier caliper, the main scale is engraved on the fixed part of the instrument. The vernier scale, engraved on the movable jaw, has ten divisions that cover the same spatial interval as nine divisions on the main scale: each vernier division is $\frac{9}{10}$ the length of a main scale division.



To measure length with a vernier caliper, close the jaws on the object and read the two scales.

- The first digit is read from the main scale. Look at where the left-most edge of the vernier scale lines up with the main scale, and read the digit from the main scale.

  In the top example above, the first digit is "0". In the second and third examples, it's "1".

- The second digit is also read from the main scale. Again, look at where the left-most edge of the vernier scale lines up, and read the closest subdivision from the main scale.

  In the second and third examples above, the second digit is "2".

- The third digit is read from the vernier scale. Read which mark on the vernier scale lines up *exactly* with a mark above it on the main scale.

  In the third example, it's clear that the left edge of the vernier scale is a little above 1.2 on the main scale, but it's hard to tell from the main scale exactly what the third digit is. (Maybe 1.25? or 1.26?) But look at the marks on the vernier scale: the 6 and 8 don't quite line up with any marks on the main scale, but the 7 lines up exactly, so the last digit is "7".

If the zero lines of the main and vernier scales do not align when the jaws are closed, all measurements will be systematically shifted. The magnitude of this shift, called the zero reading or zero correction, should be noted and recorded, so that length measurements made with the vernier caliper can be corrected, thereby removing the systematic error.

Appendix D: Introduction to Excel

Microsoft Excel is the spreadsheet program we will use for much of our data analysis and graphing. It is a powerful and easy-to-use application for graphing, fitting, and manipulating data. In this appendix, we will briefly describe how to use Excel to do some useful tasks. The current version is Excel 2013.

D.1 Data and Formulas

Figure 1 below shows a sample Excel spreadsheet containing data from a made-up experiment. The experimenter was trying to measure the density of a certain material by taking a set of cubes made of the material and measuring their masses and the lengths of the sides of the cubes. The first two columns contain her measured results. **Note that the top of each column contains both a description of the quantity contained in that column and its units.** You should make sure that all of the columns of your data tables do as well. You should also make sure that the whole spreadsheet has a descriptive title and your names at the top, as indicated in the sample spreadsheet below.

In the third column, the experimenter has figured out the volume of each of the cubes, by taking the cube of the length of a side. To avoid repetitious calculations, she had Excel do this automatically. She entered the formula `=B5^3` into cell C5. Note the equals sign, which indicates to Excel that a formula is coming. The ^ sign stands for raising to a power. After entering a formula into a cell, you can grab the square in the lower right corner of the cell with the mouse and drag it down the column, or you can just double-click on that square. (Either way, note that thing you're clicking on is the tiny square in the corner; clicking somewhere else in the cell won't work.) This will copy the cell, making the appropriate changes, into the rest of the column. For instance, in this case, cell C6 contains the formula `=B6^3`, and so forth.

Column D was similarly produced with a formula that divides the mass in column A by the volume in column C.

At the bottom of the spreadsheet we find the mean and standard deviation of the calculated densities (that is, of the numbers in cells D5 through D8). Those are computed using the formulas `=average(D5:D8)` and `=stdev(D5:D8)`.

D.2 Graphs

Here's how to make graphs in Excel. First, use the mouse to select the columns of numbers you want to graph. (If the two columns aren't next to each other, rearrange them so that they are next to each other. The variable you want on the horizontal axis needs to be to the left of the variable you want on the vertical axis). Then click on the *Insert* tab at the top of the window. In the menu that shows up, there is a section called *Charts*. Almost all of the graphs we make will be scatter plots (meaning plots with one point for each row of data), so click on the *Scatter* icon, which is the lower one in the group that looks like this . Several possible scatter plots appear. Select the first one with the same icon as before, and your graph will appear.

Next, you'll need to customize the graph in various ways, such as labeling the axes correctly.

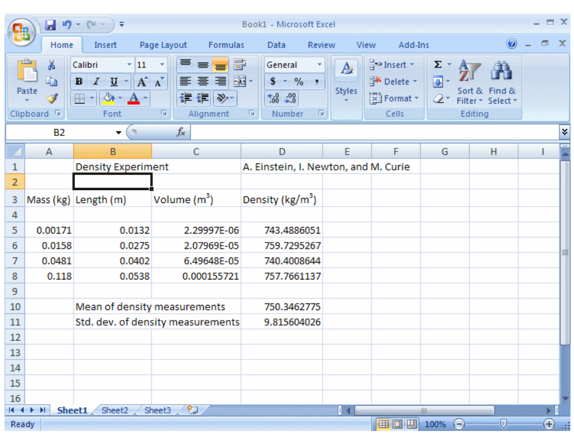


Figure 1: Sample Excel spreadsheet

This is done in the *Chart Elements* menu which is accessed by clicking on the plus sign (+) in a box, at the upper right corner of your graph. Click on *Axis Titles*, and places appear for axis titles. Edit the text inside of the two axis titles so that it indicates what's on the two axes of your graph, *including the appropriate units for each.*

Next, give your graph an overall title. Click on *Chart Title* and a box appears around it. Delete "Chart Title" and enter a proper title such as "Position vs Time" or whatever.

Usually you want your graph to contain a best-fit line passing through your data points. To do this, *right* click on one of the data points and select *Add Trendline*. A number of options appear such as "linear", "polynomial", "power", and so on. Select the one you want, and also check the *Display Equation on chart* option near the bottom. You can then drag the equation to someplace else on the chart so you can read it better. Remember that Excel won't include the correct units on the numbers in this equation, but you should. Also, Excel will always call the two variables x and y, even though they might be something else entirely. Bear these points in mind when transcribing the equation into your lab notebook.

Sometimes, you may want to make a graph in Excel where the x column is to the right of the y column in your worksheet. In these cases, Excel will make the graph with the x and y axes reversed. Here's how to fix this problem: Before you make your graph, make a copy of the y column in the worksheet and paste it so that it's to the right of the x column. Then follow the above procedure and everything will be fine.

D.3 Making Histograms

A histogram is a useful graphing tool when you want to analyze groups of data, based on the frequency at given intervals. In other words, you graph groups of numbers according to how often they appear. You start by choosing a set of 'bins', *i.e.*, creating a table of numbers that mark the edges of the intervals. You then go through your data, sorting the numbers into each bin or interval, and tabulating the number of data points that fall into each bin (this is the frequency). At the end, you have a visualization of the distribution of your data.

Start by entering your raw data in a column like the one shown in the left-hand panel of Figure 2. Look over your numbers to see what is the range of the data. If you have lots of values to sift through you might consider sorting your data is ascending or descending order. (You don't need to do this if you can easily see the range of data.) To do this task, choose the column containing your data by clicking on the letter at the top of the column,

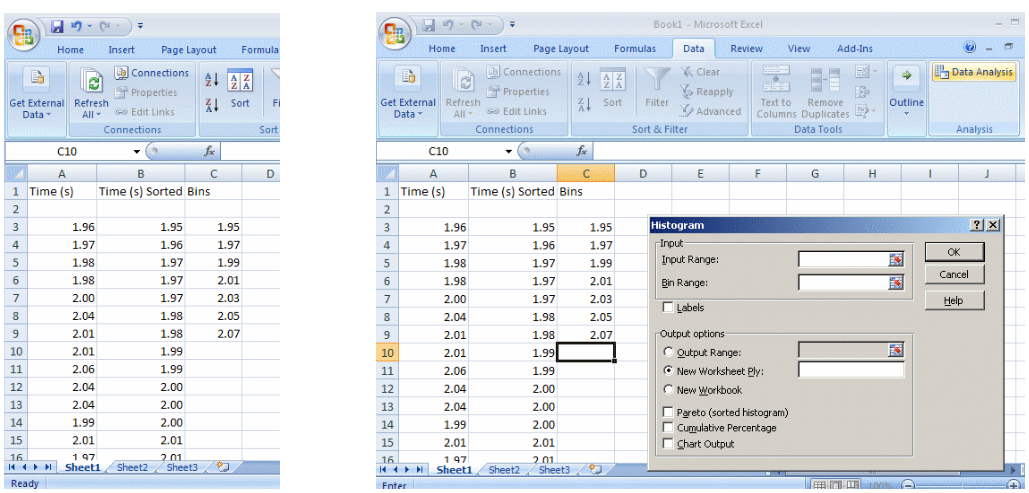


Figure 2: Column data and bins (left-hand panel) and dialog box (right-hand panel) for making a histogram in Excel.

go to *Data* in the menubar, select *Sort*, and pick ascending or descending. The data will be rearranged in the order you've chosen and it will be easier to see the range of the data. For an example, see the middle column of data in the left-hand panel of Figure 2.

Now to create your bins pick a new column on your spreadsheet and enter the values of the bin edges. Bins should be of equal size with the bin edges given by simple numbers. Make sure the bins you choose cover the range of the data. See the left-hand panel of Figure 2 again for an example.

You now have the ingredients for making the histogram. Go to *Data* in the menubar, select *Data Analysis* (on the right) and choose *Histogram*. Click *OK*. You should see a dialog box like the one in the right-hand panel of Figure 2. Click in the box labeled *Input Range* and then highlight the column on the spreadsheet containing your data. Next, click in the box labeled *Bin Range* and highlight the column on the spreadsheet containing the bins. Under *Output Options*, select *Chart Output* and click *OK*. The histogram should come up. If it doesn't, go to *Insert* in the menubar and select the histogram icon in the *Charts* section (the first icon in that section). Change the horizontal axis label to whatever you are plotting (including units). The vertical axis should already be labeled "Frequency". Change the chart title to an appropriate title for whatever you are plotting. The result should look like the right hand panel of Figure 3.

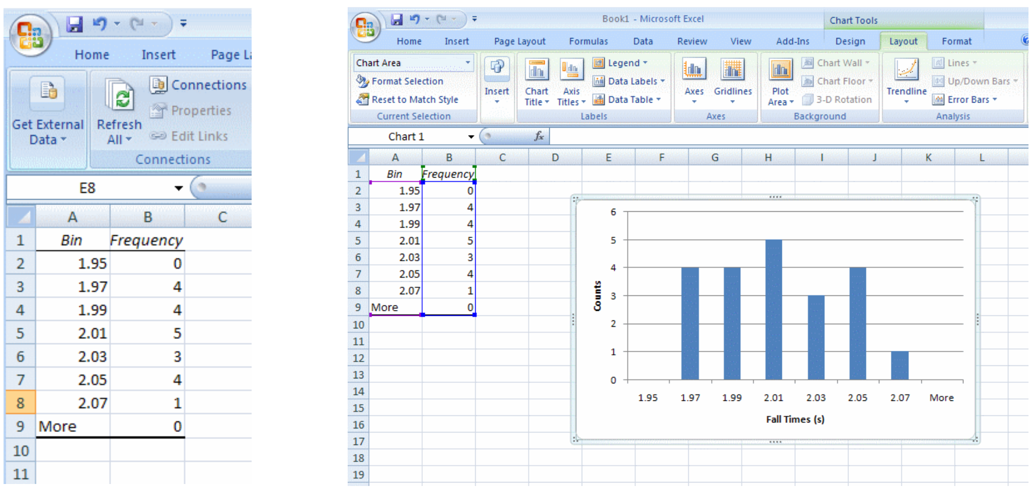


Figure 3: Newly-created worksheet (left-hand panel) and final plot (right-hand panel) for histogram worksheet in Excel.

D.4 LINEST

LINEST is a function in Excel which gives a LINear ESTimate of the slope and the uncertainty in the slope for any linear data set. It can also give the y-intercept and the uncertainty in the y-intercept. The uncertainties are based on the scatter of the data points about a perfect straight line. To use LINEST in Excel, perform the following steps:

1. Select a 2 x 2 group of cells (not including any cells with data)
2. In the function line type =LINEST(
3. Select range of y values
4. Type comma (,)
5. Select range of x values
6. Type comma (,)

7. To estimate both the slope and the y-intercept: Type 1,1)

   OR, to estimate only the slope, forcing a y-intercept of zero: Type 0,1)

8. Hold down Control AND Shift buttons, and press Enter

9. The result should look like this (if you typed "1,1"):

slope	intercept
δ slope	δ intercept

where δ slope represents the uncertainty in the slope value. You will still need to round off both numbers, depending on the relative magnitudes of the two numbers. It is also common practice to express both numbers to the same power of 10.

Appendix E: Treatment of Experimental Data

Recording Data

When performing an experiment, record all required original observations as soon as they are made. By "original observations" is meant what you actually see, not quantities found by calculation. For example, suppose you want to know the stretch of a coiled spring as caused by an added weight. You must read a scale both before and after the weight is added and then subtract one reading from the other to get the desired result. The proper scientific procedure is to record both readings as seen. Errors in calculations can be checked only if the original readings are on record.

All data should be recorded with units. If several measurements are made of the same physical quantity, the data should be recorded in a table with the units reported in the column heading.

Significant Figures

A laboratory worker must learn to determine how many figures in any measurement or calculation are reliable, or "significant" (that is, have physical meaning), and should avoid making long calculations using figures which he/she could not possibly claim to know. *All sure figures plus one estimated figure are considered significant.*

The measured diameter of a circle, for example, might be recorded to four significant figures, the fourth figure being in doubt, since it is an estimated fraction of the smallest division on the measuring apparatus. How this doubtful fourth figure affects the accuracy of the computed area can be seen from the following example.

Assume for example that the diameter of the circle has been measured as $.526\underline{4}$ cm, with the last digit being in doubt as indicated by the line under it. When this number is squared the result will contain eight digits, of which the last five are doubtful. Only one of the five doubtful digits should be retained, yielding a four-digit number as the final result.

In the sample calculation shown below, each doubtful figure has a short line under it. Of course, each figure obtained from the use of a doubtful figure will itself be doubtful. The result of this calculation should be recorded as 0.2771 cm^2, including the doubtful fourth figure. (The zero to the left of the decimal point is often used to emphasize that no significant figures precede the decimal point. This zero is not itself a significant figure.)

$$(.526\underline{4}\text{ cm})^2 = .277\underline{09696}\text{ cm}^2 = 0.277\underline{1}\text{ cm}^2$$

In multiplication and division, the rule is that a calculated result should contain the same number of significant figures as the least that were used in the calculation.

In addition and subtraction, do not carry a result beyond the first column that contains a doubtful figure.

Statistical Analysis

Any measurement is an intelligent estimation of the true value of the quantity being measured. To arrive at a "best value" we usually make several measurements of the same quantity and then analyze these measurements statistically. The results of such an analysis can be represented in several ways. Those in which we are most interested in this course are the following:

Mean - The mean is the sum of a number of measurements of a quantity divided by the number of such measurements. (In other words, the mean is the same thing as what people generally call the "average.") It generally represents the best estimate of true value of the measured quantity.

Standard Deviation - The standard deviation (σ) is a measure of the range on either side of the mean within which approximately two-thirds of the measured values fall. For example, if the mean is 9.75 m/s^2 and the standard deviation is 0.10 m/s^2, then approximately two-thirds of the measured values lie within the range 9.65 m/s^2 to 9.85 m/s^2. A customary way of expressing an experimentally determined value is: Mean$\pm\sigma$, or (9.75$\pm$ 0.10) m/s^2. Thus, the standard deviation is an indicator of the spread in the individual measurements, and a small σ implies high precision. Also, it means that the probability of any future measurement falling in this range is approximately two to one. The equation for calculating the standard deviation is

$$\sigma = \sqrt{\frac{\sum (x_i - \langle x \rangle)^2}{N-1}}$$

where x_i are the individual measurements, $\langle x \rangle$ is the mean, and N is the total number of measurements.

% Difference - Often one wishes to compare the value of a quantity determined in the laboratory with the best known or "accepted value" of the quantity obtained through repeated determinations by a number of investigators. *The % difference is calculated by subtracting the accepted value from your value, dividing by the accepted value, and multiplying by 100.* If your value is greater than the accepted value, the % difference will be positive. If your value is less than the accepted value, the % difference will be negative. The % difference between two values in a case where neither is an accepted value can be calculated by choosing either one as the accepted value.

Appendix F: A Single-Page Summary on Handling Uncertainties

Uncertainties in measurements:

When you estimate the uncertainty of a measurement, there should be a reasonable probability that the actual value of what you're measuring lies within your stated range of $x \pm \delta x$, where x is your *best guess* of its value and δx is the *uncertainty*. "Reasonable probability" usually means either 67% or 95%, called the *confidence level*. Sometimes you can only know the uncertainty from reading the manual for the measuring device. Other times you can use common sense to determine whether you would be willing to bet a cup of coffee on the final result.

Propagating uncertainties:

When a calculated quantity depends on a measurement that has uncertainty, do the calculation using both your *best-guess* value and one of the two *worst-case* values, then take the difference of the results. **Example:** calculating circumference C from measured diameter of (50 ± 2) cm.

- Best guess: $C = \pi D = \pi(50\,\text{cm}) = 157.1$ cm
- Worst case: $C = \pi D = \pi(52\,\text{cm}) = \underline{163.4\text{ cm}}$
- Difference of: 6.3 cm. $\Longrightarrow$ $\boxed{C = (157 \pm 6)\text{ cm.}}$

As long as the uncertainties are smallish, you only need to calculate one worst case.

Two or more sources of uncertainty:

When a calculated quantity depends on two or more measurements that have uncertainty, start by finding the uncertainty in the calculated quantity from each ONE of the measurements separately, as above. Then combine those individual uncertainties "in quadrature," like legs of a right triangle. **Example:** calculating $W = F\Delta x$, where $F = (45 \pm 3)$ N and $\Delta x = (100 \pm 9)$ cm.

- Uncertainty in F only: $F = (45 \pm 3)$ N, $\Delta x = 100.000$ cm $\Longrightarrow W = (45 \pm 3)$ Joules.
- Uncertainty in Δx only: $F = 45.000$ N, $\Delta x = (100 \pm 9)$ cm $\Longrightarrow W = (45 \pm 4)$ Joules.

- Combine in quadrature: $\sqrt{(3\text{ J})^2 + (4\text{ J})^2} = 5$ J. $\Longrightarrow$ $\boxed{W = (45 \pm 5)\text{ Joules.}}$

Three or more sources of uncertainty combine the same way: $\sqrt{(\ )^2 + (\ )^2 + ... + (\ )^2}$.

Correlated uncertainties:
Combining uncertainties in quadrature as above is what you do when the uncertainties are *uncorrelated*: each measurement could be either high or low, independent of the other one. But if the uncertainties are *correlated* (one measurement high means the other is high too), then calculate a single worst-case scenario with both measurements too high, or both too low. **Example:** You want the difference between two masses, $m_2 - m_1$, each measured on the same scale, which might be miscalibrated by up to 1%. Suppose $m_2 = (400 \pm 4)$ g, and $m_1 = (300 \pm 3)$ g.

- Best guess: $\Delta m = m_2 - m_1 = 400\text{ g} - 300\text{ g} = 100\text{ g}$
- Worst case: $\Delta m = m_2 - m_1 = 404\text{ g} - 303\text{ g} = \underline{101\text{ g}}$
- Difference of: 1 g. $\Longrightarrow$ $\boxed{\Delta m = (100 \pm 1)\text{ g.}}$

You can see right away that adding the uncertainties in quadrature would give (100 ± 5) g, which is crazy. (If they were measured by different scales, then you really could have $m_2 = 403$ g and $m_1 = 298$ g, so $\Delta m = (100 \pm 5)$ g would be reasonable, not crazy.) Also note that for adding the masses, a shortcut is to add the uncertainties directly, not in quadrature: $m_{total} = (700 \pm 7)$ g.

Disclaimer:
There are enough shortcuts, special techniques, definitions, and rigorous justifications to fill a book. If you want them, go find a book. This is just a single page.