

Physics 303
One-Dimensional Free Fall

1. In solving Newton's Second Law for free fall through a resistive medium we used the following expression to represent the drag force

$$\vec{F}_f = -\frac{1}{2}C_D\rho S v^2 \hat{v} \quad (1)$$

where C_D is the drag coefficient, ρ is the density, S is the cross-sectional area, v is velocity, and \hat{v} is a unit vector in the direction of \vec{v} . We found that v and t are related by

$$\frac{1}{2} \ln \left(\frac{1 + \frac{v}{v_t}}{1 - \frac{v}{v_t}} \right) = -\frac{g}{v_t} t \quad (2)$$

where

$$v_t = \frac{2mg}{C_D\rho S} \quad (3)$$

and m is the mass and g is the acceleration of gravity. Show that Equation 2 can be written in the following form.

$$v(t) = -v_t \left[\frac{1 - \exp(-\frac{2gt}{v_t})}{1 + \exp(-\frac{2gt}{v_t})} \right] \quad (4)$$

2. Show that Equation 4 can be written in the following form

$$v(t) = -v_t \tanh \left(\frac{gt}{v_t} \right) \quad (5)$$

where $\tanh x$ means the hyperbolic tangent of x defined in the following way.

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}} \quad (6)$$

3. Apply a direct integration to Equation 5 to get the general solution for $y(t)$ and then use the initial conditions that at $t = 0$, $y = y_0$ to find the particular solution.
4. Use the solution from Problem 3 to show that time it takes for Lt. Chisov to hit the ground can be written in the following way

$$t_{hit} = \frac{v_t}{g} \text{ArcCosh} \left[\exp \left(\frac{gy_0}{v_t^2} \right) \right] \quad (7)$$

where $\text{ArcCosh} x$ means the inverse hyperbolic cosine of x .