Rutherford Scattering 2

1. Use *Mathematica* to get the integral of
$$\int \frac{dr}{\sqrt{r^2(\frac{2\mu E}{l^2}r^2 + \frac{2\mu\alpha}{l^2}r - 1)}} = \theta - \theta_0$$
and show the result (known as the orbit equation) can be written in the following way
$$\frac{1}{r} = \frac{\mu\alpha}{l^2} + \frac{\mu\alpha}{l^2}\epsilon\cos(\theta - \theta_0)$$
where μ is the reduced mass (referred to as m in your text), E is the energy, l is the angular momentum, $\alpha = Gm_1m_2$ or $\alpha = -Z_1Z_2e^2/4\pi\epsilon_0$, and ϵ is the eccentricity
$$\epsilon = \sqrt{1 + \frac{2El^2}{\mu\alpha^2}} \quad .$$

2. When the projectile starts its journey it is essentially infinitely far away and its angle is equal to $\theta = 180°$. Use this information and the orbit equation (see Problem 1) to show that $\theta_0 = -\arccos(1/\epsilon)$. The negative sign is required for the projectile to start at $\theta = 180°$.

3. When the projectile finally reaches the detector the target-detector distance r is essentially infinite compared to the sizes of the particles and distances between them when they are strongly interacting. Let θ_s be the final, asymptotic scattering angle of the projectile and show the following two statements are true using the orbit equation.
$$\theta_s = \theta_0 + \arccos\left(-\frac{1}{\epsilon}\right) \qquad\qquad \sin\frac{\theta_s}{2} = \frac{1}{\epsilon}$$
Clearly point out any sign choices you make. You may find the trigonometric identity $\arccos(-x) = \pi - \arccos(x)$ helpful.

4. Show that
$$\epsilon = \sqrt{1 + \left(\frac{\mu v_{1i}^2 b}{\alpha}\right)^2}$$
where v_{i1} is the initial velocity of the projectile (referred to as v_0 in your text) and b is the impact parameter. Start with the equation in Problem 1 for ϵ.

5. Combine the results of problems 3 and 4 to prove the following expression.
$$\sin\frac{\theta_s}{2} = \frac{1}{\sqrt{1 + \left(\frac{\mu v_{1i}^2 b}{\alpha}\right)^2}}$$
Invert this relationship to get the impact parameter b in terms of the scattering angle θ_s and show the following expression is true.
$$b = \frac{|\alpha|}{\mu v_{1i}^2}\cot\frac{\theta_s}{2}$$