Rutherford Scattering 1

1. For a two-particle collision the center of mass is defined by

$$\vec{p}\,' = m_1\vec{v}_{1i}^{\,\prime} + m_2\vec{v}_{2i}^{\,\prime} = m_1\vec{v}_{1f}^{\,\prime} + m_2\vec{v}_{2f}^{\,\prime} = 0$$

   where the primes refer to quantities in the center-of-mass (CM) frame. In the laboratory frame

$$\vec{v}_1 = \vec{v}_1{}' + \vec{V}_0$$

   where $\vec{V}_0$ is the CM velocity in the lab frame. Show that in the lab frame

$$K_{lab} = \frac{1}{2}\left(m_1 v_1'^{\,2} + m_2 v_2'^{\,2}\right) + \frac{1}{2}MV_0^2$$

   where K is the kinetic energy and $M = m_1 + m_2$.

2. The relative velocity between the two particles in the collision is

$$\vec{v}_{rel} = \vec{v}_1{}' - \vec{v}_2{}'$$

   in the CM. Show the following relationships.

$$\vec{v}_1{}' = \frac{m_2}{m_1 + m_2}\vec{v}_{rel} \qquad \text{and} \qquad \vec{v}_2{}' = -\frac{m_1}{m_1 + m_2}\vec{v}_{rel}$$

   Use these last results and show

$$K_{lab} = \frac{1}{2}\mu v_{rel}^2 + \frac{1}{2}MV_0^2$$

   where

$$\mu = \frac{m_1 m_2}{m_1 + m_2} \qquad .$$

3. The kinetic energy in the CM is the following.

$$K_{cm} = \frac{1}{2}\mu v_{rel}^2$$

   Show that in polar coordinates K_{rel} can be written as

$$K_{rel} = \frac{1}{2}\mu(\dot{r}^2 + r^2\dot{\theta}^2)$$

   where r and θ are the polar components of the relative vector between the two particles in the CM.