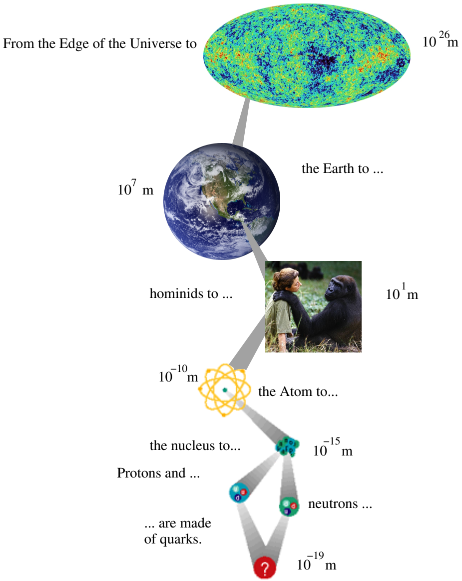
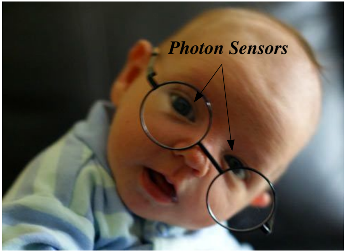
The Size of the Universe and the Stuff In It

Watch



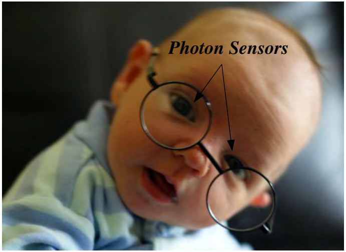
How Do We Know The Really Small Stuff? 2

SCATTERING!



SCATTERING!

RUTHERFORD SCATTERING!



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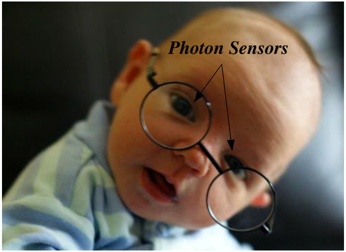
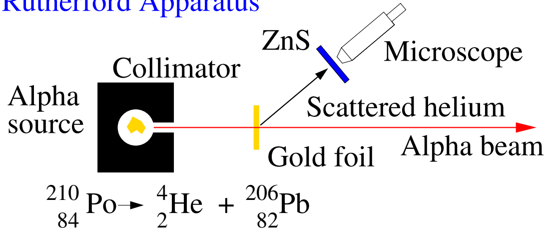


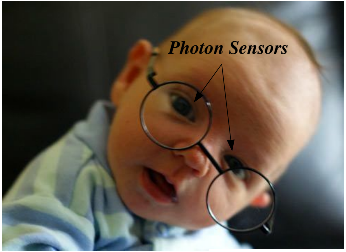
Rutherford Apparatus



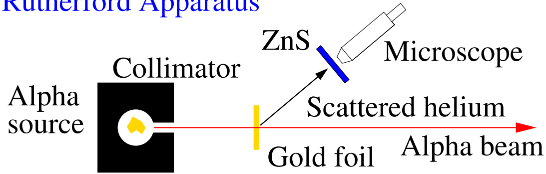
Simulation is here.

SCATTERING!

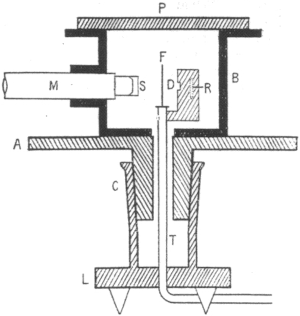
RUTHERFORD SCATTERING!



Rutherford Apparatus



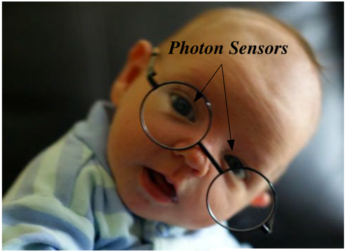
$$^{210}_{84}\mathrm{Po} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{206}_{82}\mathrm{Pb}$$

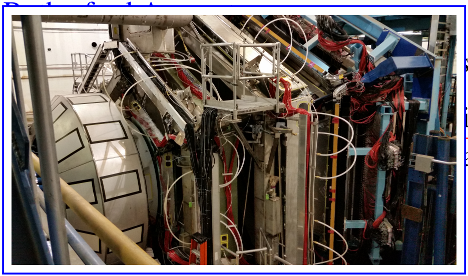


Simulation is here.

SCATTERING!

RUTHERFORD SCATTERING!

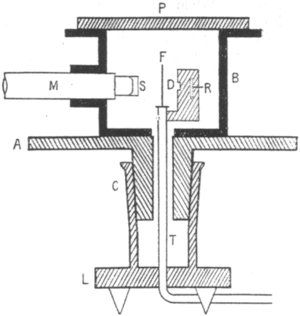




scope

ium

a beam

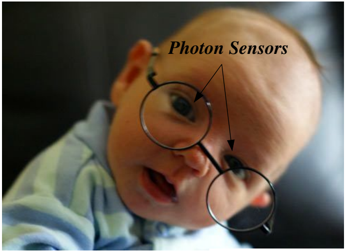


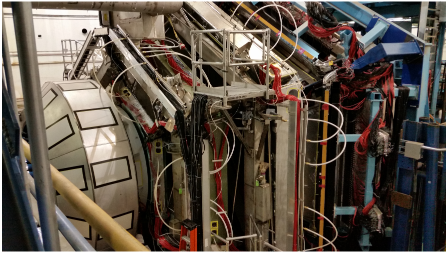
Simulation is here.

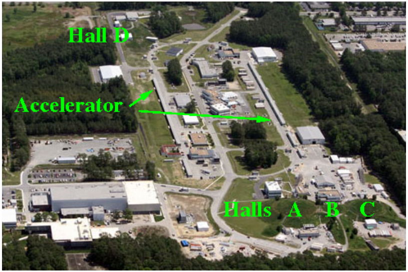
How Do We Know The Really Small Stuff?

SCATTERING!

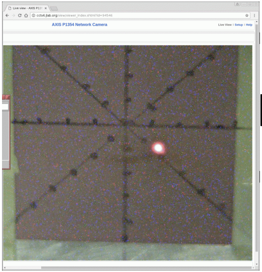
RUTHERFORD SCATTERING!

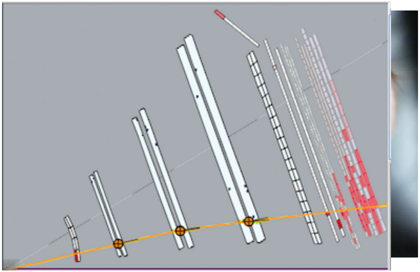


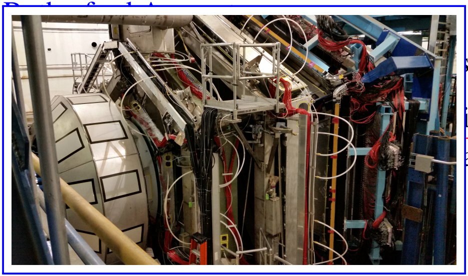


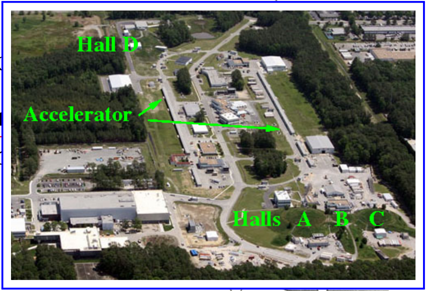


Simulation is here.



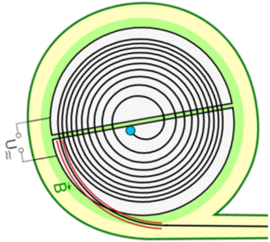




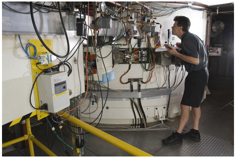


Simulation is here.

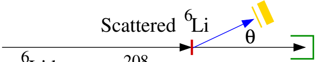
The experimental setup shown below is analogous to Rutherford's used to discover the nucleus. A beam of ${}^{6}\mathrm{Li}$-nuclei is accelerated to an energy $E_{lab} = 73.7\ \mathrm{MeV}$ in a cyclotron. It strikes a lead (${}^{208}\mathrm{Pb}$) target scattering ${}^{6}\mathrm{Li}$ into a $\Delta\mathrm{E} - \mathrm{E}$ silicon detector. The plot shows the differential cross section measured as a function of θ. (1) How do these results compare to the Rutherford cross section? (2) What is the distance of closest approach (DOCA) of the ${}^{6}\mathrm{Li}$ to the ${}^{208}\mathrm{Pb}$ target before the ${}^{6}\mathrm{Li}$ and ${}^{208}\mathrm{Pb}$ actually collide?

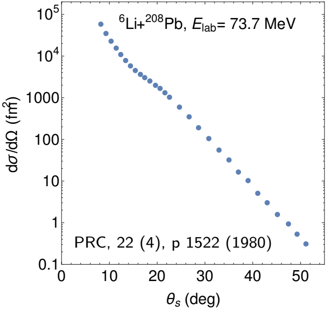


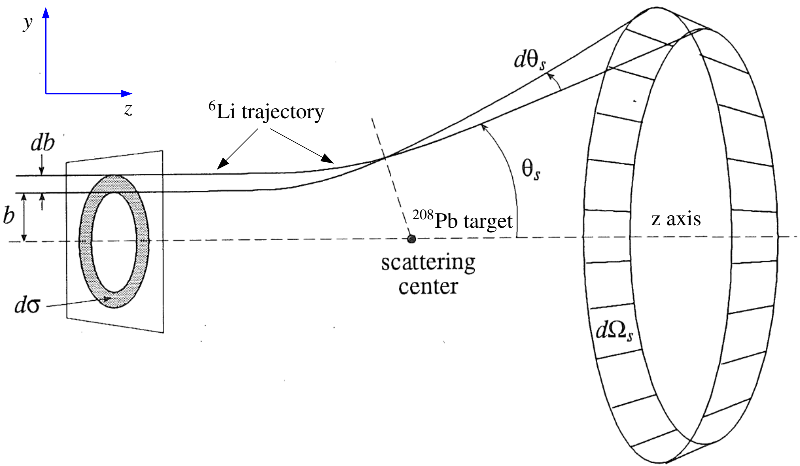
Cyclotron



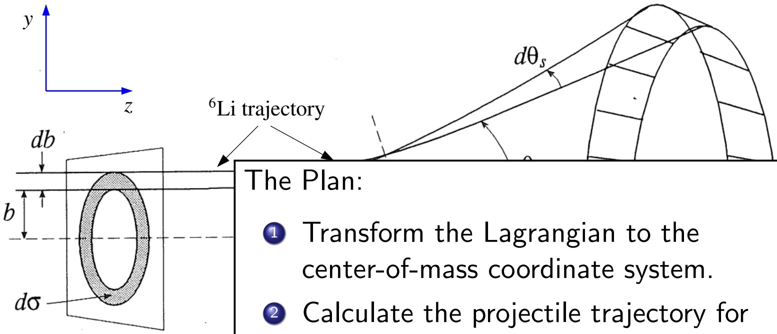
ΔE–E silicon telescope







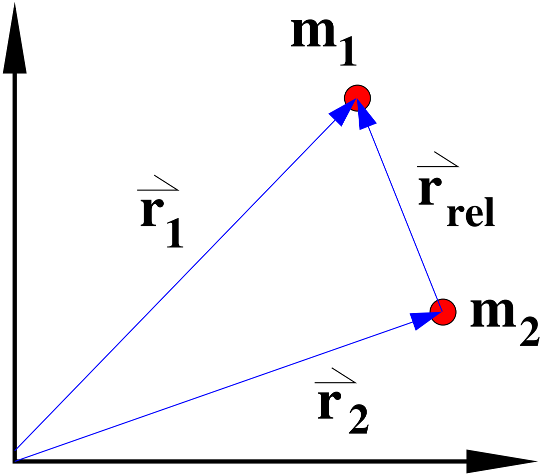
Simulation is here.



The Plan:

1. Transform the Lagrangian to the center-of-mass coordinate system.
2. Calculate the projectile trajectory for Coulomb repulsion.
3. Relate b and θ_s.
4. Construct $d\sigma/d\Omega$.
5. Get $d\sigma/d\Omega$ for random impact parameters.
6. Compare with data.

Simulation is here.



m_1
$\vec{r}_1$
$\vec{r}_{rel}$
m_2
$\vec{r}_2$

For two particles m_1 and m_2 interacting through some force we will use a particular coordinate system called the center-of-mass (CM) system. In the CM frame the total momentum is required to be zero so the CM is

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

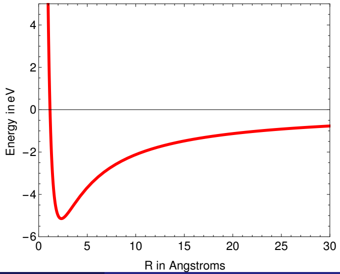
and it acts like this. The two particles in the system now behave as single particle with a different mass called the reduced mass.

$$m_r = \frac{m_1 m_2}{m_1 + m_2}$$

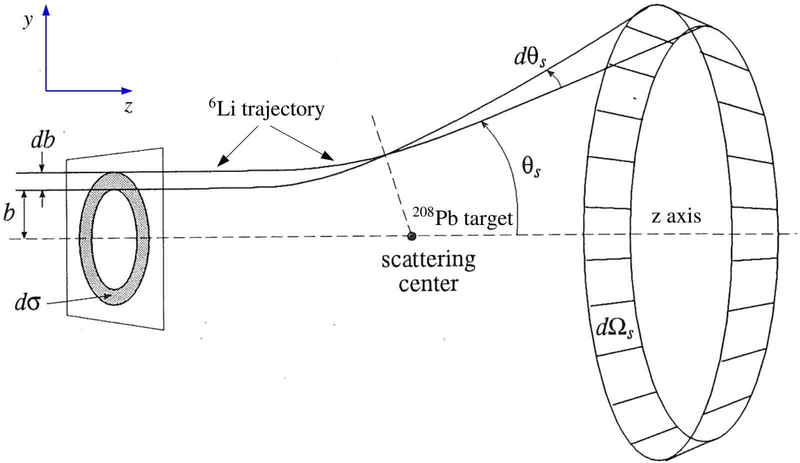
The potential energy between a Na^+ ion and a Cl^- ion is

$$V(r) = -\frac{A}{r} + \frac{B}{r^2}$$

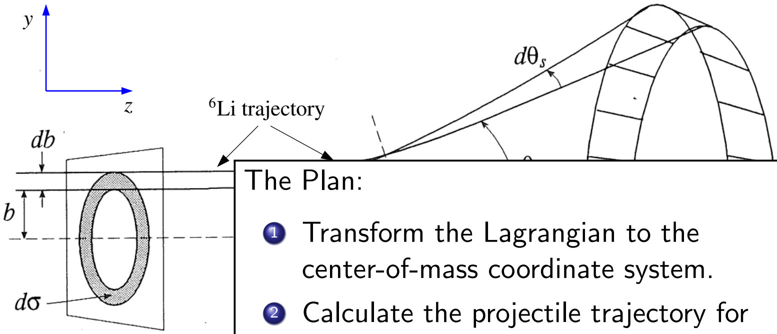
where $\mathrm{A} = 24\ \mathrm{eV} - \mathrm{\mathring{A}}$ and $\mathrm{B} = 28\ \mathrm{eV} - \mathrm{\mathring{A}}^2$. Is the attractive part of V consistent with the force between two point charges? Where is the equilibrium point? What equation describes the ions' separation near the equilibrium point? What is the energy of the system? At $t = 0$, the separation of the ions is $2.0\ \mathrm{\mathring{A}}$ and their relative velocity is zero.



We treated the vibrations of the Na-Cl system as a single harmonic oscillator with an 'average' mass.



Simulation is here.



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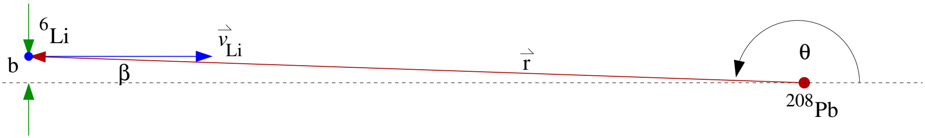
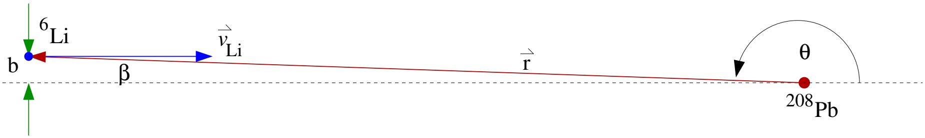
Simulation is here.

Initial Rutherford scattering geometry

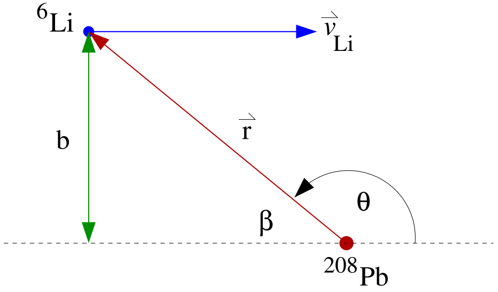


Initial Rutherford scattering geometry

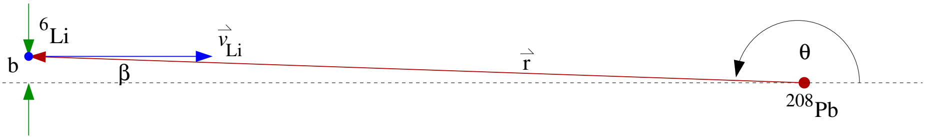


Re-scale it to see the angles better.

$$L = \mu r^2 \dot{\theta} = \mu r (r\dot{\theta})$$



Initial Rutherford scattering geometry



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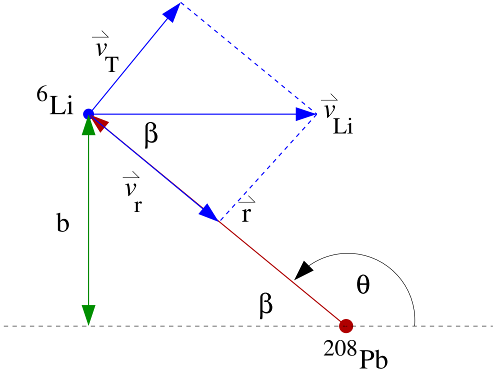
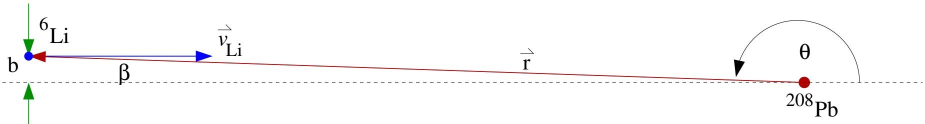
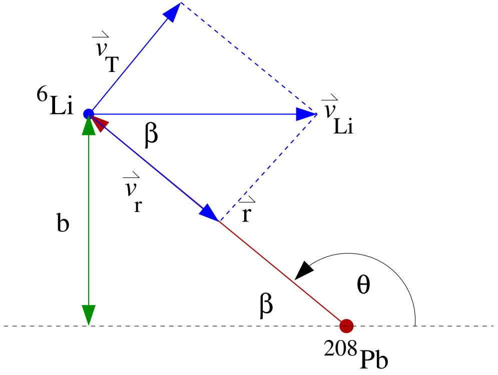


Initial Rutherford scattering geometry

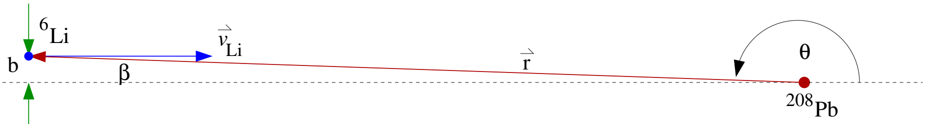


Re-scale it to see the angles better.

$L = \mu r^2 \dot{\theta} = \mu r (r\dot{\theta}) = \mu r v_T$

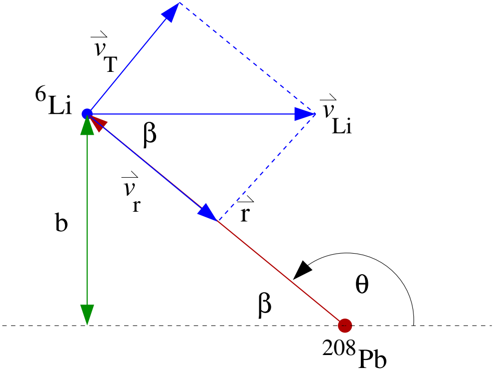


Initial Rutherford scattering geometry

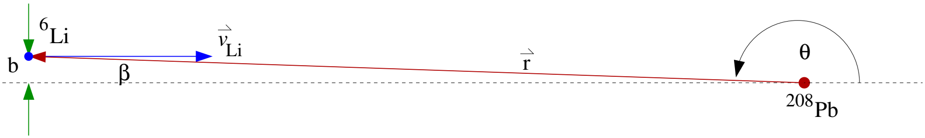


Re-scale it to see the angles better.

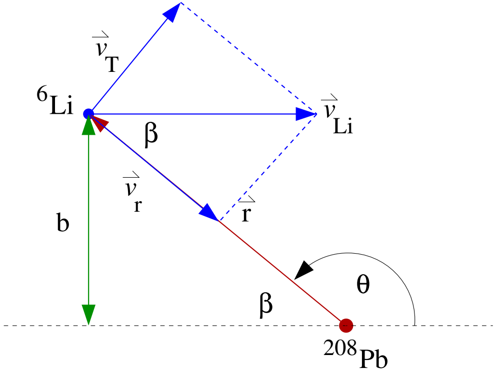
$$L = \mu r^2 \dot{\theta} = \mu r(r\dot{\theta}) = \mu r v_T$$
$$= \mu r v_{Li} \sin\beta$$



Initial Rutherford scattering geometry

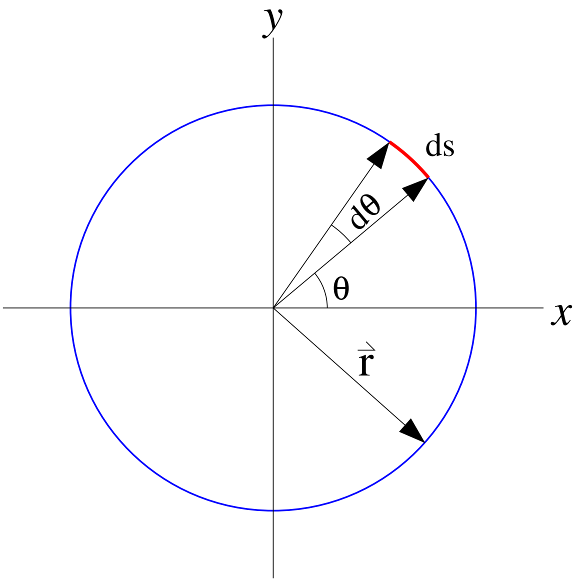


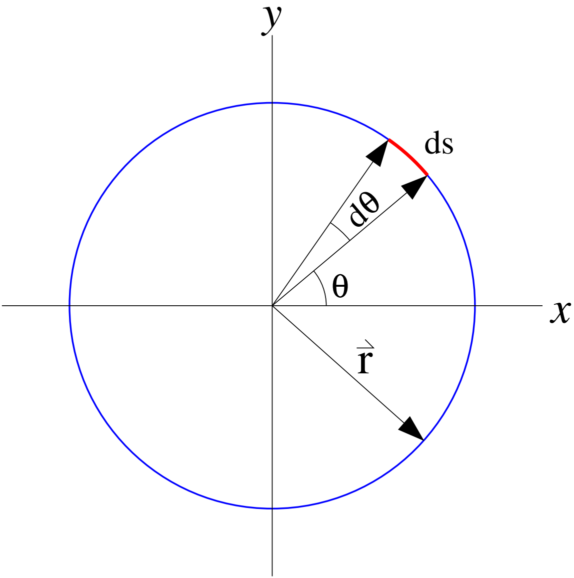
Re-scale it to see the angles better.



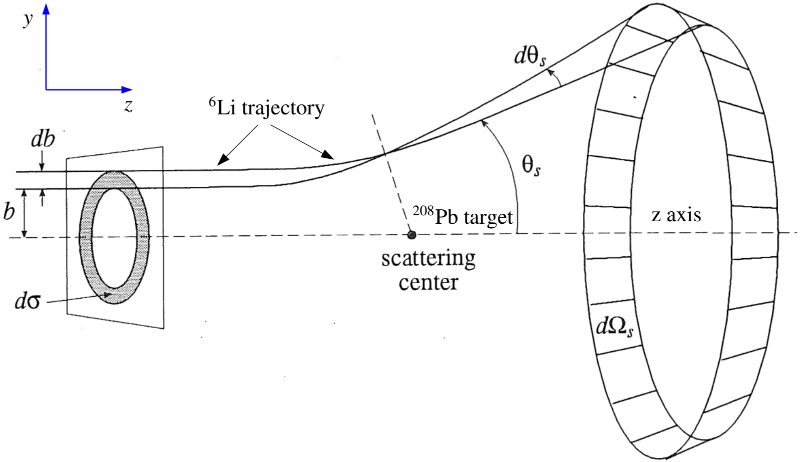
$$L = \mu r^2 \dot{\theta} = \mu r(r\dot{\theta}) = \mu r v_T$$
$$= \mu r v_{Li} \sin\beta = \mu v_{Li} b$$

What is an Angle?





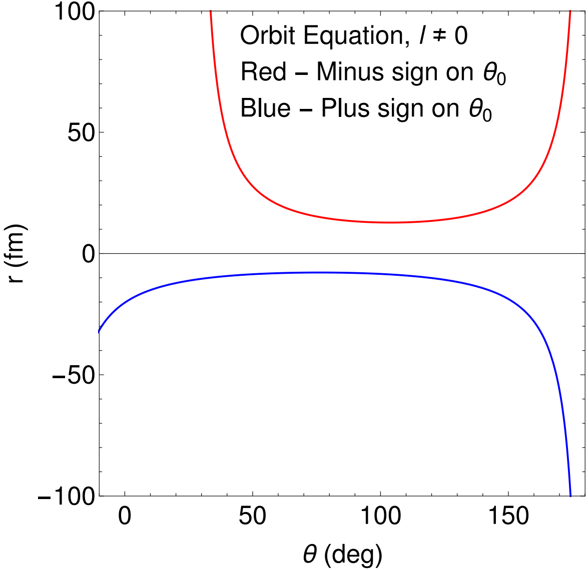
$$d\theta = \frac{ds}{|\vec{r}|}$$



y
z
^{6}Li trajectory
db
b
$d\sigma$
$d\theta_s$
θ_s
^{208}Pb target
scattering center
z axis
$d\Omega_s$

Phase Relations for Trignometric Functions

$$
\begin{array}{ll}
\sin(\pi-\theta) = +\sin\theta & \sin\left(\theta+\frac{\pi}{2}\right) = +\cos\theta \\
\cos(\pi-\theta) = -\cos\theta & \cos\left(\theta+\frac{\pi}{2}\right) = -\sin\theta \\
\tan(\pi-\theta) = -\tan\theta & \tan\left(\theta+\frac{\pi}{2}\right) = -\cot\theta \\
\csc(\pi-\theta) = +\csc\theta & \csc\left(\theta+\frac{\pi}{2}\right) = +\sec\theta \\
\sec(\pi-\theta) = -\sec\theta & \sec\left(\theta+\frac{\pi}{2}\right) = -\csc\theta \\
\cot(\pi-\theta) = -\cot\theta & \cot\left(\theta+\frac{\pi}{2}\right) = -\tan\theta
\end{array}
$$

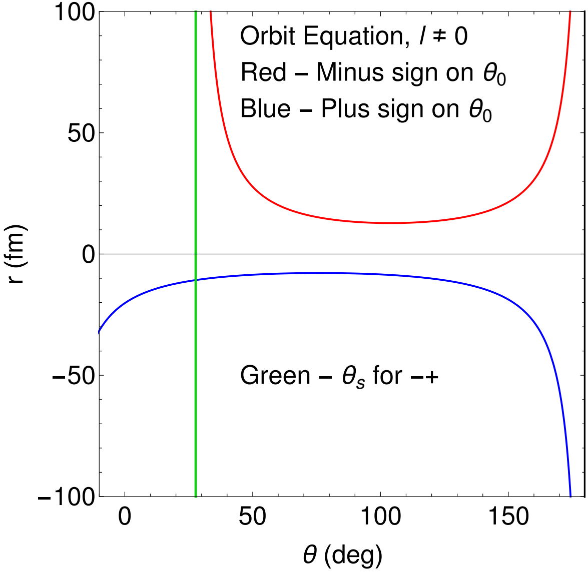

Orbit Equation, $l \neq 0$
Red – Minus sign on θ_0
Blue – Plus sign on θ_0
r (fm)
100
50
0
−50
−100
0
50
100
150
θ (deg)

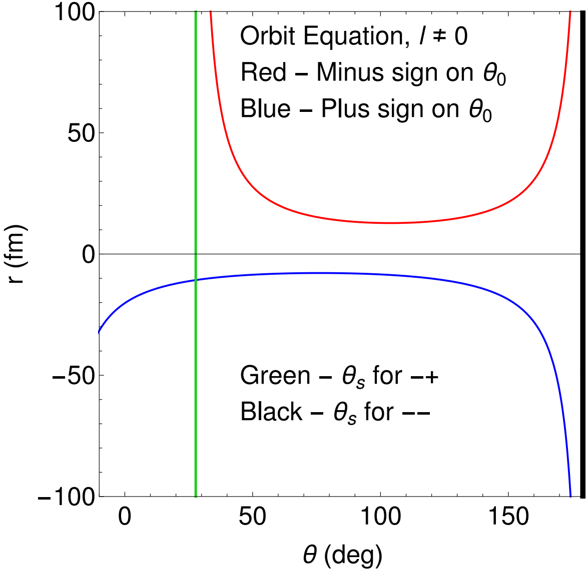
Negative Argument Formulas for Trignometric Functions

$$\sin(-\theta) = -\sin\theta$$
$$\cos(-\theta) = +\cos\theta$$
$$\tan(-\theta) = -\tan\theta$$
$$\csc(-\theta) = -\csc\theta$$
$$\sec(-\theta) = +\sec\theta$$
$$\cot(-\theta) = -\cot\theta$$

Negative Argument Formulas for Inverse Trignometric Functions

$$\arcsin(-x) = -\arcsin(x)$$
$$\arccos(-x) = \pi - \arccos(x)$$
$$\arctan(-x) = -\arctan(x)$$
$$\mathrm{arccot}(-x) = \pi - \mathrm{arccot}(x)$$
$$\mathrm{arcsec}(-x) = \pi - \mathrm{arcsec}(x)$$
$$\mathrm{arccsc}(-x) = -\mathrm{arccsc}(x)$$


Orbit Equation, $l \neq 0$
Red – Minus sign on θ_0
Blue – Plus sign on θ_0
Green – θ_s for −+
r (fm)
θ (deg)
100
50
0
−50
−100
0
50
100
150


Orbit Equation, $l \neq 0$
Red – Minus sign on θ_0
Blue – Plus sign on θ_0
Green – θ_s for −+
Black – θ_s for −−
r (fm)
100
50
0
−50
−100
0
50
100
150
θ (deg)

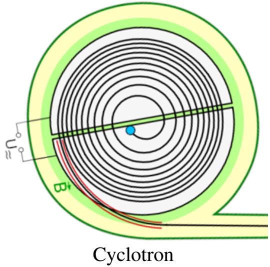
Negative Argument Formulas for Trignometric Functions

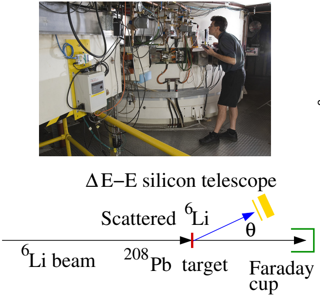
$$\sin(-\theta) = -\sin\theta$$
$$\cos(-\theta) = +\cos\theta$$
$$\tan(-\theta) = -\tan\theta$$
$$\csc(-\theta) = -\csc\theta$$
$$\sec(-\theta) = +\sec\theta$$
$$\cot(-\theta) = -\cot\theta$$

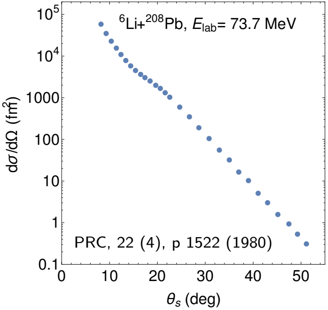
Phase Relationships for Trignometric Functions

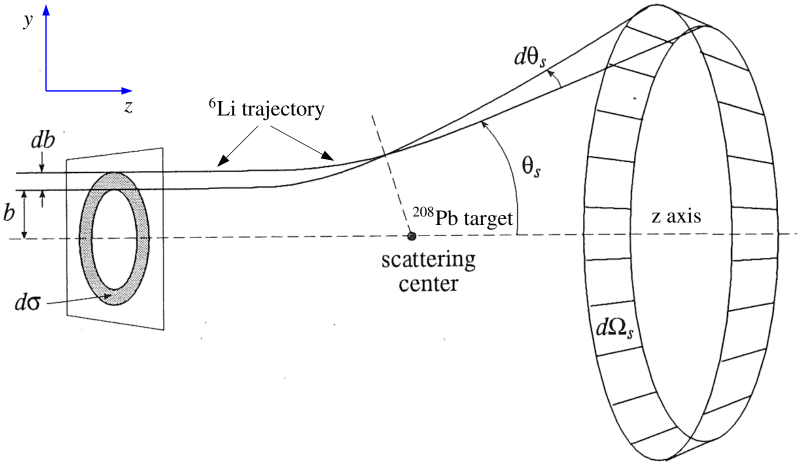
$$\begin{array}{ll}
\sin(\frac{\pi}{2} - \theta) = +\cos\theta & \sin(\theta + \pi) = -\sin\theta \\
\cos(\frac{\pi}{2} - \theta) = +\sin\theta & \cos(\theta + \pi) = -\cos\theta \\
\tan(\frac{\pi}{2} - \theta) = +\cot\theta & \tan(\theta + \pi) = +\tan\theta \\
\csc(\frac{\pi}{2} - \theta) = +\sec\theta & \csc(\theta + \pi) = -\csc\theta \\
\sec(\frac{\pi}{2} - \theta) = +\csc\theta & \sec(\theta + \pi) = -\sec\theta \\
\cot(\frac{\pi}{2} - \theta) = +\tan\theta & \cot(\theta + \pi) = +\cot\theta
\end{array}$$

The experimental setup shown below is analogous to Rutherford's used to discover the nucleus. A beam of $^{6}\mathrm{Li}$-nuclei is accelerated to an energy $E_{lab} = 73.7\ \mathrm{MeV}$ in a cyclotron. It strikes a lead ($^{208}\mathrm{Pb}$) target scattering $^{6}\mathrm{Li}$ into a $\Delta\mathrm{E}-\mathrm{E}$ silicon detector. The plot shows the differential cross section measured as a function of θ. (1) How do these results compare to the Rutherford cross section? (2) What is the distance of closest approach (DOCA) of the $^{6}\mathrm{Li}$ to the $^{208}\mathrm{Pb}$ target before the $^{6}\mathrm{Li}$ and $^{208}\mathrm{Pb}$ actually collide?

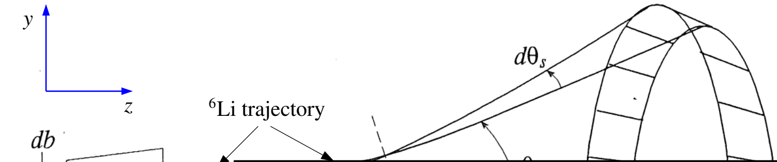


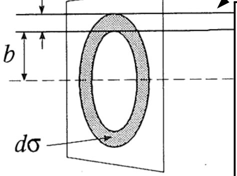






Simulation is here.





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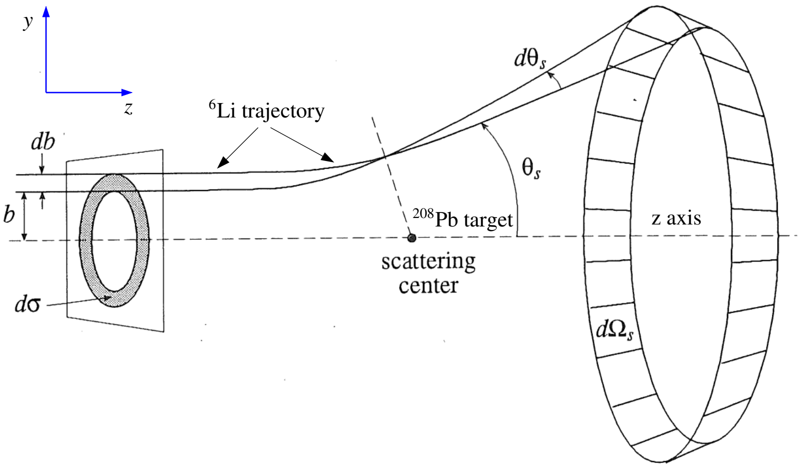
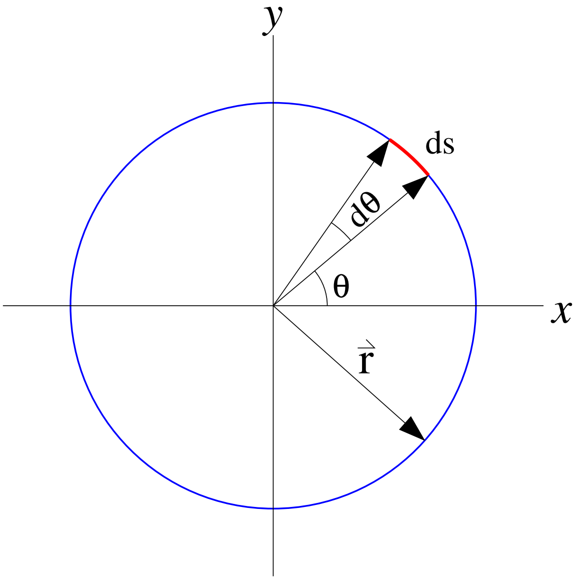
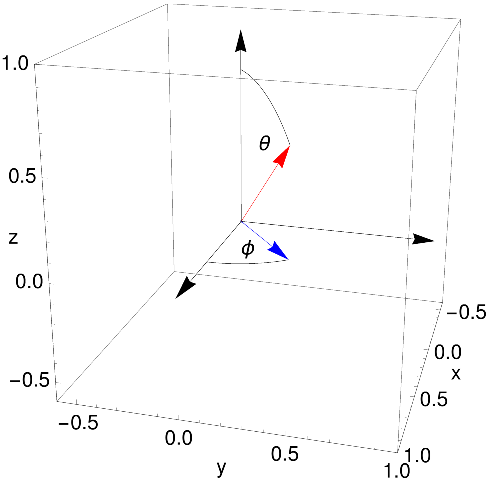
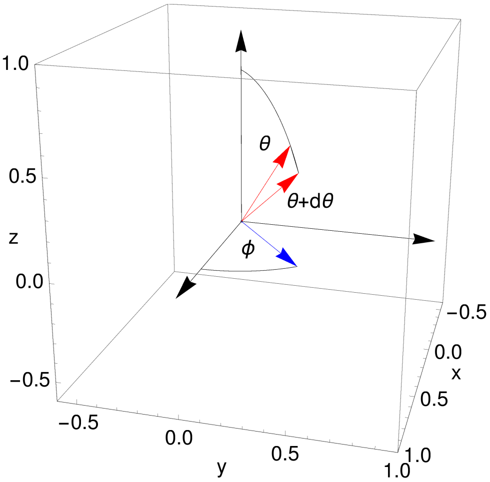

y
z
^{6}Li trajectory
db
b
dσ
$d\theta_s$
θ_s
^{208}Pb target
scattering center
z axis
$d\Omega_s$

What is an Angle?

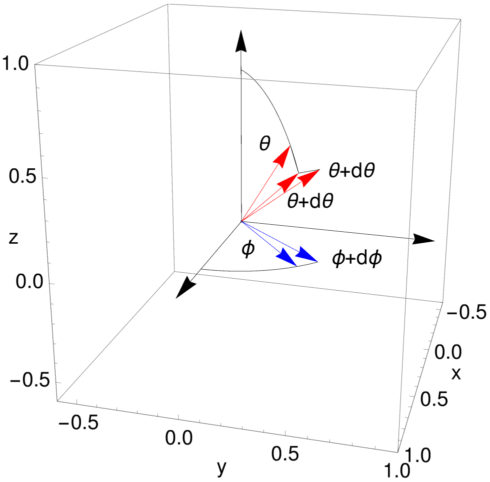


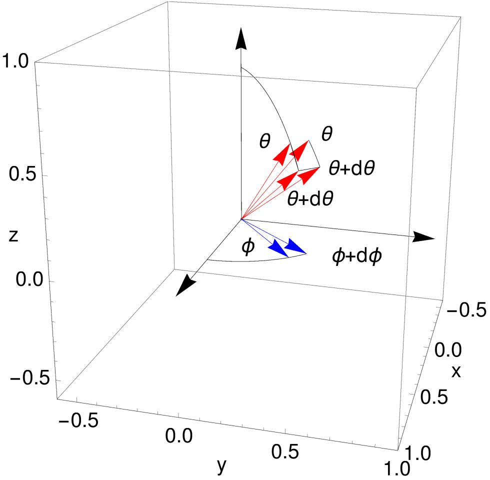
$$d\theta = \frac{ds}{|\vec{r}|}$$


1.0
0.5
z
0.0
−0.5
θ
ϕ
−0.5
0.0
0.5
1.0
y
−0.5
0.0
x
0.5
1.0

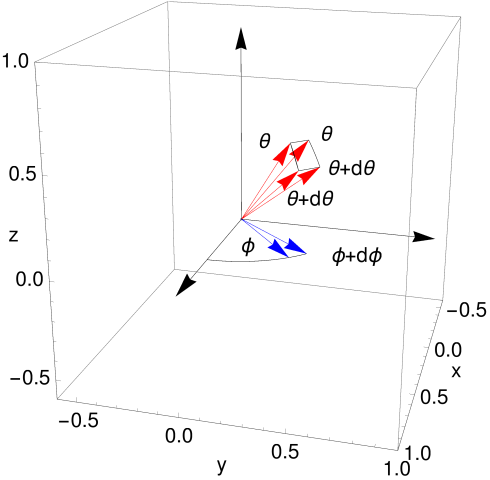

1.0
0.5
0.0
−0.5
z
θ
θ+dθ
ϕ
−0.5
0.0
0.5
1.0
y
−0.5
0.0
x
0.5
1.0

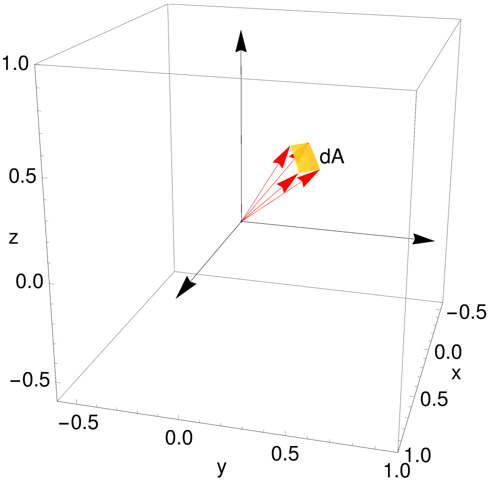
Solid Angle



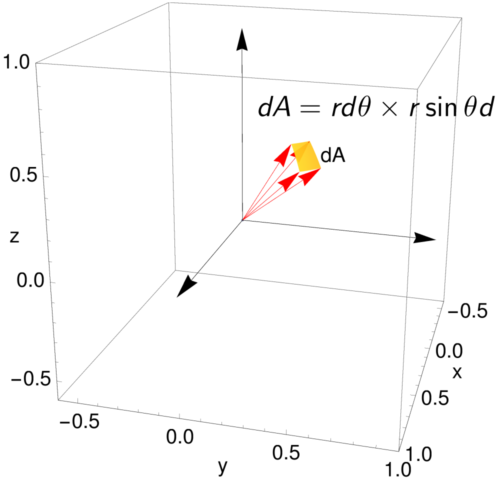


1.0
0.5
0.0
−0.5
z
θ
θ
θ+dθ
θ+dθ
ϕ
ϕ+dϕ
−0.5
0.0
x
0.5
1.0
−0.5
0.0
0.5
1.0
y

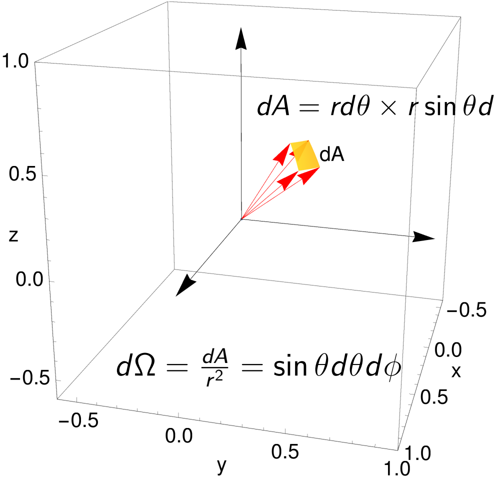

1.0
0.5
z
0.0
−0.5
θ
θ
θ+dθ
θ+dθ
ϕ
ϕ+dϕ
−0.5
0.0
x
0.5
1.0
−0.5
0.0
0.5
y
1.0


1.0
0.5
z
0.0
−0.5
dA
−0.5
0.0
y
0.5
1.0
−0.5
0.0
x
0.5
1.0

$$dA = r d\theta \times r \sin\theta d\phi = r^2 \sin\theta d\theta d\phi$$

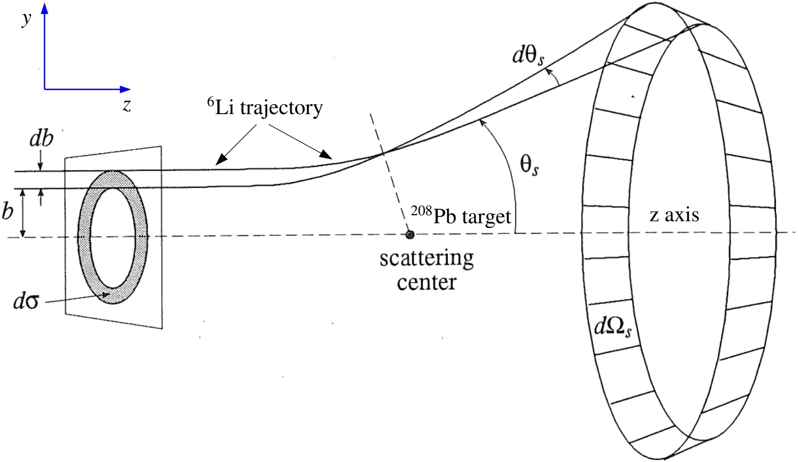


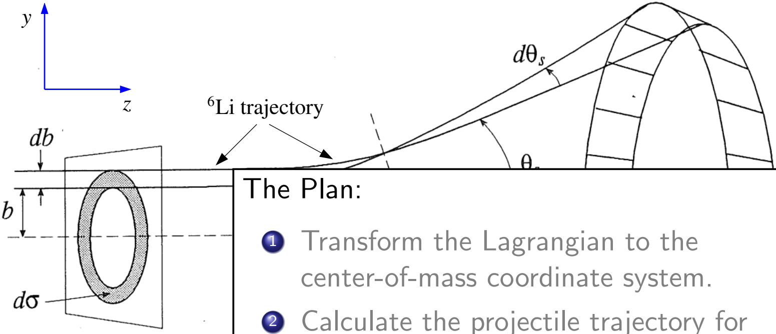
Solid Angle



$$dA = rd\theta \times r\sin\theta d\phi = r^2 \sin\theta d\theta d\phi$$

$$d\Omega = \frac{dA}{r^2} = \sin\theta d\theta d\phi$$

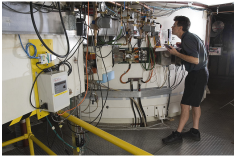

y
z
^{6}Li trajectory
$d\theta_s$
db
b
θ_s
^{208}Pb target
z axis
scattering
center
$d\sigma$
$d\Omega_s$

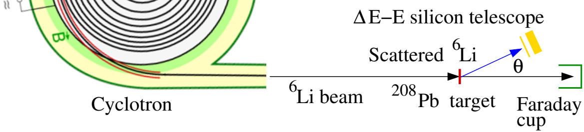


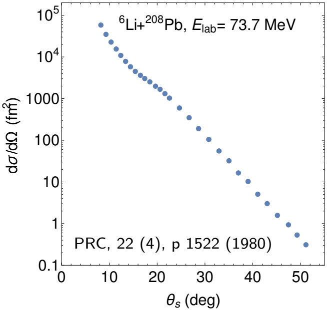
The Plan:

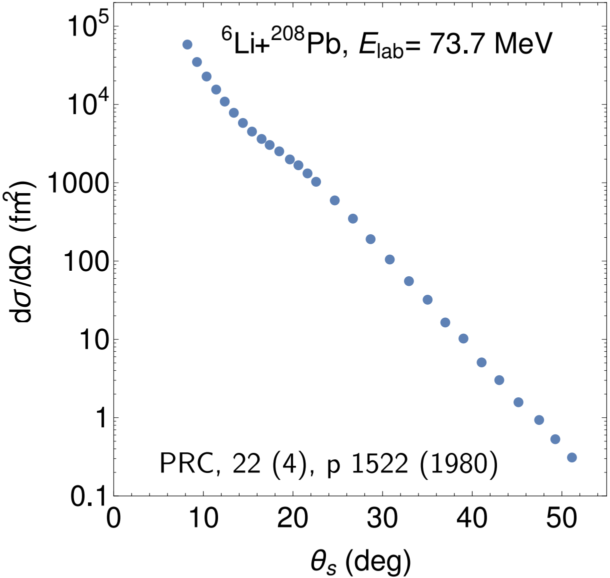
1. Transform the Lagrangian to the center-of-mass coordinate system.
2. Calculate the projectile trajectory for Coulomb repulsion.
3. Relate b and θ_s.
4. Construct $d\sigma/d\Omega$.
5. Get $d\sigma/d\Omega$ for random impact parameters.
6. Compare with data.

The experimental setup shown below is analogous to Rutherford's used to discover the nucleus. A beam of $^{6}\mathrm{Li}$-nuclei is accelerated to an energy $E_{lab} = 73.7\ \mathrm{MeV}$ in a cyclotron. It strikes a lead ($^{208}\mathrm{Pb}$) target scattering $^{6}\mathrm{Li}$ into a $\Delta\mathrm{E}-\mathrm{E}$ silicon detector. The plot shows the differential cross section measured as a function of θ. (1) How do these results compare to the Rutherford cross section calculated with only the Coulomb force active? (2) What is the distance of closest approach (DOCA) of the $^{6}\mathrm{Li}$ to the $^{208}\mathrm{Pb}$ target before the $^{6}\mathrm{Li}$ and $^{208}\mathrm{Pb}$ actually collide?

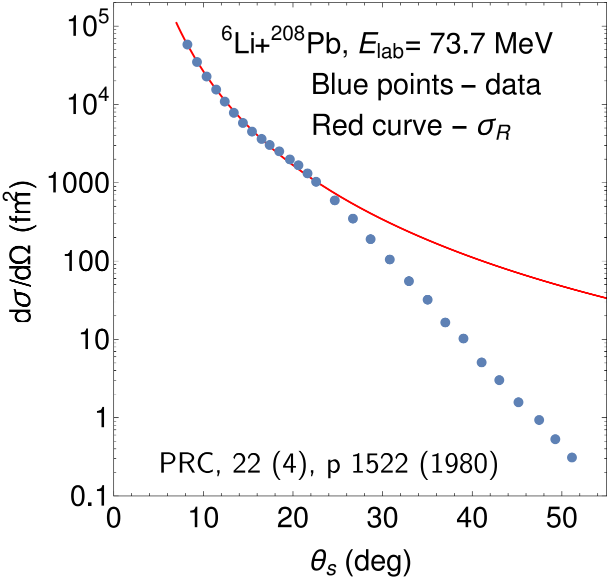


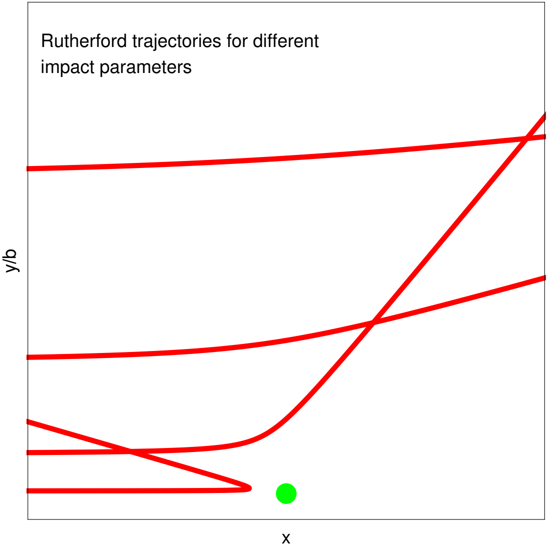


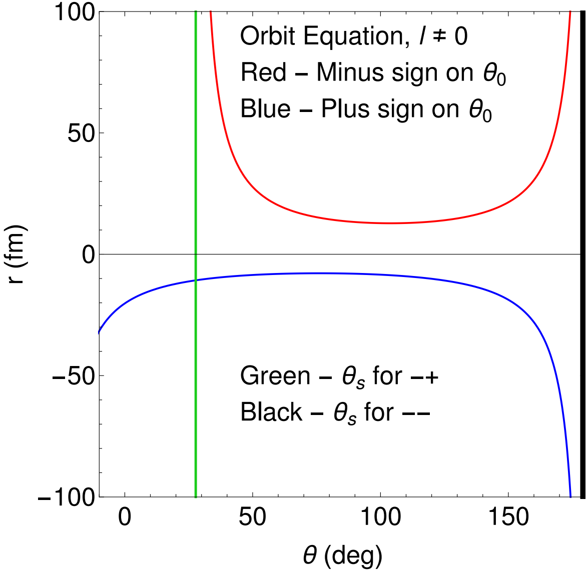




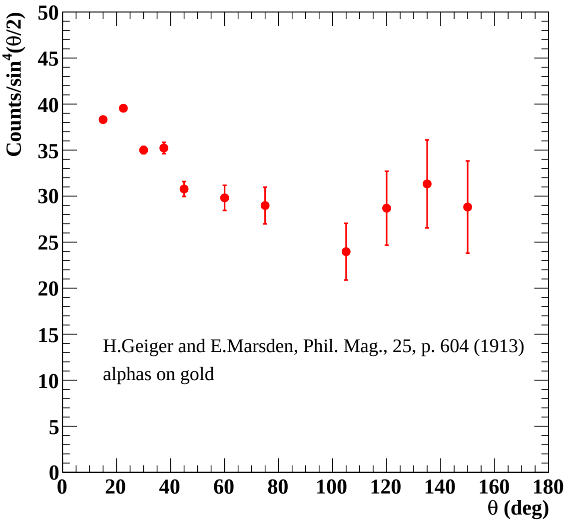
^{6}Li+^{208}Pb, E_{lab}= 73.7 MeV
PRC, 22 (4), p 1522 (1980)
$d\sigma/d\Omega$ (fm^2)
θ_s (deg)
10^5
10^4
1000
100
10
1
0.1
0
10
20
30
40
50


^{6}Li+^{208}Pb, E_{lab}= 73.7 MeV
Blue points – data
Red curve – σ_R
PRC, 22 (4), p 1522 (1980)
$d\sigma/d\Omega$ (fm^2)
θ_s (deg)
10^5
10^4
1000
100
10
1
0.1
0
10
20
30
40
50

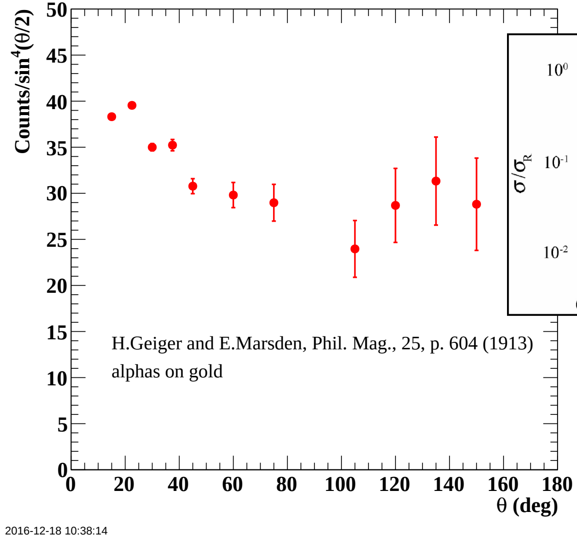

Rutherford trajectories for different
impact parameters
y/b
x

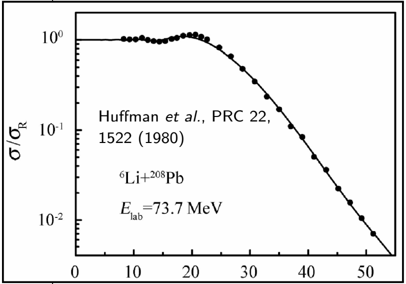

Orbit Equation, $l \neq 0$
Red – Minus sign on θ_0
Blue – Plus sign on θ_0
Green – θ_s for −+
Black – θ_s for −−
100
50
0
−50
−100
r (fm)
0
50
100
150
θ (deg)

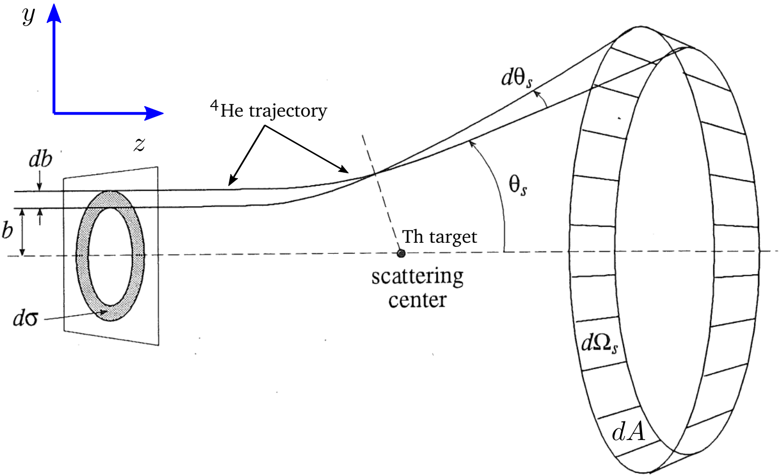
Rutherford Scattering Results From Rutherford



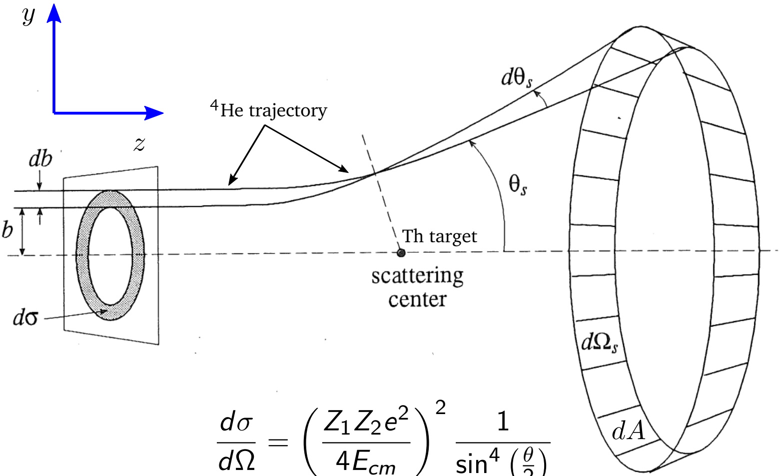
2016-12-18 10:38:14


Counts/sin^4(θ/2)
50
45
40
35
30
25
20
15
10
5
0
0
20
40
60
80
100
120
140
160
180
θ (deg)
H.Geiger and E.Marsden, Phil. Mag., 25, p. 604 (1913)
alphas on gold
2016-12-18 10:38:14


σ/σ_R
10^0
10^{-1}
10^{-2}
0
10
20
30
40
50
Huffman *et al.*, PRC 22, 1522 (1980)
^{6}Li+^{208}Pb
E_{lab}=73.7 MeV



y
z
db
b
dσ
4He trajectory
dθs
θs
Th target
scattering center
dΩs
dA



$$\frac{d\sigma}{d\Omega} = \left(\frac{Z_1 Z_2 e^2}{4E_{cm}}\right)^2 \frac{1}{\sin^4\left(\frac{\theta}{2}\right)}$$

$$\begin{array}{l} \text{particle rate} \\ \text{scattered into} \\ dA \text{ of detector} \end{array} = \frac{dN_s}{dt} \propto \begin{array}{l} \text{incident} \\ \text{beam} \\ \text{rate} \end{array} \times \begin{array}{l} \text{areal} \\ \text{target} \\ \text{density} \end{array} \times \begin{array}{l} \text{angular} \\ \text{detector} \\ \text{size} \end{array}$$

$$\frac{dN_s}{dt} \propto \frac{dN_{inc}}{dt} \times n_{tgt} \times d\Omega$$

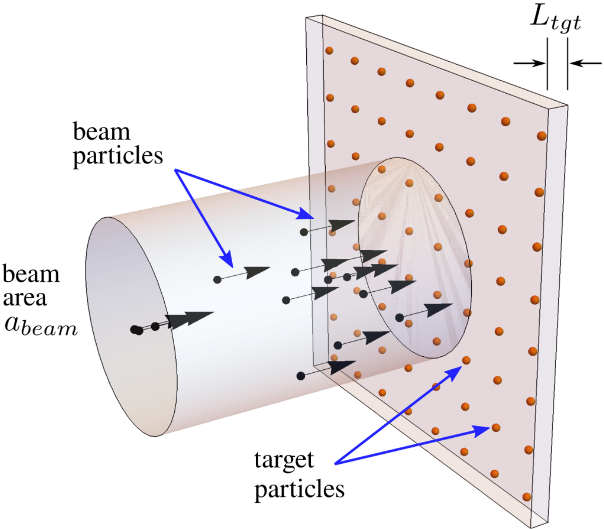
$$\frac{dN_s}{dt} = \frac{d\sigma}{d\Omega} \times \frac{dN_{inc}}{dt} \times n_{tgt} \times d\Omega$$

$$\frac{dN_{inc}}{dt} = \frac{\Delta N_{inc}}{\Delta t} = \frac{I_{beam}}{Ze}$$

I_{beam} - beam current
Z - beam charge

$$n_{tgt} = \frac{\rho_{tgt}}{A_{tgt}} N_A V_{hit} \frac{1}{a_{beam}} = \frac{\rho_{tgt}}{A_{tgt}} N_A L_{tgt}$$

ρ_{tgt} - target density
A_{tgt} - molar mass
V_{hit} - beam-target overlap
L_{tgt} - target thickness



molar mass A_{tgt}

density ρ_{tgt}

Avogadro's N_A

beam current I_{beam}

beam charge q_{beam}

target thickness L_{tgt}

beamtime on target Δt

beam area a_{beam}

$$\begin{array}{l}\text{particle rate}\\ \text{scattered into}\\ dA \text{ of detector}\end{array} = \frac{dN_s}{dt} \propto \begin{array}{l}\text{incident}\\ \text{beam}\\ \text{rate}\end{array} \times \begin{array}{l}\text{areal}\\ \text{target}\\ \text{density}\end{array} \times \begin{array}{l}\text{angular}\\ \text{detector}\\ \text{size}\end{array}$$

$$\frac{dN_s}{dt} \propto \frac{dN_{inc}}{dt} \times n_{tgt} \times d\Omega$$

$$\frac{dN_s}{dt} = \frac{d\sigma}{d\Omega} \times \frac{dN_{inc}}{dt} \times n_{tgt} \times d\Omega$$

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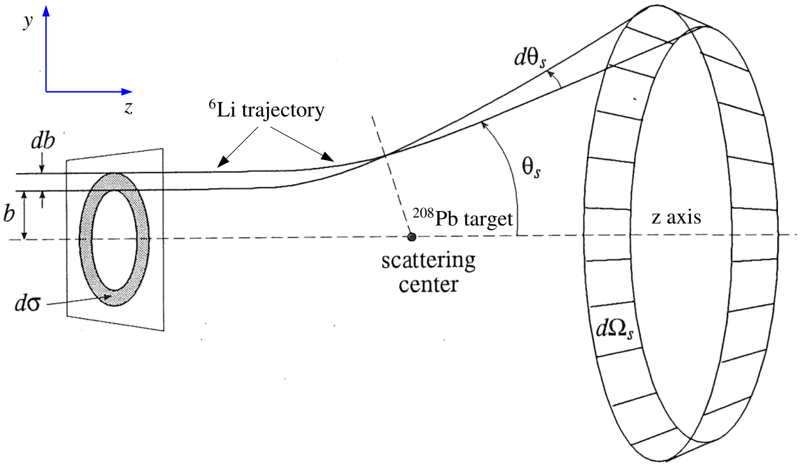
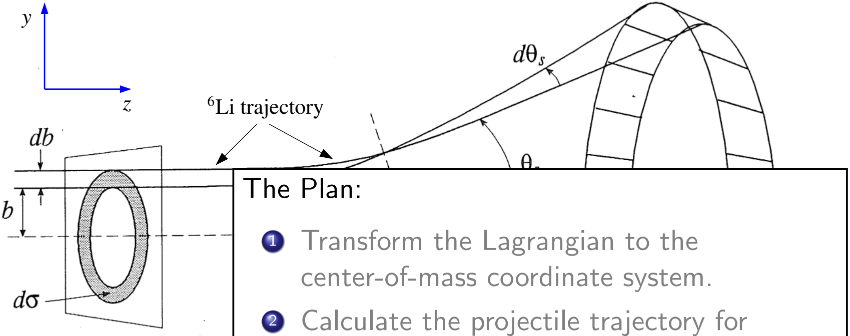
ρ_{tgt} - target density
A_{tgt} - molar mass
V_{hit} - beam-target overlap
L_{tgt} - target thickness

$$d\Omega = \frac{dA_{det}}{r_{det}^2} = \frac{\Delta A_{det}}{r_{det}^2} = \sin\theta d\theta d\phi$$

dA_{det} - detector area
r_{det} - target-detector distance


y
z
^{6}Li trajectory
db
b
$d\sigma$
$d\theta_s$
θ_s
^{208}Pb target
scattering center
z axis
$d\Omega_s$



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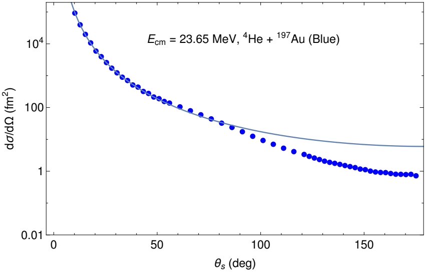
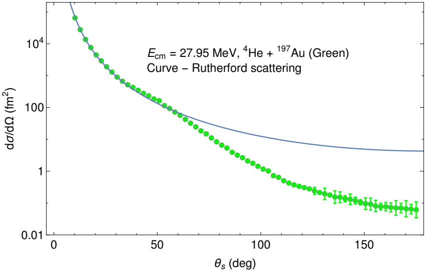
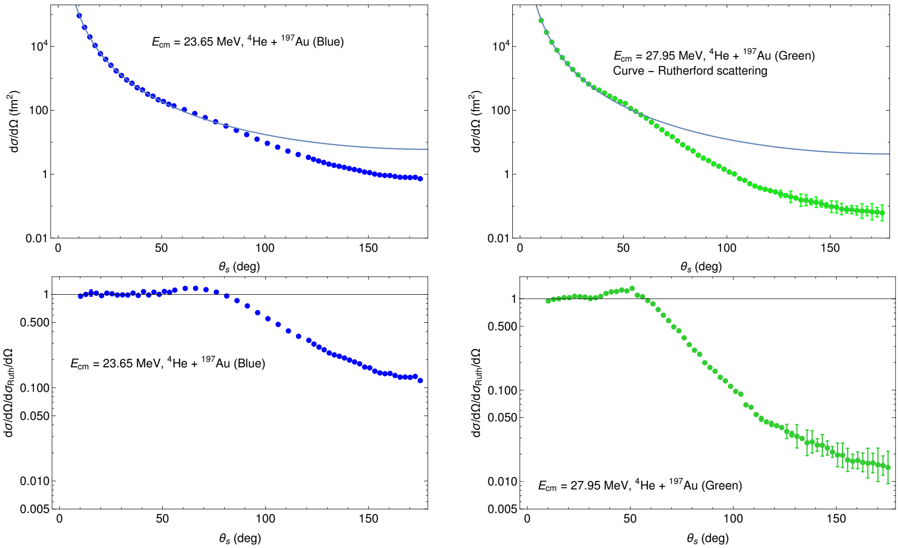
3. Relate b and θ_s.
4. Construct $d\sigma/d\Omega$.
5. Get $d\sigma/d\Omega$ for random impact parameters.
6. Compare with data.


E_{cm} = 23.65 MeV, ^{4}He + ^{197}Au (Blue)
$d\sigma/d\Omega$ (fm^2)
θ_s (deg)
10^4
100
1
0.01
0
50
100
150


E_{cm} = 27.95 MeV, ^{4}He + ^{197}Au (Green)
Curve – Rutherford scattering
$d\sigma/d\Omega$ (fm^2)
θ_s (deg)
10^4
100
1
0.01
0
50
100
150



E_{cm} = 23.65 MeV, ^{4}He + ^{197}Au (Blue)
E_{cm} = 27.95 MeV, ^{4}He + ^{197}Au (Green)
Curve – Rutherford scattering
dσ/dΩ (fm^2)
dσ/dΩ/dσ$_{Ruth}$/dΩ
θ_s (deg)
E_{cm} = 23.65 MeV, ^{4}He + ^{197}Au (Blue)
E_{cm} = 27.95 MeV, ^{4}He + ^{197}Au (Green)