Physics 303

The Simple Pendulum

1. We have used the approximation that $\sin\theta \approx \theta$ to simplify the differential equation describing the motion of a simple pendulum. Recall the definition of a Taylor series

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n \tag{1}$$

where $f^{(n)}(x_0)$ is the nth derivative of $f(x)$ evaluated at $x = x_0$. Use the Taylor's to expand $\sin\theta$ about zero out to sixth order and justify the approximation we have made.

2. The general solution to the simple pendulum problem is

$$\theta(t) = A' \cos(\omega_0 t + \phi) \tag{2}$$

where A' is the amplitude, ω_0 is the angular frequency, and ϕ is the phase angle. Apply the following initial conditions to the problem

$$\theta(t = 0) = \theta_0 \qquad \text{and} \qquad \dot{\theta}(t = 0) = 0 \tag{3}$$

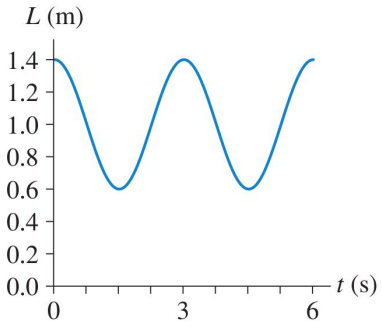
and determine the particular solution.

3. Another form of the general solution to the differential equation for the simple pendulum is

$$\theta(t) = c_1 e^{i\omega_0 t} + c_2 e^{-i\omega_0 t} \tag{4}$$

where c_1 and c_2 are unknown coefficients. Use the same initial conditions from Problem 2 to show that you obtain the same particular solution to simple pendulum as you found in Problem 2.

4. Astronauts in space cannot weigh themselves by standing on a bathroom scale. Instead, they use a large spring. Suppose an astronaut attaches one end of a large spring to her belt and the other end to a hook on the wall of the space craft. A fellow astronaut then pulls her away from the wall and releases her. The spring's length as a function of time is shown in the figure. Ignore the mass of the belt and the spring. What is her mass m if the spring constant is $k = 220\ N/m$?



5. Bungee jumpers bounce up and down freely on the bungee cord through many cycles. After a few cycles though, the cord does not go slack. Consider the following method to determine the mass of each person. An object of mass m_0 is oscillating freely on a light vertical spring with a period T_0. An object of unknown mass m_1 on the same spring oscillates with a period T_1. What is the spring constant and the unknown mass?