Physics 303

Molecular Vibrations

1. The Coulomb force between two point charges is

$$\vec{F}_C = k_C \frac{q_1 q_2}{r^2} \hat{r} \tag{1}$$

where q_i is the electric charge on each particle, k_C is the Coulomb constant, r is the separation between the two particles, and $\hat{r}$ is a unit vector that points outward in the radial direction. Using the definition of the potential energy

$$V(r) = \int_{r_1}^{r_2} \vec{F}_c \cdot d\vec{r} \tag{2}$$

and assuming the potential at infinity is zero show the Coulomb potential is

$$V_C = k_C \frac{q_1 q_2}{r} \qquad . \tag{3}$$

2. The potential energy between Na^+ and Cl^- ions is

$$V(r) = -\frac{A}{r} + \frac{B}{r^2} \tag{4}$$

where $A = 24\ eV - \text{Å}$ and $B = 28\ eV - \text{Å}^2$. Is the attractive part of the potential consistent with the force between two point charges?

3. In one dimension Hooke's Law can be written as

$$F_s = -k(r - r_e) \tag{5}$$

where k is the spring constant and x_e is the equilibrium position. Show the potential associated with this force is the following.

$$V_s = \frac{k}{2}(r - r_e)^2 \tag{6}$$

4. Write out the first four terms of the Taylor series expansion of

$$\tan\theta \quad \text{about } \theta = \frac{\pi}{4} \qquad \text{and} \qquad e^x \quad \text{about } x = 0 \qquad . \tag{7}$$

5. Use a Taylor series to estimate to third order the following integral for small x

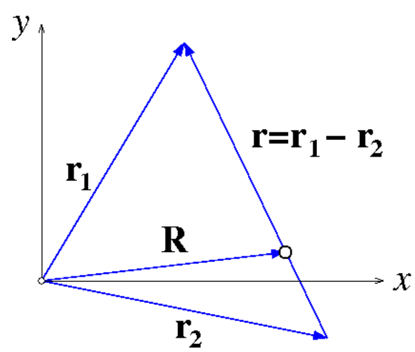
$$\int_0^1 \sin(x^2)dx \tag{8}$$

and compare your value with an expected one.

6. In studying rotational motion, we take advantage of the center-of-mass system to make life easier. Consider the two-particle system shown in the figure including the center-of-mass vector $\mathbf{R}$. For convenience we will place our origin at the center-of-mass of the system ($\mathbf{R} = \mathbf{0}$). Show the classical mechanical energy of the two-particle system in the center-of-mass frame can be written as

$$E_{cm} = \frac{1}{2}\mu v^2 + V(r) \qquad \text{where} \qquad \mu = \frac{m_1 m_2}{m_1 + m_2} \qquad \text{and} \qquad v = \frac{dr}{dt}$$

and r is the relative coordinate between the two particles as shown in the figure. Notice that $V(r)$ depends only on the relative coordinate.



7. Consider the oscillating $Na^+ - Cl^-$ system we studied in class. We found the distance between the ions was described by

$$x(t) = A\sin(\omega_0 t + \phi) \tag{9}$$

where $x = r - r_e$, and $r_e = 2.33$ Å. What is the final form of $r(t)$? What are the amplitude and phase angle if the initial position is a distance 2.0 Å from the equilibrium point and it is released from rest? What are the numerical values for the amplitude A, phase ϕ, angular frequency ω_0, and total mechanical energy.