Physics 303

Collisions

1. Consider an elastic collision between a projectile of mass m_1 and a stationary target of mass m_2. The projectile is scattered at an angle θ to the original beam direction and the target moves off at an angle ϕ.

   (a) Show the ratio of the projectile's outgoing to incident speed is the following.

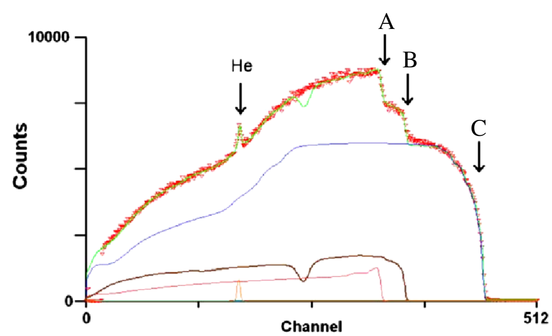
   $$\frac{v_{1f}}{v_{1i}} = \frac{m_1 \cos\theta \pm \sqrt{m_2^2 - m_1^2 \sin^2\theta}}{m_1 + m_2} \tag{1}$$

   (b) Show the ratio of the projectile's outgoing to incident kinetic energy is the following.

   $$\frac{K_1}{K_0} = \frac{1}{\left(1+\frac{m_2}{m_1}\right)^2}\left[\cos\theta \pm \sqrt{\left(\frac{m_2}{m_1}\right)^2 - \sin^2\theta}\,\right]^2 \tag{2}$$

2. Consider the result in Equation 2. There is an ambiguity we have to resolve in choosing the positive or negative sign. Calculate Equation 2 in the following limits (1) $m_2 >> m_1$, (2) $m_2 << m_1$, and (3) $m_2 = m_1$. One of these cases will reveal a difference between the positive and negative solutions that will allow you to pick one over the other. Find that difference.

3. A beam of protons of energy $E_{proj} = 2.964$ MeV strikes a fragment of the painting 'La Bohemienne'. The scattered protons are detected at an angle $\theta = 150°$ and their energies measured producing the spectrum shown here. The peak labeled 'He' is from protons elastically scattered off ^{4}He nuclei and has energy $E_{He} = 1.133$ MeV. The shoulders labeled A, B, and C are from protons elastically scattering off other, unknown nuclei.



L.Beck *et al.*, NIM B266, 1871 (2008).

   (a) What is the ratio of the elastically scattered proton kinetic energy K_1 to its incident energy K_0?

   (b) The shoulders at A, B, and C correspond to scattered proton energies $E_A = 2.214$ MeV, $E_B = 2.377$ MeV, and $E_C = 2.911$ MeV. What are the masses of the nuclei that create these shoulders?

4. Consider an asteroid that collides at an angle $\theta = 30°$ to the velocity of the Earth $\vec{v}_E$ and sticks to the surface (a perfectly inelastic collision). Assume the velocity of the asteroid $\vec{v}_A$ is towards the center of the Earth. How much does the velocity of the Earth change? How much energy is released in the collision? How does this compare

with the energy released by the Hiroshima atomic bomb ($6.8 \times 10^{13}\ J$)? Ignore the effects of potential energy here. Why is this a good assumption?

Asteroid:	$m_A = 3.4 \times 10^{14}\ kg$	$v_A = 2.5 \times 10^4\ m/s$
Earth:	$m_E = 6.0 \times 10^{24}\ kg$	$v_E = 3.0 \times 10^4\ m/s$
Collision:	$\theta = 30°$	

5. A proton moving with velocity $\vec{v}_0 = v_0\hat{i}$ collides elastically with another proton initially at rest. If the two protons have the same speed after the collision, what is the speed of each proton in terms of v_0? What is the direction of the velocity vectors after the collision?

6. An unstable nucleus of mass $m_0 = 17 \times 10^{-27}\ kg$ initially at rest decays into three particle. One of the particles of mass $m_1 = 5 \times 10^{-27}\ kg$ moves in the y direction with speed $v_1 = 6 \times 10^6\ m/s$. Another particle of mass $m_2 = 8.4 \times 10^{-27}\ kg$ moves in the x direction with speed $v_2 = 4 \times 10^6\ m/s$. What is the velocity $\vec{v}_3$ of the third particle? What is the total kinetic energy increase in the process? Assume mass is NOT converted into energy.

7. A neutron of mass $m_n = 1\ u$ and known kinetic energy K_0 is scattered through an angle $\theta = 90°$ in an elastic collision with a deuteron of mass $m_d = 2\ u$ that is initially at rest. Starting from the conservation of momentum, derive an expression for the ratio of the scattered neutron's final kinetic energy K_n to the kinetic energy of the recoiling deuteron K_d in terms of known quantities, *e.g.* the masses m_n, m_d.