In January, 1942 a Soviet Ilyushin Il-4 bomber carrying navigator Lieutenant I.M.Chisov was badly damaged by German gunfire. At an altitude of about 21,980 feet Lieutenant Chisov bailed out. He planned to open his parachute when he cleared the battle zone, but due to the thin atmosphere at that altitude he lost consciousness and was unable to pull the rip cord. He landed on the slopes of a snow-covered ravine and slid to the bottom. He suffered a fractured pelvis, severe spinal damage, and other injuries, but lived. How fast was Lieutenant Chisov moving when he hit the ravine? How long did he have to think about it? Is air drag important?

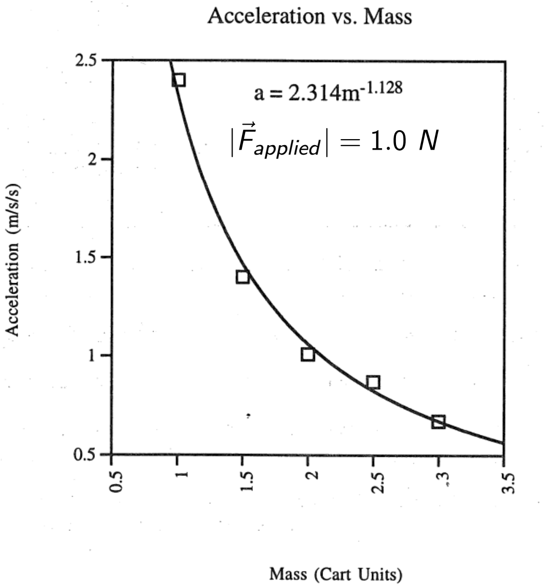
1. Consider a body with no net force acting on it. If it is at rest it will remain at rest. If it is moving with a constant velocity it will continue to move at that velocity.
2. For all the different forces acting on a body

$$\sum \vec{F}_i = m\vec{a} = \frac{d\vec{p}}{dt} \qquad .$$

3. For every action there is an equal and opposite reaction.

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

Why Does $\vec{F} = m\vec{a}$?



$$\vec{F}_G = -\frac{GM_1M_2}{r^2} \qquad \vec{F}_g = -mg\hat{y}$$

$$\vec{F}_C = k_e\frac{e_1e_2}{r^2}\hat{r} \qquad \vec{F}_B = e_1\vec{v}_1 \times \left(\frac{\mu_0}{4\pi}\ \frac{e_2\vec{v}_2\times\vec{r}}{r^2}\right)$$

$$\vec{F}_S = -g_S\frac{e^{-\mu_S r}}{r}\hat{r} \qquad \vec{F}_W = -g_W\frac{e^{-\mu_W r}}{r}\hat{r}$$

$$\vec{F}_c = 3 \text{ tons} \qquad \vec{F}_S = -kr\hat{r}$$

$$|F_f| \leq \mu_s N \qquad |F_f| = \mu_k N$$

$$\vec{F}_f = -bv\hat{v} \qquad \vec{F}_f = -\frac{1}{2}C_D S\rho v^2\hat{v}$$

In January, 1942 a Soviet Ilyushin Il-4 bomber carrying navigator Lieutenant I.M.Chisov was badly damaged by German gunfire. At an altitude of about 21,980 feet Lieutenant Chisov bailed out. He planned to open his parachute when he cleared the battle zone, but due to the thin atmosphere at that altitude he lost consciousness and was unable to pull the rip cord. He landed on the slopes of a snow-covered ravine and slid to the bottom. He suffered a fractured pelvis, severe spinal damage, and other injuries, but lived. How fast was Lieutenant Chisov moving when he hit the ravine? How long did he have to think about it? Is air drag important?

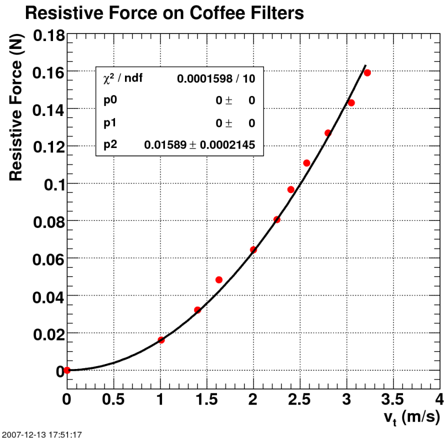
$$|\vec{F}_f| = \frac{1}{2} C_D S \rho v^2$$

C_D - drag coefficient (dimensionless).

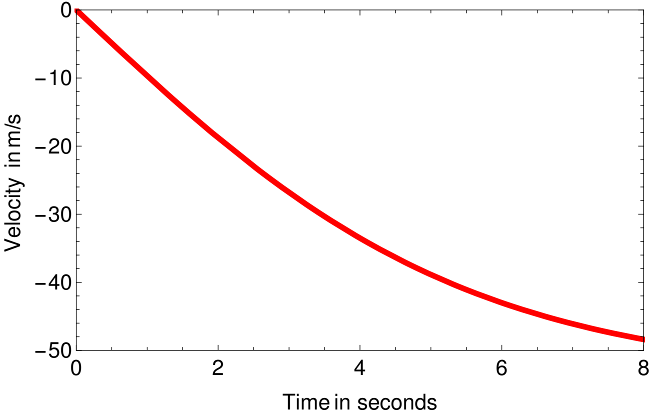
ρ - air density (kg/m^3).

S - Cross sectional area (m^2).

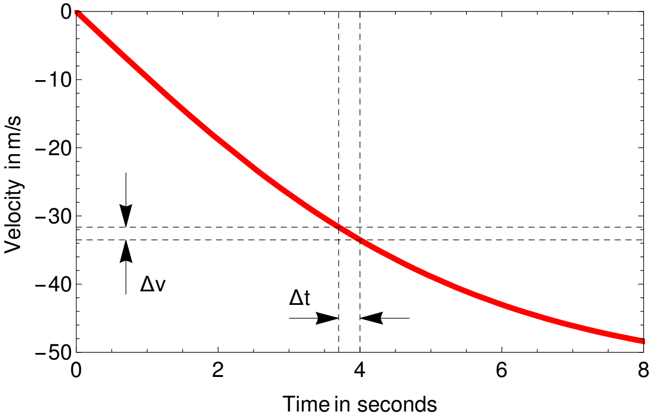
v - speed (m/s).



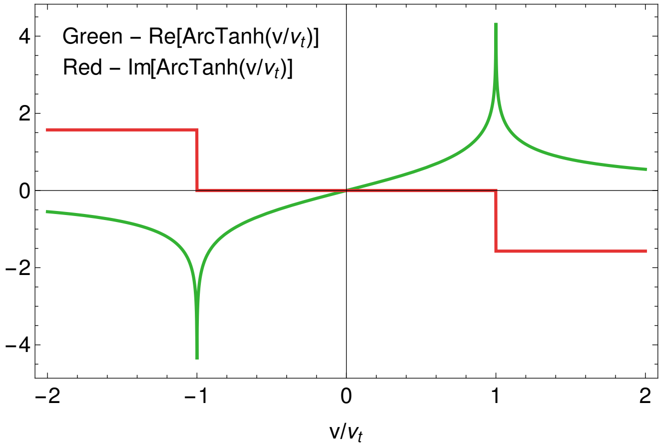
What Happens to Δv as $\Delta t \to 0$?



What Happens to Δv as $\Delta t \to 0$? 8



$$ArcTanh\left(\frac{v}{v_t}\right) = -\frac{g}{v_t}t + C_1$$



- Hyperbolic sine:

$$\sinh x = \frac{e^x - e^{-x}}{2} = \frac{e^{2x} - 1}{2e^x} = \frac{1 - e^{-2x}}{2e^{-x}}.$$

- Hyperbolic cosine:

$$\cosh x = \frac{e^x + e^{-x}}{2} = \frac{e^{2x} + 1}{2e^x} = \frac{1 + e^{-2x}}{2e^{-x}}.$$

- Hyperbolic tangent:

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

- Hyperbolic cotangent: $x \neq 0$

$$\coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

- Hyperbolic secant:

$$\operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}} = \\ = \frac{2e^x}{e^{2x} + 1} = \frac{2e^{-x}}{1 + e^{-2x}}$$

- Hyperbolic cosecant: $x \neq 0$

$$\operatorname{csch} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}} = \\ = \frac{2e^x}{e^{2x} - 1} = \frac{2e^{-x}}{1 - e^{-2x}}$$

$$\operatorname{arsinh}(x) = \ln\left(x + \sqrt{x^2 + 1}\right)$$
$$\operatorname{arcosh}(x) = \ln\left(x + \sqrt{x^2 - 1}\right); x \geq 1$$
$$\operatorname{artanh}(x) = \frac{1}{2}\ln\left(\frac{1 + x}{1 - x}\right); |x| < 1$$
$$\operatorname{arcoth}(x) = \frac{1}{2}\ln\left(\frac{x + 1}{x - 1}\right); |x| > 1$$

Odd and even functions:

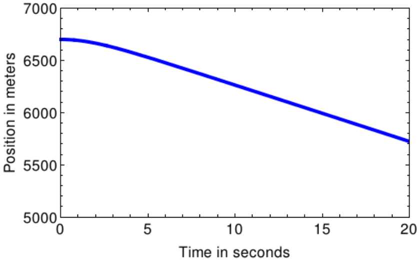
$$\sinh(-x) = -\sinh x$$
$$\cosh(-x) = \cosh x$$

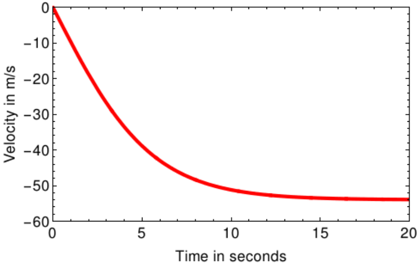
Hence:

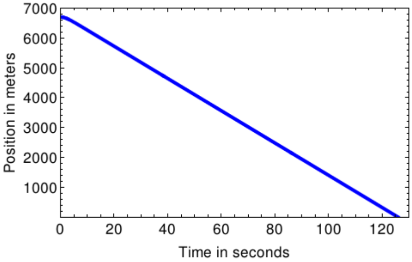
$$\tanh(-x) = -\tanh x$$
$$\coth(-x) = -\coth x$$
$$\operatorname{sech}(-x) = \operatorname{sech} x$$
$$\operatorname{csch}(-x) = -\operatorname{csch} x$$

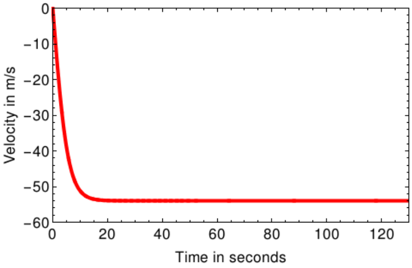
Falling object	Mass	Area	Terminal velocity
Skydiver	75 kg	0.7 m^2	54 m/s or 121mi/hr
Baseball	145 g	42 cm^2	33 m/s or 74mi/hr
Golf ball	46 g	14 cm^2	32 m/s or 72mi/hr
Hail stone (0.5 cm radius)	0.48 g	0.79 cm^2	14 m/s or 31mi/hr
Raindrop (0.2 cm radius)	0.034 g	0.13 cm^2	9 m/s or 20mi/hr

Lieutenant Chisov's Fall






7000
6000
5000
4000
3000
2000
1000
Position in meters
0
20
40
60
80
100
120
Time in seconds


0
−10
−20
−30
−40
−50
−60
Velocity in m/s
0
20
40
60
80
100
120
Time in seconds

Sample Homework

Style Points on Problems

1. Draw a picture. Write out knowns and unknowns.
2. Choose appropriate notation - don't have the same variable mean different things.
3. Use your words. Use short sentences to describe what you are doing.
4. Clearly separate different sections. Use a sentence, a line, a number, *etc.*
5. Do it symbolically - no numbers until the end.
6. Imagine someone will read this this solution who isn't in the class, but is a physics major.
7. Use your text for examples of mathematical writing.
8. Be legible. Use a pen only if you are perfect. Scratch outs annoy the reader.

An athlete can throw a javelin 60 m from a standing position. If he can run 100 m at constant velocity in 10 s, how far could he hope to throw the javelin while running? Neglect air resistance and the height of the thrower in the interest of simplicity. (*Hint: derive an expression for the distance R in terms of the initial angle θ to the horizontal and maximize R.*) Compare your answer with a world-class throw of 105 m for the javelin.

Style Points on Problems

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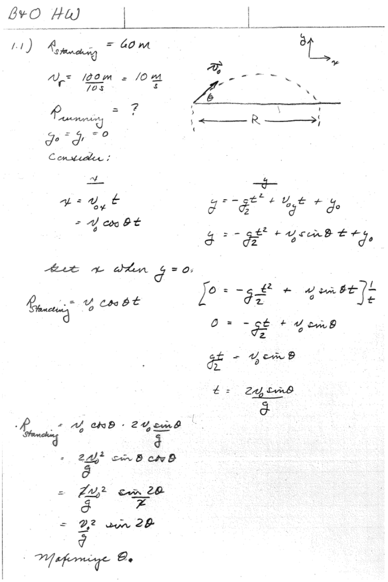

B+O HW

1.1) $R_{standing} = 60\,m$

$v_r = \frac{100\,m}{10\,s} = 10\,\frac{m}{s}$

$R_{running} = ?$

$y_0 = y_1 = 0$

Consider:

x: $x = v_{0x} t = v_0 \cos\theta\, t$

y: $y = -\frac{g}{2}t^2 + v_{0y} t + y_0$

$y = -\frac{g}{2}t^2 + v_0 \sin\theta\, t + y_0$

Get x when $y = 0$.

$R_{standing} = v_0 \cos\theta\, t$

$\left[0 = -g\frac{t^2}{2} + v_0 \sin\theta\, t\right]\frac{1}{t}$

$0 = -\frac{gt}{2} + v_0 \sin\theta$

$\frac{gt}{2} = v_0 \sin\theta$

$t = \frac{2v_0 \sin\theta}{g}$

$R_{standing} = v_0 \cos\theta \cdot \frac{2v_0 \sin\theta}{g} = \frac{2v_0^2}{g}\sin\theta\cos\theta = \frac{2v_0^2}{g}\frac{\sin 2\theta}{2} = \frac{v_0^2}{g}\sin 2\theta$

Maximize θ.

2

$$\frac{dR}{d\theta} = \frac{v_0^2}{g} 2\cos 2\theta = 0$$

$$\therefore \cos 2\theta = 0$$

$$2\theta = \frac{\pi}{2}$$

$$\theta = \frac{\pi}{4}$$

$$\therefore R_{standing} = \frac{v_0^2}{g} \sin\left(2 \cdot \frac{\pi}{4}\right)$$

$$v_0 = \sqrt{g R_{standing}}$$

$$= \sqrt{(9.8\ m/s^2)(60\ m)}$$

$$v_0 = 24.2\ m/s$$

Now add v_r to v_{0x}.

$$R_{running} = (v_0 \cos\theta + v_r) \frac{2 v_0 \sin\theta}{g}$$

Maximize again

$$\frac{dR}{d\theta} = (-v_0 \sin\theta) \frac{2 v_0 \sin\theta}{g} + (v_0\cos\theta + v_r)\frac{2 v_0 \cos\theta}{g}$$

$$0 = -\frac{2 v_0^2 \sin^2\theta}{g} + \frac{2 v_0^2 \cos^2\theta}{g} + \frac{2 v_0 v_r \cos\theta}{g}$$

$$\frac{g}{2 v_0}\left[0 = \frac{2 v_0^2 (\cos^2\theta - \sin^2\theta)}{g} + \frac{2 v_0 v_r \cos\theta}{g}\right]$$

$$0 = v_0 (\cos^2\theta - \sin^2\theta) + v_r \cos\theta$$

$$0 = v_0 (\cos^2\theta \underbrace{- \sin^2\theta - \cos^2\theta}_{-1} + \cos^2\theta) + v_r \cos\theta$$

3

$$0 = v_0 \left[2\cos^2\theta - 1\right] + v_r \cos\theta$$

$$0 = 2 v_0 \cos^2\theta - v_0 + v_r \cos\theta$$

$$0 = 2 v_0 \cos^2\theta + v_r \cos\theta - v_0$$

Use the quadratic formula

$$\cos\theta = \frac{-v_r \pm \left[v_r^2 - 4(2v_0)(-v_0)\right]^{1/2}}{2(2v_0)}$$

$$= \frac{-v_r \pm \left[v_r^2 + 8 v_0^2\right]}{4 v_0}$$

$$\cos\theta = -\frac{v_r}{4v_0} \pm \left[\left(\frac{v_r}{4v_0}\right)^2 + \frac{8 v_0^2}{16 v_0^2}\right]^{1/2}$$

$$= -\frac{v_r}{4v_0} \pm \left[\left(\frac{v_r}{4v_0}\right)^2 + \frac{1}{2}\right]^{1/2}$$

$$\cos\theta = -\frac{10\ m/s}{4(24.2\ m/s)} \pm \left[\left(\frac{10\ m/s}{4 \cdot 24.2\ m/s}\right)^2 + \frac{1}{2}\right]^{1/2}$$

$$= -0.1033 \pm 0.7146$$

$$= 0.6113 \text{ or } -0.8179$$

$$\theta = 52.3° \text{ or } 145°$$

Choose the (+) result to keep $0 < \theta < 90°$.

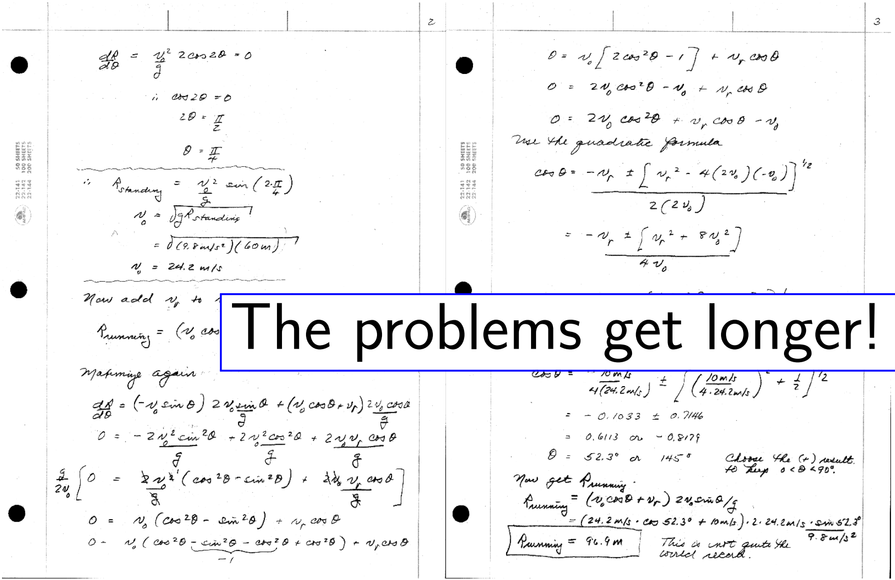
Now get $R_{running}$.

$$R_{running} = (v_0 \cos\theta + v_r)\, 2 v_0 \sin\theta / g$$

$$= (24.2\ m/s \cdot \cos 52.3° + 10\ m/s) \cdot 2 \cdot 24.2\ m/s \cdot \frac{\sin 52.3°}{9.8\ m/s^2}$$

$$\boxed{R_{running} = 96.4\ m}$$

This is not quite the world record.



2

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$$v_0 = \sqrt{g R_{standing}}$$

$$= \sqrt{(9.8\ m/s^2)(60\ m)}$$

$$v_0 = 24.2\ m/s$$

Now add v_r to ...

$R_{running} = (v_0 \cos$ ...

Maximize again

$$\frac{dR}{d\theta} = (-v_0 \sin\theta)\, \frac{2 v_0 \sin\theta}{g} + (v_0\cos\theta + v_r)\frac{2 v_0 \cos\theta}{g}$$

$$0 = -\frac{2v_0^2 \sin^2\theta}{g} + \frac{2v_0^2\cos^2\theta}{g} + \frac{2 v_0 v_r \cos\theta}{g}$$

$$\frac{g}{2v_0}\left[0 = \frac{2v_0^2(\cos^2\theta - \sin^2\theta)}{g} + \frac{2v_0 v_r \cos\theta}{g}\right]$$

$$0 = v_0(\cos^2\theta - \sin^2\theta) + v_r\cos\theta$$

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3

$$0 = v_0\left[2\cos^2\theta - 1\right] + v_r\cos\theta$$

$$0 = 2v_0\cos^2\theta - v_0 + v_r\cos\theta$$

$$0 = 2v_0\cos^2\theta + v_r\cos\theta - v_0$$

Use the quadratic formula

$$\cos\theta = \frac{-v_r \pm \left[v_r^2 - 4(2v_0)(-v_0)\right]^{1/2}}{2(2v_0)}$$

$$= \frac{-v_r \pm \left[v_r^2 + 8v_0^2\right]}{4v_0}$$

$$\cos\theta = \frac{-10\ m/s}{4(24.2\ m/s)} \pm \left[\left(\frac{10\ m/s}{4 \cdot 24.2\ m/s}\right)^2 + \frac{1}{2}\right]^{1/2}$$

$$= -0.1033 \pm 0.7146$$

$$= 0.6113 \text{ or } -0.8179$$

$\theta = 52.3°$ or $145°$ — Choose the (+) result to keep $0 < \theta < 90°$.

Now get $R_{running}$.

$$R_{running} = (v_0\cos\theta + v_r)\, 2v_0\sin\theta / g$$

$$= (24.2\ m/s \cdot \cos 52.3° + 10\ m/s) \cdot 2 \cdot 24.2\ m/s \cdot \frac{\sin 52.3°}{9.8\ m/s^2}$$

$$R_{running} = 96.4\ m$$

This is not quite the world record.

The problems get longer!