Critique of "Time of flight between a source and a detector observed from a satellite" (arxiv:1110.2685v3)

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Ronald A.J. van Elburg [1] presents a possible explanation of the OPERA experiment's surprising faster-than-light neutrino result [2]. His analysis contains at least one error, which seems to invalidate his conclusion.

van Elburg's calculation involves analyzing the time of flight of a light between a source and a detector at rest on Earth.[1] He analyzes the time of flight in two different reference frames, one attached to the Earth and one attached to a GPS satellite. It is unclear (at least to me) why he chooses to do the analysis in the way he does: his final result is a relationship between distances and times both measured in the Earth frame, and I can't see any reason not to do the entire calculation in the Earth frame. In fact, as I'll show, the calculation he does contains an error which, when corrected, leads to a final result that is exactly the same as a simple calculation done entirely in the Earth's frame.

The calculation starts by relating the source-detector distance in the two frames via the usual length contraction formula:

$$S_s = \frac{S_b}{\gamma}. \qquad (vE1)$$

[vE refers to an equation number in van Elburg's paper.] Here s refers to the satellite frame and b refers to the "baseline" frame (in which source and detector are at rest). He next calculates the time of flight of a light signal from source to detector in the satellite frame. In any time interval dt_s, the light signal advances $c\,dt_s$, and the detector moves toward it by $v\,dt_s$. So the distance from signal to detector decreases at a rate $(c + v)$, and the time required to reach the detector is

$$\tau_s = \frac{S_s}{c+v} = \frac{S_b}{\gamma}\frac{1}{c+v}. \qquad (vE3).$$

This is also correct.

Now we come to the difficulty. We wish to calculate the time of flight of the signal in the baseline frame. Two methods are proposed for doing this. One is to work entirely in the baseline frame,

$$\tau_b = \frac{S_b}{c}. \qquad (vE4)$$

For reasons that are not clear to me, van Elburg claims that this is incorrect. If this equation is not correct, then I have no idea what quantity he's trying to calculate. He explicitly refers to the "time-of-fligh estimate for photons in the baseline reference frame." There is absolutely no doubt that time time of flight for a photon is related to the distance in precisely this way. Indeed, this is just an application of the fundamental assumption of special relativity that the speed of light is invariant.

[1] I originally misread his article to be about a signal from a satellite to an Earthbound detector. If you saw an earlier version of this document, I apologize for the confusion.

Instead, van Elburg proposes a different calculation of the time of flight, namely taking the satellite-frame time of flight and converting it to the baseline frame via the time dilation formula

$$\tau_o = \gamma \tau_s. \qquad (vE5).$$

This equation is incorrect. The time dilation formula applies only if the time on the right side is the time between two events located at the same place in the given reference frame. For instance, it is correct to use this formula to relate the age of the rocket-borne twin in the twin paraox to her stay-at-home sibling. In this case, the time interval on the right is the time between, say, two of the rocket-borne twin's birthdays, which are at the same place in the rocket frame.

The quantity τ_s is a time interval between two events (photon leaves source and photon arrives at detector), which are not at the same place in either frame. In this situation, the time dilation multiply-by-gamma rule doesn't work. Instead, we need to use the Lorentz transformation.

The simplest way to use the Lorentz transformation to relate τ_b and τ_s is[2]

$$\tau_s = \gamma \left(\tau_b - \frac{v}{c^2} S_b \right). \qquad (1)$$

Solving for τ_b and substituting in equation $(vE3)$, we get

$$\tau_b = \frac{\tau_s}{\gamma} + \frac{v}{c^2} S_b = S_b \left(\frac{1}{\gamma^2 (c+v)} + \frac{v}{c^2} \right) = \frac{S_b}{c} \qquad (2)$$

in agreement with equation $(vE4)$.

In summary, equation $(vE5)$ is an incorrect application of special relativity. A correct application of special relativity leads to a time of flight that is the same as equation $(vE4)$. van Elburg's claimed error is the difference between the quantities in these two equations, but when correctly calculated there is no difference.

References

[1] R.A.J. van Elburg, preprint arxiv:1110.2685v3 (2011).

[2] OPERA Collaboration, arxiv:1109.4897v1 (2011).

[2] One might be tempted to use the inverse relation $\tau_b = \gamma(\tau_s + (v/c^2) S_s)$, but this is incorrect, because S_s is not the final position of the detector in the satellite frame. S_s is the detector position when the signal was emitted. The final position is smaller by a factor $c/(c+v)$, because the detector moved toward the source during the time of flight. It's straightforward to check that the same result arises from such a calculation.