Answer to Question # 67: The Electric Field Outside a Black Hole.

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In a recent issue of this journal, Singham[1] poses a question that can be summarized as follows. Given that electromagnetic information propagates at the speed of light, how does the information about the charge of a black hole propagate across the event horizon in order to produce an electric field on the outside?

Singham poses the question in quantum-mechanical terms, asking, "Is the electric force not mediated by photons?" However, the problem is just as puzzling when viewed from the perspective of classical electrodynamics. Let us therefore address the question classically and see if the answer sheds any light on the quantum version.

In order to make the statement of the puzzle precise, recall that the electromagnetic field at any point x in spacetime can be calculated from the retarded four-potential

$$A^{\nu}(x) = \int j^{\mu}(x') G_{\mu}{}^{\nu}(x'; x)\, d^4x', \tag{1}$$

where j is the four-current density.[2] The retarded Green's function $G(x'; x)$ has support only on the past light cone of x. If x is a spacetime point outside the horizon of a charged black hole, then the past light cone consists only of points outside the event horizon. How, then, does the charge inside the black hole produce an electromagnetic field at x?

Consider first the case of a charged black hole that formed by gravitational collapse (as opposed to one that has existed for all time). At early times (before the hole formed), all of the charged material that would eventually constitute the black hole was outside the event horizon. The integral in (1) over the past light cone contains contributions from this matter. In this case, then, the mystery is

[1] M. Singham, "Question #67. The electric field outside a black hole," Am.J.Phys. **65**(12), 1133 (1997).

[2] See, for example, J.D. Jackson, Classical Electrodynamics (John Wiley & Sons, New York, 1975), 2nd ed., p. 611, for a discussion of the flat-space retarded potential; see R.M. Wald, General Relativity (University of Chicago Press, Chicago, 1984), pp. 267-268, for the generalization to curved spacetime.

solved. The electromagnetic field at x has as its source the charged matter that would *eventually* form the black hole.[3]

For completeness, we should also consider the less physically realistic case of a black hole that has existed for all time. Specifically, let us assume a static, isolated, charged black hole embedded in asymptotically flat spacetime. Such a black hole is described by the Reissner-Nordstrøm geometry.[4] In this vacuum solution to the Einstein-Maxwell equations, there is no infalling charged matter, so the above resolution of the paradox does not work. However, the Reissner-Nordstrøm solution contains a white-hole singularity in the past of every external observer.[5] The integral (1) always hits this singularity, so the retarded potential is ill-defined. In this situation, we should probably say that the electric field at x has as its "source" the white-hole singularity on x's past light cone.

Having dispensed with the classical problem, let us now regard the electromagnetic field as quantized. (From now on, we consider only black holes that formed via collapse, not eternally-existing Reissner-Nordstrøm holes.) Then, as Singham observes, the electromagnetic force on our test charge is mediated by virtual photons. We could in principle calculate the transition amplitude to any particular final state of the test charge by considering virtual photon exchange between the test charge and the charged particles that collapsed to form the black hole. Since the force is determined by virtual photon exchanges with the infalling charged matter, we would be justified in saying that the infalling matter is the "source" of the electric field, just as in the classical case.

There is, however, an important difference between the classical and quantum treatments. Virtual photons, unlike real ones, need not propagate along the light cone. When we compute transition amplitudes for our test charge, we must include the contributions of *all* virtual photons, including off-shell photons that propagate out from inside the event horizon. In other words, the electromagnetic interaction between a test charge and the black hole is mediated by both horizon-crossing and non-horizon-crossing virtual photons.[6]

It is tempting to suppose that, in some appropriate semiclassical limit, "most" of the contribution to the external electric field comes from nearly-on-shell, non-horizon-crossing virtual photons. However, such a statement is difficult to make precise, since the amount a particular virtual photon contributes to the final field is a gauge-dependent quantity. In particular, in the Coulomb gauge, no

[3] This is of course closely related to the fact that an external observer never sees the charged matter fall all the way into the black hole. Since the world line of the infalling matter always intersects the past light cone of an outside observer, the observer can always see the infalling matter.

[4] See, for example, C.W. Misner, K.S. Thorne, and J.A. Wheeler, Gravitation (W.H. Freeman and Co., New York, 1973), pp. 840ff.

[5] See, for example, Misner *et al.*, *op. cit.*, p. 921 for a helpful conformal diagram of the Reissner-Nordstrøm geometry.

[6] Although these horizon-crossing virtual photons must be accounted for when computing transition amplitudes, it is important to remember that they carry neither information nor energy across the event horizon.

such statement is true. On the contrary, at times long after the formation of the hole, the electric field is computed entirely from the static Coulomb potential ϕ. Since ϕ is determined by data on a slice at constant t, one might somewhat fancifully say that the field in Coulomb gauge is produced by "virtual photons of infinite speed." In a covariant gauge, one might find that the dominant contribution to the field comes from photons near the light cone, but as far as we know nobody has done such a calculation.

Emory F. Bunn
Department of Physics and Astronomy
44 Campus Ave.
Bates College
Lewiston, ME 04240
Electronic mail: ebunn@abacus.bates.edu

Matthew McIrvin
105 Trowbridge St. Apt. #6B
Cambridge, MA 02138
Electronic mail: mmcirvin@world.std.com