Top-Down Parsing
and
Intro to Bottom-Up Parsing

Lecture 7

Predictive Parsers

- Like recursive-descent but parser can "predict" which production to use
- Predictive parsers are never wrong
  - Always able to guess correctly which production will lead to a successful parse, provided a string is in L(G).
- Two strategies allow this:
  - By looking at next few tokens
    - Lookahead
  - By restricting the form of the grammar

Predictive Parsers

- Advantage: No backtracking
  - So parsing is completely “deterministic”
- Predictive parsers accept LL(k) grammars
  - L means “left-to-right” scan of input
    - We always do this, so all our techniques would have “L” in first position
  - L means “leftmost derivation”
  - k means “predict based on k tokens of lookahead”
    - Theory is developed for arbitrary k, but…
  - In practice, LL(1) is used

LL(1) vs. Recursive Descent

- In recursive-descent,
  - At each step, many choices of production to use
  - Backtracking used to undo bad choices
- In LL(1),
  - At each step, only one choice of production
  - That is
    - When a non-terminal A is leftmost non-terminal in a derivation…
    - And the next input symbol is token t
    - There is a unique production $A \to \alpha$ to use
      - Or no production to use (an error state)
      - Any other production is guaranteed to be incorrect…
      - But even the single production $A \to \alpha$ might not end up succeeding
      - Put another way, in LL(1), there is AT MOST one production to be used in a given situation

LL(1) vs. Recursive Descent

- In recursive-descent,
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  - Backtracking used to undo bad choices

- In LL(1),
  - At each step, only one choice of production
  - That is
    - When a non-terminal A is leftmost non-terminal in a derivation…
    - And the next input symbol is token t
    - There is a unique production $A \rightarrow \alpha$ to use
      - Or no production to use (an error state)

- LL(1) is a recursive descent variant without backtracking

Predictive Parsing and Left Factoring

- Recall our favorite grammar

  $E \rightarrow T + E \mid T$

  $T \rightarrow \text{int} \mid \text{int} * T \mid (E)$

- Hard to predict because
  - For T two productions start with int
    - With lookahead 1, can't choose which production
  - For E it is not clear how to predict
    - What's more T is a non-terminal so how do we even do the prediction?
    - Regardless T starts both productions of E, so with single token of lookahead, not going to be easy to know what to do

Predictive Parsing and Left Factoring

- Recall our favorite grammar

$$E \to T + E \mid T$$
$$T \to int \mid int * T \mid (E)$$

This grammar is unacceptable for LL(1) parsing

- Hard to predict because
  - For T two productions start with int
    - With lookahead 1, can't choose which production
  - For E it is not clear how to predict
    - What's more T is a non-terminal so how do we even do the prediction?
- We need to left-factor the grammar

The Idea Behind Left Factoring

- Eliminate the common prefixes of multiple productions for a given non-terminal
  - In English: If for some non-terminal there are multiple productions that have the same prefix, we want to get rid of that (somehow)
- Ex:

$$E \rightarrow T + E \mid T$$

$$T \rightarrow \text{int} \mid \text{int} * T \mid (E)$$

E has two productions with prefix T

T has two productions with prefix int

Left-Factoring Example

- Recall the grammar

$$E \rightarrow T + E \mid T$$
$$T \rightarrow int \mid int * T \mid (E)$$

- Factor out common prefixes of productions
  - So the prefix appears in only one production

$$E \rightarrow T X$$
$$X \rightarrow + E \mid \varepsilon$$
$$T \rightarrow (E) \mid int Y$$
$$Y \rightarrow * T \mid \varepsilon$$

But multiple suffixes!
New nonterminals X
and Y handle suffixes

Left-Factoring Example

- Recall the grammar

$$E \rightarrow T + E \mid T$$
$$T \rightarrow \text{int} \mid \text{int} * T \mid (E)$$

- Factor out common prefixes of productions
  - So the prefix appears in only one production

$$E \rightarrow T\,X$$
$$X \rightarrow + E \mid \varepsilon$$
$$T \rightarrow (E) \mid \text{int}\,Y$$
$$Y \rightarrow * T \mid \varepsilon$$

Effectively delays the decision about which production we're using

LL(1) Parsing Table Example

- Left-factored grammar

$E \rightarrow T\,X$ $\quad\quad$ $X \rightarrow +\,E \mid \varepsilon$

$T \rightarrow (\,E\,) \mid \text{int}\ Y$ $\quad\quad$ $Y \rightarrow *\,T \mid \varepsilon$

- The LL(1) parsing table:

next input token

	int	*	+	(	)	$
E	T X			T X		
X			+ E		ε	ε
T	int Y			(E)		
Y		* T	ε		ε	ε

leftmost non-terminal

rhs of production to use

LL(1) Parsing Table Example (Cont.)

- Consider the [E, int] entry
  - "When current non-terminal is E and next input is int, use production $E \to T X$"
  - This can generate an int in the first position

- Consider the [Y,+] entry
  - "When current non-terminal is Y and current token is +, get rid of Y"
  - Y can be followed by + only if $Y \to \varepsilon$

LL(1) Parsing Tables. Errors

- Blank entries indicate error situations

- Consider the [E,*] entry
  - "There is no way to derive a string starting with * from non-terminal E"

Using Parsing Tables

- Method similar to recursive descent, except
  - For the leftmost non-terminal S
  - We look at the next input token a
  - And choose the production shown at [S,a]

- A stack records frontier of parse tree
  - Non-terminals that have yet to be expanded
  - Terminals that have yet to matched against the input
  - Top of stack = leftmost pending terminal or non-terminal

- Reject on reaching error state
- Accept on end of input & empty stack

LL(1) Parsing Algorithm

```
initialize stack = <S $> and next
repeat
   case stack of
      <X, rest>  : if T[X,*next] = Y1…Yn
                        then stack ← <Y1… Yn rest>;
                        else  error ();
      <t, rest>   : if t == *next ++
                        then  stack ← <rest>;
                        else error ();
until stack == < >
```

LL(1) Parsing Algorithm

$ marks bottom of stack

```
initialize stack = <S $> and next
repeat
   case stack of
      <X, rest>   : if T[X,*next] = Y1...Yn
                       then stack ← <Y1... Yn rest>;
                       else  error ();
      <t, rest>   : if t == *next ++
                       then  stack ← <rest>;
                       else error ();
until stack == < >
```

For non-terminal X on top of stack, lookup production

Pop X, push production rhs on stack. Note leftmost symbol of rhs is on top of the stack.

For terminal t on top of stack, check t matches next input token.

LL(1) Parsing Example

Stack	Input	Action
E $	int * int $	T X
T X $	int * int $	int Y
int Y X $	int * int $	terminal
Y X $	* int $	* T
* T X $	* int $	terminal
T X $	int $	int Y
int Y X $	int $	terminal
Y X $	$	ε
X $	$	ε
$	$	ACCEPT

Our Short Term Goal

- How do we construct LL(1) parse tables?

- What are the conditions necessary for constructing LL(1) parse tables?

Constructing Parsing Tables: The Intuition

- Consider non-terminal A, production $A \rightarrow \alpha$, & token t

The question: Given A and t, under what conditions will we make the move $A \rightarrow \alpha$?

That is, under what conditions is $T[A,t] = \alpha$?

Constructing Parsing Tables: The Intuition

- Consider non-terminal A, production $A \rightarrow \alpha$, & token t
- $T[A,t] = \alpha$ in two cases:

Note this is $\rightarrow^*$

- If $\alpha \rightarrow^* t\ \beta$
  - α can derive a t in the first position
  - We say that $t \in First(\alpha)$

Constructing Parsing Tables: The Intuition

- Consider non-terminal A, production $A \rightarrow \alpha$, & token t
- $T[A,t] = \alpha$ in two cases:

note β and δ can be anything

- Now, assume $t \notin \text{First}(\alpha)$
  - Doesn't sound very promising to use α
  - But it turns out it may not be hopeless to use $A \rightarrow \alpha$
- If $A \rightarrow \alpha$ and $\alpha \rightarrow^* \varepsilon$ and $S \rightarrow^* \beta\ A\ t\ \delta$
  - Useful if stack has A, input is t, and A cannot derive t
  - In this case only option is to get rid of A (by deriving ε)
    - Can work only if t can follow A in at least one derivation
  - We say $t \in \text{Follow}(A)$
  - I.e., t is one of the things that can come after A in the grammar

Often A Point of Confusion

- We are NOT talking about A deriving t
  - A does not produce t
  - We ARE talking about t appearing in a derivation directly after A
  - So this has nothing to do with what A produces
  - Has to do with where A can appear in derivations

But for right now, let's concentrate on First sets

(We'll get to Follow sets in a bit)

Computing First Sets

Definition

$$\text{First}(X) = \{\, t \mid X \rightarrow^{*} t\alpha \} \cup \{\varepsilon \mid X \rightarrow^{*} \varepsilon\}$$

- Note: X can be a single terminal, it could be a single non-terminal, or it could be a string of grammar symbols
- t however, must be a terminal
- For technical reasons, ε needs to be in First(X) if it's the case that X can go to ε in zero or more steps
  - Need to keep track of this in order to compute all of the terminals that are in the first set of a given grammar symbol

Computing First Sets

Definition

$$\text{First}(X) = \{\, t \mid X \rightarrow^* t\alpha\} \cup \{\varepsilon \mid X \rightarrow^* \varepsilon\}$$

Algorithm sketch:

1. For t a terminal, $\text{First}(t) = \{\, t \,\}$
2. For X a non-terminal $\varepsilon \in \text{First}(X)$
   - if $X \rightarrow \varepsilon$
   - if $X \rightarrow A_1 \ldots A_n$ and $\varepsilon \in \text{First}(A_i)$ for $1 \leq i \leq n$
   - Note this can only happen if all of the A_i are non-terminals, since if there are any terminals on the R.H.S. then it can never completely go to ε

Computing First Sets

Definition

$$\text{First}(X) = \{\, t \mid X \to^* t\alpha \} \cup \{\varepsilon \mid X \to^* \varepsilon\}$$

Algorithm sketch:

1. For t a terminal, $\text{First}(t) = \{\, t \,\}$
2. For X a non-terminal $\varepsilon \in \text{First}(X)$
   - if $X \to \varepsilon$
   - if $X \to A_1 \ldots A_n$ and $\varepsilon \in \text{First}(A_i)$ for $1 \leq i \leq n$
3. $\text{First}(\alpha) \subseteq \text{First}(X)$ if $X \to A_1 \ldots A_n\, \alpha$ and $\varepsilon \in \text{First}(A_i)$ for $1 \leq i \leq n$

Make sure it's clear to you why (3) is true

Computing First Sets

Definition

$$\text{First}(X) = \{ t \mid X \rightarrow^* t\alpha \} \cup \{\varepsilon \mid X \rightarrow^* \varepsilon\}$$

Algorithm sketch:

1. For t a terminal, $\text{First}(t) = \{ t \}$
2. For X a non-terminal $\varepsilon \in \text{First}(X)$
   - if $X \rightarrow \varepsilon$
   - if $X \rightarrow A_1 \ldots A_n$ and $\varepsilon \in \text{First}(A_i)$ for $1 \leq i \leq n$
3. $\text{First}(\alpha) \subseteq \text{First}(X)$ if $X \rightarrow A_1 \ldots A_n\ \alpha$ and $\varepsilon \in \text{First}(A_i)$ for $1 \leq i \leq n$

Note: Rule (1) covers terminals; (2) and (3) cover non-terminals

First Sets. Example

- Recall the grammar

$E \to T\,X$ $\qquad\qquad$ $X \to +\,E \mid \varepsilon$

$T \to (\,E\,) \mid \text{int}\ Y$ $\qquad\qquad$ $Y \to *\ T \mid \varepsilon$

- First sets

First(( ) = { (} $\qquad$ First(T) = {int, (}

First( )) = {) } $\qquad$ First(E) = {int, (}

First(int) = { int } $\qquad$ First(X) = {+, ε }

First(+) = { + } $\qquad$ First(Y) = {*, ε }

First(*) = { * }

Computing Follow Sets

- Definition:

$$\text{Follow}(X) = \{ t \mid S \rightarrow^* \beta\, X\, t\, \delta \}$$

- Recall that the definition of the Follow set for a given symbol in the grammar is not about what that symbol can generate, but on where that symbol can appear
- In words, t is in $\text{Follow}(X)$ if there is some derivation where terminal t can appear immediately after the symbol X

Computing Follow Sets

- Definition:

$$\text{Follow}(X) = \{ t \mid S \rightarrow^* \beta\, X\, t\, \delta \}$$

- Intuition
  - If $X \rightarrow A\ B$ then $\text{First}(B) \subseteq \text{Follow}(A)$ and
    $\text{Follow}(X) \subseteq \text{Follow}(B)$
    - if $B \rightarrow^* \varepsilon$ then $\text{Follow}(X) \subseteq \text{Follow}(A)$
  - If S is the start symbol then $\$ \in \text{Follow}(S)$

Recall that $ is special symbol marking end of input

Computing Follow Sets

- Definition:

$$\text{Follow}(X) = \{\, t \mid S \rightarrow^* \beta\, X\, t\, \delta \,\}$$

- Intuition
  - If $X \rightarrow A\ B$ then $\text{First}(B) \subseteq \text{Follow}(A)$ and $\text{Follow}(X) \subseteq \text{Follow}(B)$
    - if $B \rightarrow^* \varepsilon$ then $\text{Follow}(X) \subseteq \text{Follow}(A)$
  - If S is the start symbol then $\$ \in \text{Follow}(S)$
  - That is, $ is in the Follow of the start symbol
    - Always added as an initial condition

Computing Follow Sets (Cont.)

Algorithm sketch:

1. $\$ \in \text{Follow}(S)$
2. $\text{First}(\beta) - \{\varepsilon\} \subseteq \text{Follow}(X)$
   - For each production $A \rightarrow \alpha\ X\ \beta$
3. $\text{Follow}(A) \subseteq \text{Follow}(X)$
   - For each production $A \rightarrow \alpha\ X\ \beta$ where $\varepsilon \in \text{First}(\beta)$

When is \$ in Follow(α)?

Students are often confused about \$, so let's discuss exactly when \$ is in Follow(α)

(This is important, since we'll be using Follow sets to build the parse table)

First, Some New Language

- If S is the start symbol of a grammar G, and α is such that $S \rightarrow^* \alpha$, then α is a *sentential form* of G
  - Note α may contain both terminals and non-terminals
- A *sentence* of G is a sentential form that contains no nonterminals.
- Technically speaking, the language of G, $L(G)$, is the set of sentences of G

So, the rule:

- $ is in Follow(α) if and only if α can appear at the end of a sentential form
- EX: Consider the following grammar

  $E \to TX$

  $X \to Ta \mid Cb$

  $T \to Tc \mid \varepsilon$

  $C \to a \mid b$

  - Note that neither B nor C can end a sentential form (why?), so $ is not in Follow(B) or Follow(C). But $ is in Follow(X).

Follow Sets. Example

- Recall the grammar

$E \to T X$ $\quad\quad$ $X \to + E \mid \varepsilon$

$T \to (E) \mid \text{int } Y$ $\quad\quad$ $Y \to * T \mid \varepsilon$

- Follow sets

Follow(+) = { int, (} $\quad$ Follow(*) = { int, (}

Follow(( ) = { int, (} $\quad$ Follow(E) = {), $}

Follow(X) = {$,) } $\quad$ Follow(T) = {+,) , $}

Follow( )) = {+,) , $} $\quad$ Follow(Y) = {+,) , $}

Follow(int) = {*, +,) , $}

Note, unlike with First sets, Follow sets for terminals can actually be interesting.

Follow Sets. Example

- Recall the grammar

$E \rightarrow T X$ $\qquad$ $X \rightarrow + E \mid \varepsilon$

$T \rightarrow (E) \mid \text{int } Y$ $\qquad$ $Y \rightarrow * T \mid \varepsilon$

- Follow sets

Follow(+) = { int, (} $\qquad$ Follow(*) = { int, (}

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Follow( )) = {+,) , $} $\qquad$ Follow(Y) = {+,) , $}

Follow(int) = {*, +,) , $}

Note Follow(( ). It makes sense: what can follow an (in the language? A nested (or an int

Follow Sets. Example

- Recall the grammar

$E \to T\,X \qquad\qquad X \to +\,E \mid \varepsilon$

$T \to (\,E\,) \mid \text{int}\;Y \qquad\qquad Y \to *\,T \mid \varepsilon$

- Follow sets

Follow(+) = { int, (}   Follow(*) = { int, (}

Follow(( ) = { int, (}   Follow(E) = {), $}

Follow(X) = {$,) }   Follow(T) = {+,) , $}

Follow( )) = {+,) , $}   Follow(Y) = {+,) , $}

Follow(int) = {*, +,) , $}

Similarly, Follow(+). What can follow + in the language? A new (or an int. Can't be $ (end input)

So Now

- We're going to pull together what we know about first and follow sets to construct LL(1) parsing tables.
- This is done one production at a time, eventually considering every production in the grammar

Recall:

$$\mathrm{First}(X) = \{\, t \mid X \rightarrow^{*} t\alpha \} \cup \{\varepsilon \mid X \rightarrow^{*} \varepsilon\}$$

$$\mathrm{Follow}(X) = \{\, t \mid S \rightarrow^{*} \beta\, X\, t\, \delta \,\}$$

Constructing LL(1) Parsing Tables

- Construct a parsing table T for CFG G

- For each production $A \to \alpha$ in G do:
  - For each terminal $t \in First(A)$ do
    - $T[A, t] = \alpha$
  - If $\varepsilon \in First(\alpha)$, for each $t \in Follow(A)$ do
    - $T[A, t] = \alpha$
  - If $\varepsilon \in First(\alpha)$ and $\$ \in Follow(A)$ do
    - $T[A, \$] = \alpha$
    - (note this is a special case, since $ is technically not a terminal symbol)

Constructing LL(1) Parsing Tables

- Construct a parsing table T for CFG G

Note: α might be ε

- For each production $A \rightarrow \alpha$ in G do:
  - For each terminal $t \in First(A)$ do
    - $T[A, t] = \alpha$
  - If $\varepsilon \in First(\alpha)$, for each $t \in Follow(A)$ do
    - $T[A, t] = \alpha$
  - If $\varepsilon \in First(\alpha)$ and $\$ \in Follow(A)$ do
    - $T[A, \$] = \alpha$
    - (note this is a special case, since $ is technically not a terminal symbol)

This is the algorithm for building LL(1) tables!

LL(1) Parsing Table Example

- Left-factored grammar

  $E \rightarrow T\,X$ $\qquad$ $X \rightarrow +\,E \mid \varepsilon$

  $T \rightarrow (\,E\,) \mid \text{int}\ Y$ $\qquad$ $Y \rightarrow *\,T \mid \varepsilon$

- The LL(1) parsing table:

next input token

	int	*	+	(	)	$
E	T X			T X		
X			+ E		ε	ε
T	int Y			(E)		
Y		* T	ε		ε	ε

leftmost non-terminal

rhs of production to use

Reference

First(( ) = { (}

First( )) = {) }

First(int) = { int }

First(+) = { + }

First(*) = { * }

First(T) = {int, (}

First(E) = {int, (}

First(X) = {+, ε }

First(Y) = {*, ε }

Follow(+) = { int, (}

Follow(( ) = { int, (}

Follow(X) = {$,) }

Follow( )) = {+,) , $}

Follow(int) = {*, +,) , $}

Follow(*) = { int, (}

Follow(E) = {), $}

Follow(T) = {+,) , $}

Follow(Y) = {+,) , $}

Another Example

$S \rightarrow Sa \mid b$

First(S) = {b}, Follow(S) = {$, a}

- The LL(1) parsing table:

	a	b	$
S		b Sa	ε

The problem: both productions
can produce a b in the first position,
giving entry with multiple moves

Notes on LL(1) Parsing Tables

- If any entry is multiply defined then G is not LL(1)
  - If G is ambiguous
  - If G is left recursive
  - If G is not left-factored
  - And in other cases as well
  - In fact, definition of LL(1) is that a grammar is NOT LL(1) iff the built LL(1) table has a multiply defined entry
- Most programming language CFGs are not LL(1)

These amount to quick checks for NOT LL(1)-ness

Bottom Line on LL(1) Parsing

- Most programming language CFGs are not LL(1)
  - LL(1) grammars are just too weak to capture all of the interesting and important structures in real-world programming languages
- There are more powerful formalisms for describing practical grammars
- So, why bother with LL(1)?
  - Well, it turns out that these formalisms build on everything we've learned here for LL(1) grammars, so our effort is not wasted.
  - Ideas just assembled in a more sophisticated way to build more powerful parts

Bottom-Up Parsing

- Bottom-up parsing is more general than (deterministic) top-down parsing
  - And just as efficient
  - Builds on ideas in top-down parsing

- Bottom-up is the preferred method
  - Used in most parser generator tools (including CUP)

- Concepts today, algorithms next time

Recall where we are:

We talked about Recursive Descent parsing, which is completely general but requires backtracking.

Then we talked about LL(1), which is not completely general, but requires no backtracking.

We are now embarking on bottom-up parsing.

An Introductory Example

- Bottom-up parsers don't need left-factored grammars
  - Recall what left-factored means: (no two productions with the same prefix)
- Revert to the "natural" grammar for our example:

$$E \rightarrow T + E \mid T$$

$$T \rightarrow \text{int} * T \mid \text{int} \mid (E)$$

- "natural" in quotes because we still have to deal with precedence of + and *

An Introductory Example

- Bottom-up parsers don't need left-factored grammars
  - Recall what left-factored means

- Revert to the "natural" grammar for our example:

$$E \rightarrow T + E \mid T$$
$$T \rightarrow \text{int} * T \mid \text{int} \mid (E)$$

- Consider the string: int * int + int

The Idea

Bottom-up parsing *reduces* a string to the start symbol by inverting productions (running them backwards)

int * int + int	T → int
int * T + int	T → int * T
T + int	T → int
T + T	E → T
T + E	E → T + E
E	

Left column is a sequence of states.
right side is productions used.

Observation

- Read the productions in reverse (from bottom to top)
  - Read in the order we did them, called *reductions*
- This is a rightmost derivation!

int * int + int	T → int
int * T + int	T → int * T
T + int	T → int
T + T	E → T
T + E	E → T + E
E	

Important Fact #1

Important Fact #1 about bottom-up parsing:

A bottom-up parser traces a rightmost derivation in reverse

If you're ever having trouble with bottom-up parsing, it's good to come back to this basic fact

The Idea

- A top-down parser begins with the start symbol and produces the tree incrementally by expanding some non-terminal at the frontier

- A bottom-up parser begins with all ALL the leaves of the parse tree (the entire input) and builds little trees on top of those
  - It pastes all the subtrees that it's built so far together to create the entire parse tree

A Bottom-up Parse

int * int + int

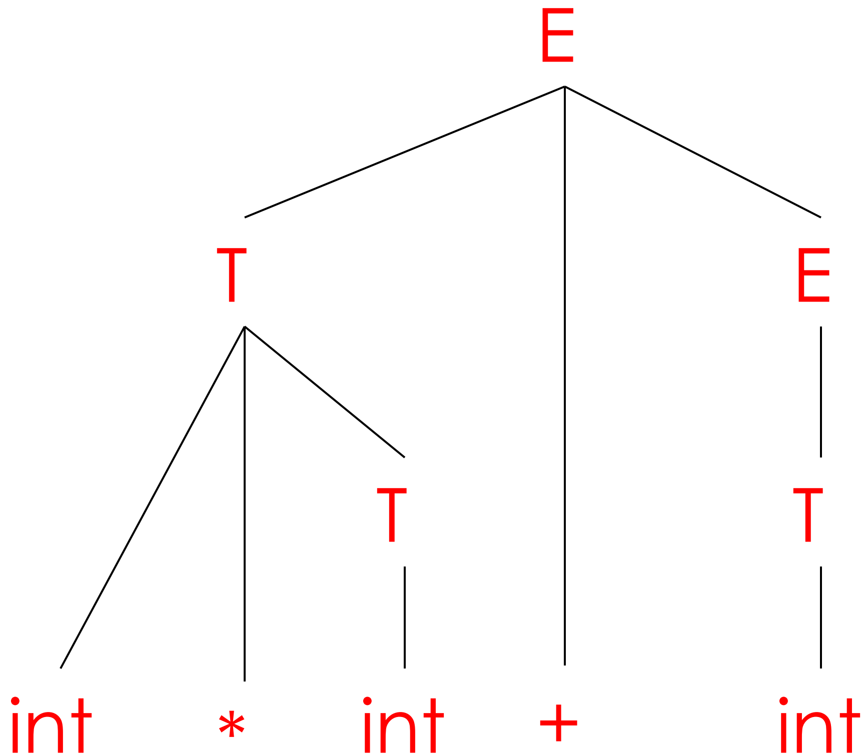
int * T + int

T + int

T + T

T + E

E



A Bottom-up Parse in Detail (1)

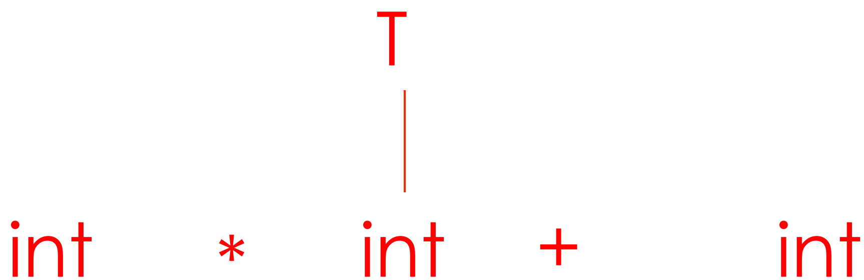
int * int + int

int * int + int

A Bottom-up Parse in Detail (2)

int * int + int

int * T + int

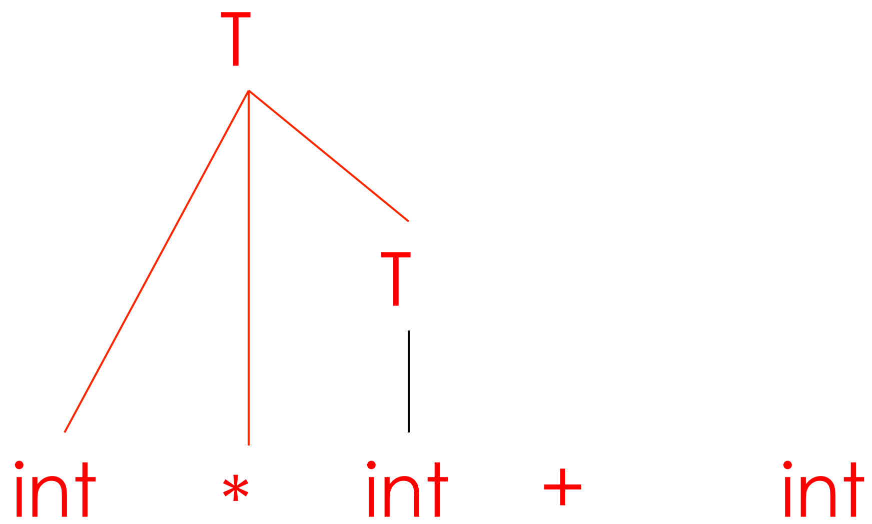


A Bottom-up Parse in Detail (3)

int * int + int

int * T + int

T + int



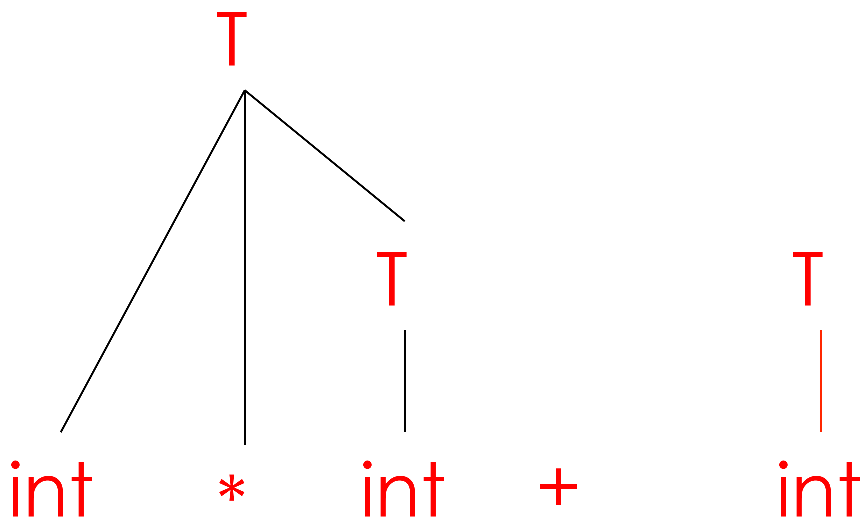
A Bottom-up Parse in Detail (4)

int * int + int

int * T + int

T + int

T + T



A Bottom-up Parse in Detail (5)

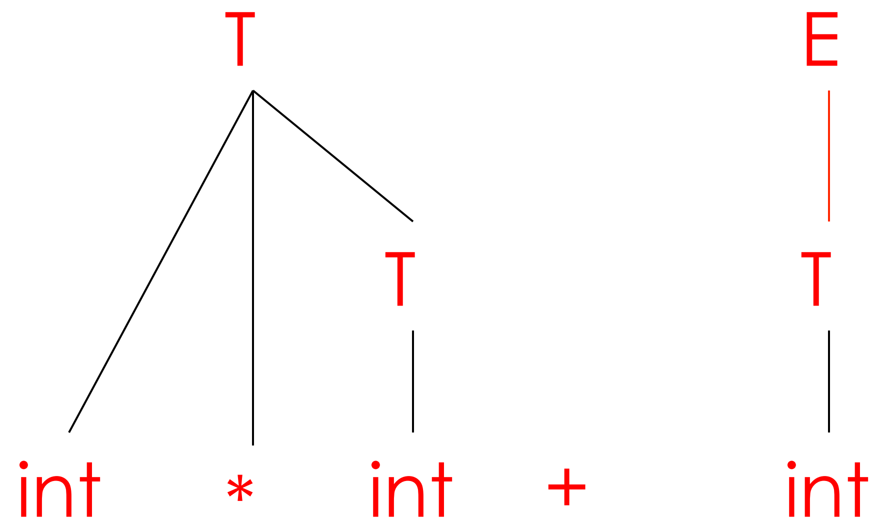
int * int + int

int * T + int

T + int

T + T

T + E



A Bottom-up Parse in Detail (6)

int * int + int

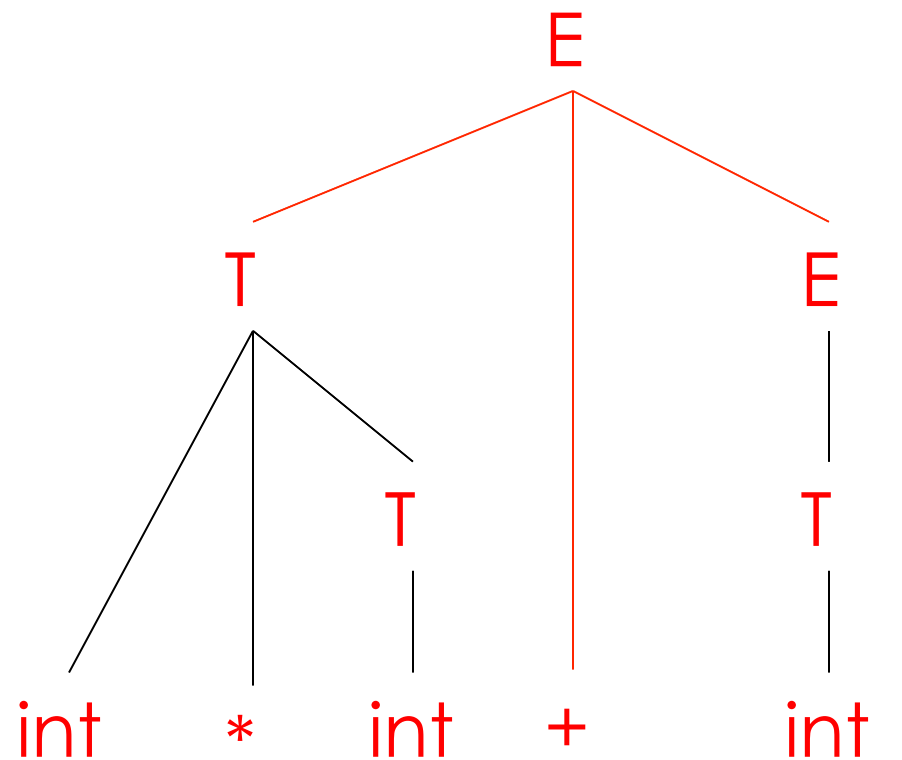
int * T + int

T + int

T + T

T + E

E



A Trivial Bottom-Up Parsing Algorithm

Let I = input string

repeat

pick a non-empty substring β of I

where $X \rightarrow \beta$ is a production

if no such β, backtrack

replace one β by X in I

until I = "S" (the start symbol) or all possibilities are exhausted

Questions

- Does this algorithm terminate?
- How fast is the algorithm?
- Does the algorithm handle all cases?
- How do we choose the substring to reduce at each step?

Where Do Reductions Happen?

Important Fact #1 has an interesting consequence:

- Let $\alpha\beta\omega$ be a step of a bottom-up parse
- Assume the next reduction is by $X \rightarrow \beta$
- Then ω is a string of terminals

Why? Because $\alpha X\omega \rightarrow \alpha\beta\omega$ is a step in a right-most derivation

Recall Fact #1: Bottom-up parser traces a rightmost derivation in reverse.

Notation

- Idea: Split string into two substrings
  - Right substring is as yet unexamined by parsing (a string of terminals)
    - Turns out terminal symbols to right of right most non-terminal are exactly the unexamined input in bottom-up parsing
  - Left substring has terminals and non-terminals
  - E.g., if we have $\alpha X \omega$, and X is the rightmost non-terminal, then ω is the input we have not read yet

Notation

- The dividing point is marked by a |
  - The | is not part of the string
- Initially, all input is unexamined $|x_1 x_2 \ldots x_n$

- After some input has been examined:

$$\underbrace{x_1 x_2 x_3}_{\text{processed}} | \underbrace{x_4 \ldots x_n}_{\text{unprocessed}}$$

Shift-Reduce Parsing

Bottom-up parsing uses only two kinds of actions:

Shift moves

Reduce moves

Shift

- *Shift:* Move | one place to the right
  - Shifts a terminal to the left string
  - Equivalently reads one token of input
  - In example below, the shift indicates that token x can now be considered as part of processing
  - y and z remain unprocessed and unread at this point

$$ABC|xyz \Rightarrow ABCx|yz$$

Reduce Move

- Apply an inverse production at the right end of the left string
  - If $A \rightarrow xy$ is a production, then

$$Cbxy \mid ijk \Rightarrow CbA \mid ijk$$

Previously Seen Example with Reductions Only

$E \rightarrow T + E \mid T$
$T \rightarrow \text{int} * T \mid \text{int} \mid (E)$

int * int | + int	reduce T → int
int * T | + int	reduce T → int * T
T + int |	reduce T → int
T + T |	reduce E → T
T + E |	reduce E → T + E
E |	

The Example with Shift-Reduce Parsing

$E \rightarrow T + E \mid T$
$T \rightarrow int * T \mid int \mid (E)$

| int * int + int	shift
int | * int + int	shift
int * | int + int	shift
int * int | + int	reduce T → int
int * T | + int	reduce T → int * T
T | + int	shift
T + | int	shift
T + int |	reduce T → int
T + T |	reduce E → T
T + E |	reduce E → T + E
E |	

- Note: In this derivation, and in the details of it that follows, all I'm showing is that there exists a sequence of shift and reduce moves that can successfully parse the input string.
- I do not (yet) explain how we choose whether to perform a shift or reduce move.

A Shift-Reduce Parse in Detail (1)

|int * int + int

int * int + int

↑

A Shift-Reduce Parse in Detail (2)

|int * int + int

int | * int + int

int * int + int

A Shift-Reduce Parse in Detail (3)

|int * int + int

int | * int + int

int * | int + int

int * int + int

↑

A Shift-Reduce Parse in Detail (4)

|int * int + int

int | * int + int

int * | int + int

int * int | + int

int * int + int

A Shift-Reduce Parse in Detail (5)

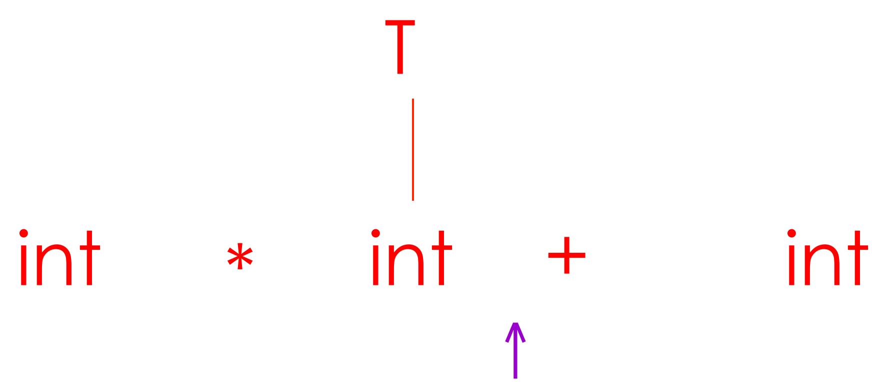
|int * int + int

int | * int + int

int * | int + int

int * int | + int

int * T | + int



A Shift-Reduce Parse in Detail (6)

|int * int + int

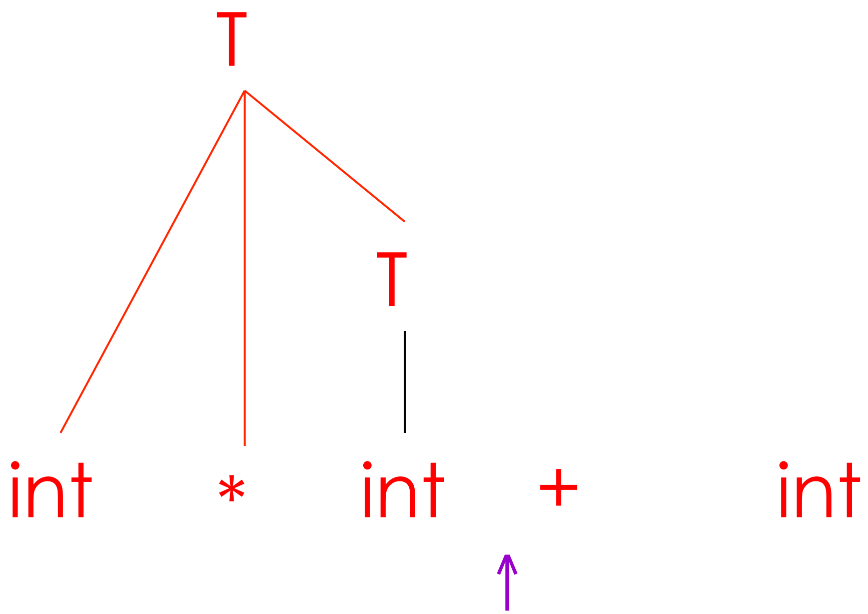
int | * int + int

int * | int + int

int * int | + int

int * T | + int

T | + int



A Shift-Reduce Parse in Detail (7)

|int * int + int

int | * int + int

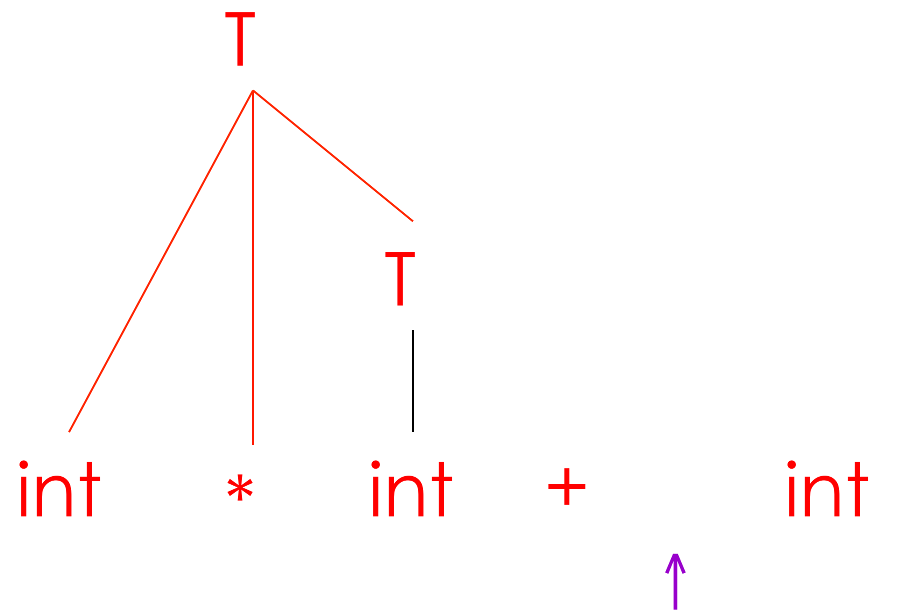
int * | int + int

int * int | + int

int * T | + int

T | + int

T + | int



A Shift-Reduce Parse in Detail (8)

|int * int + int

int | * int + int

int * | int + int

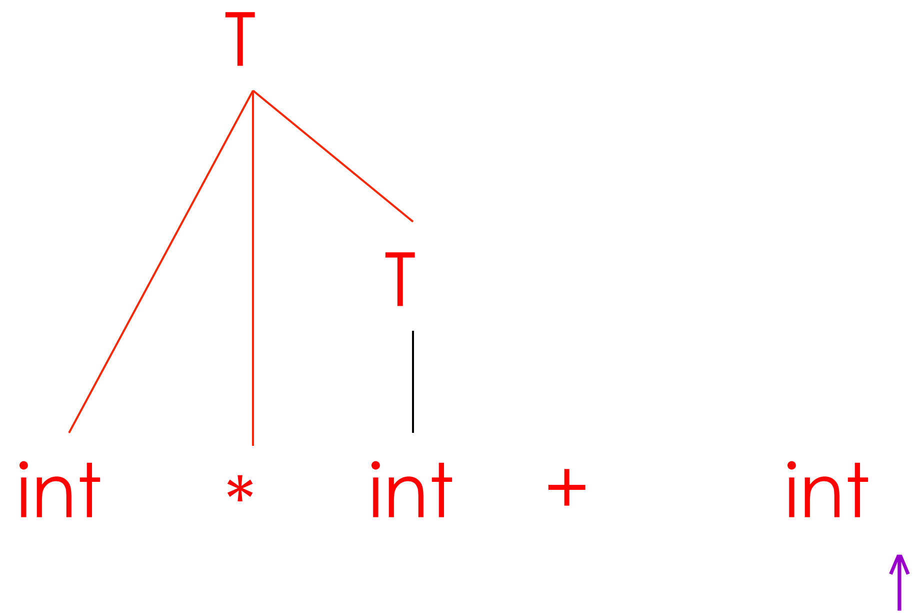
int * int | + int

int * T | + int

T | + int

T + | int

T + int |



A Shift-Reduce Parse in Detail (9)

|int * int + int

int | * int + int

int * | int + int

int * int | + int

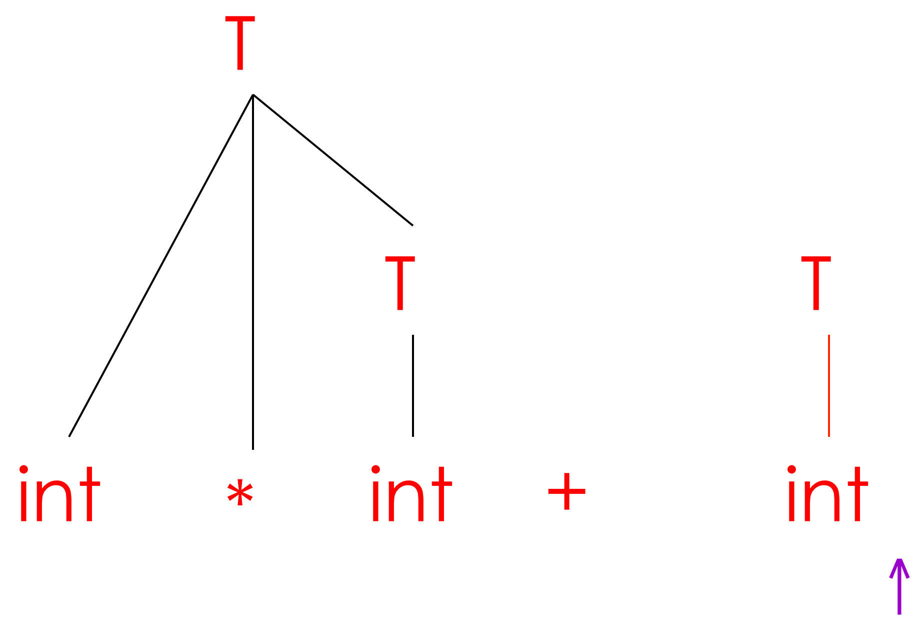
int * T | + int

T | + int

T + | int

T + int |

T + T |



A Shift-Reduce Parse in Detail (10)

|int * int + int

int | * int + int

int * | int + int

int * int | + int

int * T | + int

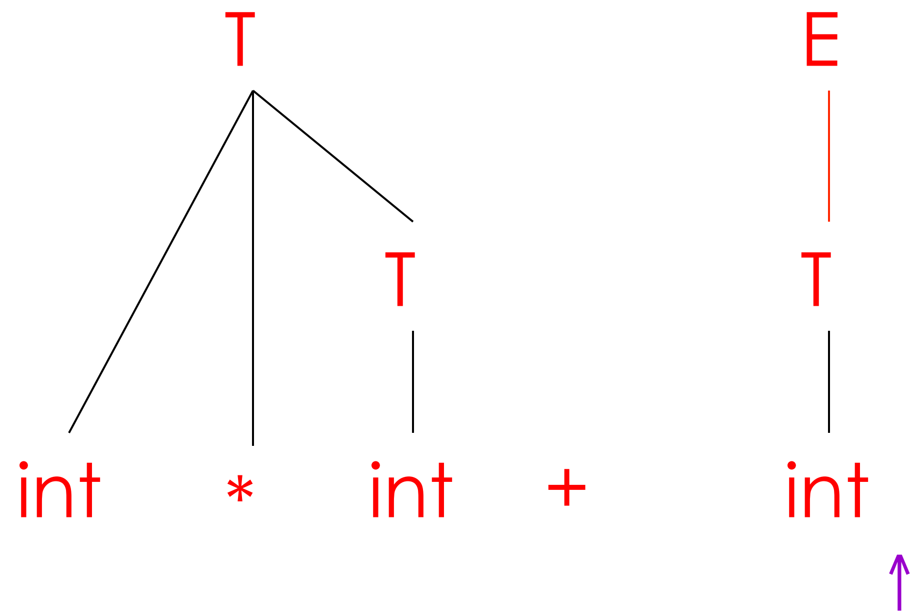
T | + int

T + | int

T + int |

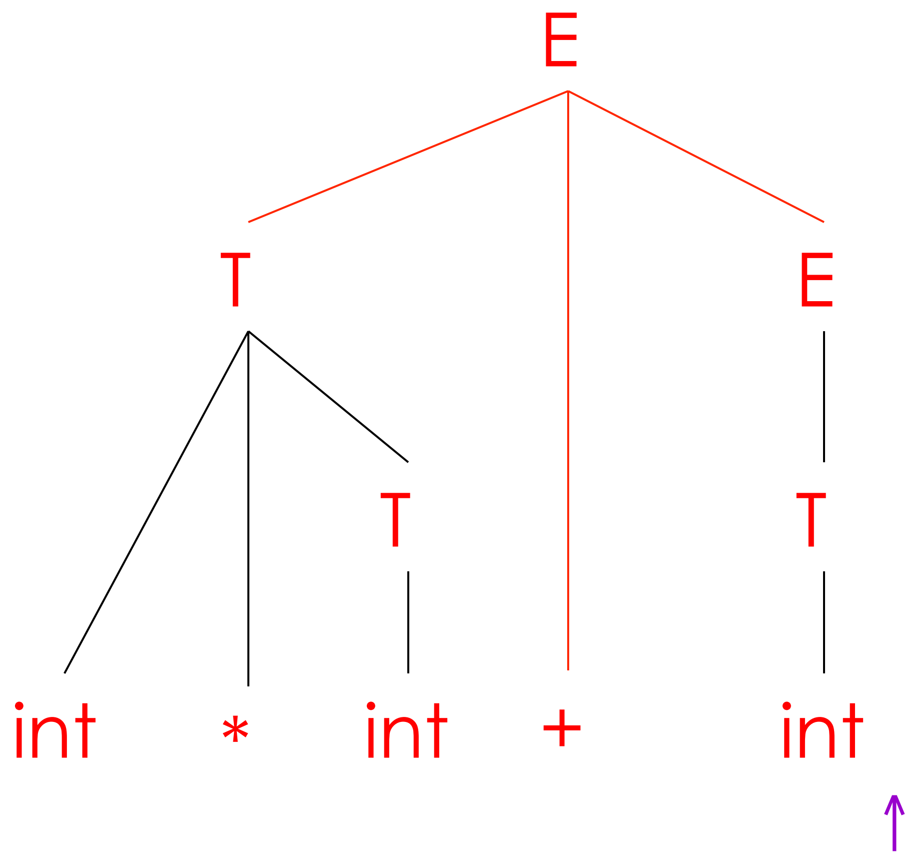
T + T |

T + E |



A Shift-Reduce Parse in Detail (11)

| int * int + int
int | * int + int
int * | int + int
int * int | + int
int * T | + int
T | + int
T + | int
T + int |
T + T |
T + E |
E |



The Stack

- Left string can be implemented by a stack
  - Top of the stack is the |

- Shift pushes a terminal on the stack

- Reduce pops 0 or more symbols off of the stack (production rhs) and pushes a non-terminal on the stack (production lhs)

Conflicts

- In a given state, more than one action (shift or reduce) may lead to a valid parse

- If both a shift and a reduce are possible at some juncture, there is a *shift-reduce* conflict

- If it is legal to reduce by two different productions, there is a *reduce-reduce* conflict

- You will see such conflicts in your project!
  - More next time . . .