This assignment covers counting and basic probability. You must cite any conversations that have contributed to your solutions and turn in only work that you understand and have written up yourself. Solutions to optional questions do not need to be submitted, but you are responsible for any material covered. You must submit this homework in \LaTeX. Only submit your final pdfs, not your .tex source files.

1. (Optional) I have 10 shirts, 12 pants, and 7 belts. I have to pick 4 shirts, 4 pants, and 2 belts to put in my suitcase. How many different ways can I pick what goes in the suitcase?

Solution:

2. (Optional) How many binary *palindromes* of size 9 are there? Justify your answer.

Solution:

3. (Optional) A valid password at a company must be 10 characters long, made of upper case letters, lower case letters, and numbers, and must contain at least one of each character type. How many possible passwords are there?

Solution:

4. (Optional) \( n \) kids randomly line up for recess. Two kids are named Celia and Felicity.
   
   (a) What is the probability that Celia is first in line?

   Solution:

   (b) What is the probability that Celia is third in line?

   Solution:

   (c) What is the probability that Celia is first and Felicity is last in line?

   Solution:

   (d) What is the probability that Celia and Felicity are next to each other in line?

   Solution:
5. (50 points) We are counting 20 character strings over the 26 lowercase letters with certain properties.

(a) (10 points) How many such strings have the property that no two letters that are adjacent are the same? Thus substrings like “aa” are forbidden.

Solution:

(b) (10 points) How many such strings have only the property that no two letters that are 2-apart are the same? Thus substrings like “aa” are now allowed but substrings like “aba” are forbidden. Justify your answer.

Hint: There are many ways to approach this. One is much easier than the others.

Solution:

(c) (10 points) How many such strings have the property that no two adjacent letters are the same, AND also no two letters that are 2-apart are the same. Thus substrings like “aa” are forbidden, as are substrings like “aba”. For example, ‘adcbadcbba’ meets the criteria, but ‘adcbadcbba’ and ‘adccabdbd’ do not. Justify your answer.

Hint: There are many ways to approach this. One is much easier than the others.

Solution:

(d) (10 points) How many such strings have property that no two adjacent letters are the same, OR no two letters that are 2-apart are the same. Thus you may have substrings like “aa”, or like “aba”, or both.

Solution:

(e) (10 points) How many such strings have property that EXACTLY ONE of the two properties is true? Either no two adjacent letters are the same, (XOR) no two letters that are 2-apart are the same. Thus you must have either substrings like “aa”, or like “aba”, but not both.

Solution:

6. (50 points) Recall that a standard deck of cards consists of 52 distinct cards, each of which has one of four suits (clubs, diamonds, hearts, spades) and one of four ranks/values (A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K). In the mystical land of Tootwen-Teetoo, people play a variant of poker with 10-card hands.

(a) (10 points) Twotwen-Teetoo poker features the “Monochrome”, a 10-card hand that consists entirely of red cards (Diamond, Heart) or entirely of black cards (Clubs, Spades). What is the probability of getting dealt a Monochrome? Justify your answer.

Solution:

(b) (10 points) Twoten-Teetoo poker also features the “Four-Three-Two-One”, which consists of getting four cards of one rank, three of another, two of another, and one of another. Whare is the probability of getting dealt a Four-Three-Two-One?
Solution:

(c) (10 points) Twotwen-Teetoo poker also features the “Hat Trick”, a 10-card hand that consists three each of three different ranks, and one extra card. What is the probability of getting dealt a Hat-Trick? Justify your answer.

Solution:

(d) (10 points) Twotwen-Teetoo poker also features the “Three-pair”, a 10-card hand that consists of three ranks that appear twice and four ranks that appear once. What is the probability of getting a Three-pair? Justify your answer.

Solution:

(e) (10 points) Twotwen-Teetoo poker features the “Fuller House”, a 10-card hand that consists of two ranks that appear twice each (two pairs) and two ranks that appear three times each (two three-of-a-kinds). What is the probability of getting a Fuller House? Justify your answer.

Solution:

7. (Optional) Say $n$ people in a classroom all go to a White Elephant Christmas party, where each person brings a gift (numbered $1, 2, \ldots n$), and must give their gift to a different person (no one can give their gift to themselves). How many possible ways can this happen? Give an expression and justify it.

Hint: Start with the total number of possible ways that the gifts can be distributed without this restriction, and exclude the situations you don’t want.