This assignment covers propositional logic, rules of inference, some corresponding set theoretic concepts, predicates, and quantifiers. (Rosen 1.1-1.6, 2.2). Cite any conversations that have contributed to your solutions and turn in only work that you understand and have written up yourself. Solutions to optional questions do not need to be submitted, but you are responsible for any material covered. You do not need to submit this homework in LATEX, but you will get 1 bonus point if you do. (See Syllabus regarding bonus points, and the LaTeX guide regarding how to write and submit in LaTeX). If submitting LATEX, then please only submit your final pdfs, not your .tex source files.

1. (OPTIONAL) The order of operations for boolean logic operations is
   - Parenthesis
   - Not
   - Ands, then Xors, then Ors (but it’s often useful to add parentheses when these three are at the same level)
   - Implies, then IFF

   Write a truth table for each of the following expressions in terms of the variables given.
   (a) \( x \land (\neg y \lor z) \)
   (b) \( (x) \to ((x \to y)) \)
   (c) \( (y) \to ((x \to y)) \)
   (d) \( (x \land (x \to y)) \land y \)
   (e) \( (x \land (x \to y)) \leftrightarrow y \)

2. (OPTIONAL) Translate each English sentence into a logical expression using the propositional variables defined below. Then negate the logical expression, applying De Morgan’s law to the resulting expression as necessary to distribute the "nots" fully. In other words, in your final logical statement, there should be no (not)s being applied to parenthetical expressions, only nots applied to individual variables. Then translate the negated version of the original expression back into english.
   - p: the applicant has written permission from their parents
   - q: the applicant is at least 18 years old
   - r: the applicant is at least 16 years old

   (a) The applicant is either 16 or 17 years old.
   (b) The applicant is either at least 16 years old and has written permission from their parents, or is at least 18 years old.
   (c) If the applicant is not at least 16 years old, then the applicant has written permission from their parents.
3. (Optional) Prove the following logical equivalences with a truth table.

(a) \( p \land (q \lor r) \equiv (p \land q) \lor (p \land r) \)

Solution:

(b) \( p \lor (q \land r) \equiv (p \lor q) \land (p \lor r) \)

Solution:

4. (20 points) Prove the following logical statements using the laws of inference, and NOT truth tables. Express all your steps either in a table (like in 1.6 examples 6 and 7) or using sentences (like in 1.6 example 8).

(a) Example: \([ (p \land q) \lor r ] \land [ r \implies s]\)  \[\implies p \lor s\]

Solution: Rewrite \([ (p \land q) \lor r ]\) as \([- (p \land q) \implies r].\) Then the statements \([- (p \land q) \implies r]\) and \([ r \implies s]\) imply \([- (p \land q)] \implies s\) by Hypothetical syllogism. We can rewrite this as \((p \land q) \lor s\). We can rewrite this as \((p \lor s) \land (q \lor s)\). By simplification, we conclude that this implies \(p \lor s\), as desired.

(b) \([ [a \implies b] \land [ \neg c \implies \neg b] \land [\neg c]]\)

\[\implies \neg a\]

Solution:

(c) \([ [a] \land [ a \implies (b \land \neg c)] \land [(b \land \neg d) \implies (e \lor f)] \land [d \implies c] \land [\neg f]]\)

\[\implies e\]

Solution:

5. (30 points) Translate the following English sentences into logical propositions and translate the following logical propositions into English statements. The domain for each of the following predicates is the set of employees of a company \(E\), which includes the board of directors and also Dan. Expressions like \(E(Dan)\) and \(x \neq Dan\) are valid. Expressions like \(x \in B\) are not, since \(B\) is not a set.

- \(B(x) \equiv \text{"x is on the board of directors"}\)
- \(E(x) \equiv \text{"x earns more than $100,000 per year"}\)
- \(W(x) \equiv \text{"x works more than 60 hours per week"}\)

(a) Example: Dan is not on the board of directors.

Solution: \(\neg B(Dan)\)

(b) No one works more than 60 hours per week.
(c) Every board of directors member earns more than $100,000 per year.

Solution:

(d) No one on the board of directors works more than 60 hours per week.

Solution:

(e) Someone other than Dan earns more than $100,000 per year.

Solution:

(f) Every member of the board of directors makes over 100,000 per year, but no one else does.

Solution:

(g) Dan is the only board of directors member who works more than 60 hours per week.

Solution:

6. (50 points) At a certain chess club, everyone is placed either in the advanced A class, or the beginner B class. People play other people in this club repeatedly all the time. Any two people potentially have played multiple games against each other or no games against each other, with potentially different winners in the different games. Alice is a member of class A, and Bob is a member of class B.

The implied set domain for the following predicates is the set of members of the chess club, C.

- $A(x) \equiv \text{“} x \text{ is in Class A”}$
- $B(x) \equiv \text{“} x \text{ is in Class B”}$
- $W(x, y) \equiv \text{“} x \text{ has won at least one game against } y \text{”}$

Naturally, no player can play a game against themselves.

Write logical statements equivalent to each of the below English sentences.

(a) Example: Alice has beaten everyone.

Solution: $\forall x \in C \ (x = Alice) \lor W(Alice, x)$

(b) Alice is undefeated. Note, this is not the same as the above.

Solution:

(c) All the Class A members have beaten all the Class B members.
Solution:

(d) The Class A members have beaten only Class B members.

Solution:

(e) Bob has won against only one player.

Solution:

(f) There are at least two, different members who are undefeated.

Solution: