

## Example: Multiple Standard Additions

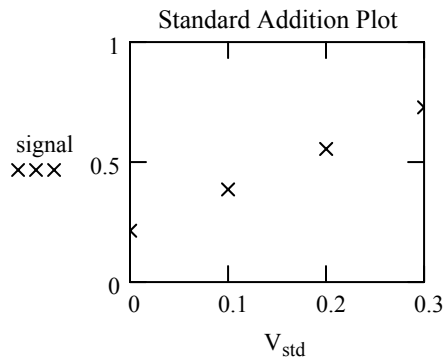
$g_{\text{sample}} := 3.12$  mass of sample, in g

$C_{\text{std}} := 300.0$  conc of added standard, in ppm

$V_a := 25$  volume of sample solutions, in mL

$V_{\text{std}} := (0 \ 0.1 \ 0.2 \ 0.3)^T$  signal := (0.214 0.386 0.554 0.728)<sup>T</sup> std addition data

Since all the sample solutions were diluted to the same volume, there is no need to correct the signal for dilution. Let's take a look at the standard addition plot (to verify linearity).



Looks linear; here are the LS estimates

$$b_0 := \text{intercept}(V_{\text{std}}, \text{signal}) \quad b_0 = 0.2140$$

$$b_1 := \text{slope}(V_{\text{std}}, \text{signal}) \quad b_1 = 1.7100$$

$$S_{xx} := 3 \cdot \text{Var}(V_{\text{std}}) \quad \text{xbar} := \text{mean}(V_{\text{std}})$$

Point Estimate of Analyte Concentration:

$$V_{\text{prime}} := \frac{b_0}{b_1} \quad V_{\text{prime}} = 0.1251 \quad C_a := C_{\text{std}} \cdot \frac{V_{\text{prime}}}{V_a} \quad C_a = 1.5018 \quad \text{concentration in sample solution, in ppm}$$

$$\mu g_{\text{analyte}} := C_a \cdot \frac{25}{5} \cdot 100 \quad \mu g_{\text{analyte}} = 750.8772 \quad \text{mass of analyte in chewing gum sample, in ug (assumes 1 ppm = 1 ug/mL)}$$

$$\text{conc} := \frac{\mu g_{\text{analyte}}}{g_{\text{sample}}} \quad \text{conc} = 240.6658 \quad \text{concentration of analyte in original sample, in ppm}$$

Std Error of Point Estimate, from Residuals

$$\text{fit} := b_1 \cdot V_{\text{std}} + b_0 \quad \text{res} := \text{signal} - \text{fit} \quad s_{\text{res}} := \sqrt{\frac{1}{2} \sum \text{res}^2} \quad s_{\text{res}} = 1.7321 \times 10^{-3}$$

$$s(V_{\text{prime}}) := \frac{s_{\text{res}}}{b_1} \cdot \sqrt{1 + \frac{1}{4} + \frac{(V_{\text{prime}} - \text{xbar})^2}{S_{xx}}} \quad s(V_{\text{prime}}) = 1.6840 \times 10^{-3} \quad \text{std error of } V_{\text{prime}}$$

$$se_{C_a} := s(V_{\text{prime}}) \cdot \frac{C_{\text{std}}}{V_a} \quad se_{C_a} = 0.0202 \quad \text{std error of solution point estimate, in ppm}$$

$$se := se_{C_a} \cdot \frac{25}{5} \cdot \frac{100}{g_{\text{sample}}} \quad se = 3.2385 \quad \text{standard error of analyte conc estimate in original sample, in ppm}$$

$$t := \text{qt}(.975, 2) \quad t = 4.3027 \quad t \cdot se = 13.9340 \quad \text{width of 95\% CI}$$

**The concentration of aluminum in the chewing gum is 241 ± 14 ppm (95% CL)**