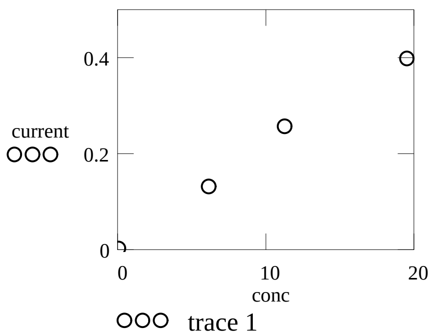
Linear Calibration Example 2

$conc := (0\ \ 6.1\ \ 11.2\ \ 19.4)^T$ $current := (0.003\ \ 0.134\ \ 0.259\ \ 0.398)^T$

Let's look at a plot of the calibration measurements.



Looks reasonably linear! Let's calculate the least-squares estimates of the slope and intercept of a best-fit line.

$b_0 := intercept(conc, current)$ $b_0 = 0.0101$

$b_1 := slope(conc, current)$ $b_1 = 0.0205$

We need to calculate the residuals and a few other items

$fit := b_1 \cdot conc + b_0$ $res := current - fit$ $res^T = \left[-7.0698 \cdot 10^{-3}\ \ -1.3476 \cdot 10^{-3}\ \ 0.0189\ \ -0.0105\right]$ residuals

$s_{res} := \sqrt{\frac{1}{2} \cdot \sum \overrightarrow{res^2}}$ $s_{res} = 0.0161$ $S_{xx} := 3 \cdot Var(conc)$ $S_{xx} = 202.2875$ $xbar := mean(conc)$

Now we can calculate a confidence interval for the analyte concentration in the "unknown"

$y_u := 0.175$ signal from the "unknown" sample, in microamps

$x_u := \frac{y_u - b_0}{b_1}$ $x_u = 8.0307$ point estimate of analyte concentration, in ppb

$se_u := \frac{s_{res}}{b_1} \cdot \sqrt{1 + \frac{1}{4} + \frac{(x_u - xbar)^2}{S_{xx}}}$ $se_u = 0.8797$ std error of point estimate, in ppb

$t := qt(.975, 2)$ $t = 4.3027$ $t \cdot se_u = 3.7852$ width of 95% confidence interval

The concentration of arsenic in the sample is 8.0 ± 3.8 ppb (95% CL).

This confidence interval assumes homogeneous noise in the calibration measurements, and that the measurements are normally distributed.