Hypothesis Testing

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On the Agenda

1. Null and Alternative Hypotheses
2. Type I and Type II Errors
3. Steps of Hypothesis Testing
4. Population Mean: One-Sample σ Known
5. Population Mean: One-Sample σ Unknown
6. Tests About Population Proportion

On the Agenda

Null and Alternative Hypotheses

- Hypothesis testing: determine whether a statement about the value of a population parameter should or should not be rejected
- Null hypothesis: is a tentative assumption about a population parameter. Denoted by H_0
- Alternative hypothesis: is the opposite of what is stated in the null hypothesis. Denoteb by H_a
- The hypothesis testing procedure uses data from a sample to test the two competing statements indicated by H_0 and H_a

Null and Alternative Hypotheses

- It is not always obvious how the null and alternative hypotheses should be formulated
- The context of the situation is very important in determining how the hypotheses should be stated
- In some cases it is easier to identify the alternative hypothesis first. In other cases the null is easier

Example: Null and Alternative Hypotheses

- The label on a soft drink bottle states that it contains 67.6 fluid ounces
- Null Hypothesis:

  The label is correct: $H_0: \ \mu \geq 67.6$

- Alternative Hypothesis:

  The label is incorrect: $H_a: \ \mu < 67.6$

Null and Alternative Hypotheses

- As a general rule, the equality part of the hypotheses always appears in the null hypothesis

- In general, a hypothesis test about the value of a population mean must take one of the following three forms:

lower-tail	upper-tail	Two-tailed
$H_0 : \mu \geq \mu_0$	$H_0 : \mu \leq \mu_0$	$H_0 : \mu = \mu_0$
$H_a : \mu < \mu_0$	$H_a : \mu > \mu_0$	$H_a : \mu \neq \mu_0$

On the Agenda

Type I and Type II Errors

- Because hypothesis tests are based on sample data, we must allow for the possibility of errors
- Type I error: is rejecting H_0 when it is true
- Level of Significance: The probability of making a Type I error when the null hypothesis is true
- Applications of hypothesis testing that only control the Type I error are often called significance tests

Type I and Type II Errors

- Type II error: is accepting H_0 when it is false
- It is difficult to control the probability of making a Type II error
- Statisticians avoid the risk of making a Type II error by using "do not reject H_0" and not "accept H_0"

Type I and Type II Errors

	Do not reject null hypothesis	Reject null hypothesis
Null hypothesis is true	Good Job!	
Null hypothesis is false		Good Job!

Type I and Type II Errors

	Do not reject null hypothesis	Reject null hypothesis
Null hypothesis is true	Good Job!	Type I error
Null hypothesis is false	Type II error	Good Job!

On the Agenda

Steps of Hypothesis Testing

- We can use two methods to test hypotheses
  - p-Value Approach
  - Critical Value Approach

- There are three steps that we always follow:
  1. Develop the null and alternative hypotheses
  2. Specify the level of significance α
  3. Collect the sample data and compute the value of the test statistic

Steps of Hypothesis Testing

- p-Value Approach

  4. Use the value of the test statistic to compute the p-value
  5. Reject H_0 if p-value $\leq \alpha$

- Critical Value Approach

  4. Use the level of significance a to determine the critical value and the rejection rule
  5. Use the value of the test statistic and the rejection rule to determine whether to reject H_0

Example: Steps of Hypothesis Testing

- Hilltop Coffee
- FTC wants to check Hilltop's claim that its large can of coffee contains 3 pounds of coffee
- The FTC director wants to perform a hypothesis test, with a .01 level of significance, to check Hilltop's claim
- A random sample of 36 cans provides a sample mean of $\bar{x} = 2.92$
- Previous FTC tests show that the population standard deviation can be assumed known with a value of $\sigma = 0.18$
- Also, the population of filling weights can be assumed to have a normal distribution

Example: Steps of Hypothesis Testing

- Hilltop Coffee
  1. Develop the hypotheses

$$H_0 : \mu \geq 3$$
$$H_a : \mu < 3$$

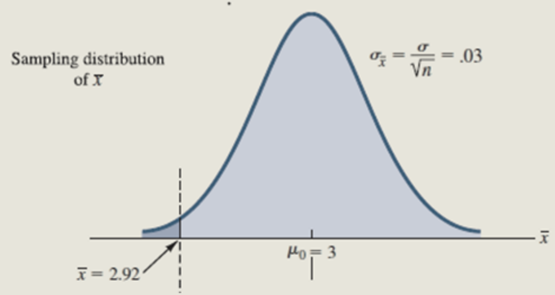
  2. Specify the level of significance

$$\alpha = 0.01 = 1\%$$

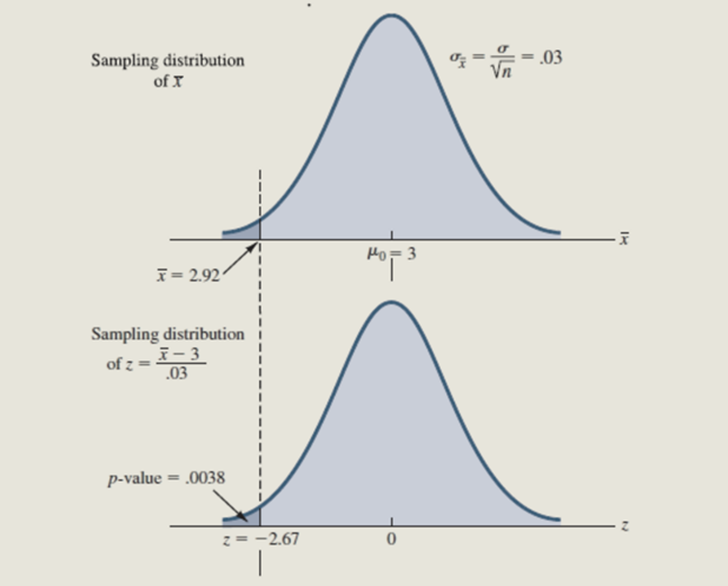
  3. Compute the value of the test statistic

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = \frac{2.92 - 3}{0.18/\sqrt{36}} = -2.67$$

Example: Steps of Hypothesis Testing



Example: Steps of Hypothesis Testing



Example: Steps of Hypothesis Testing

- Hilltop Coffee

  p –Value Approach

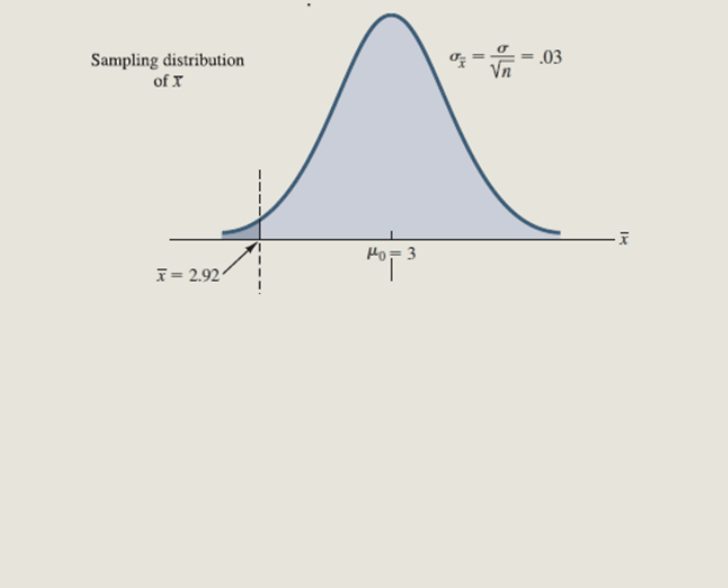
  4. Compute the p-value

     From the z-table, For z = - 2.67, the lower tail area is 0.0038

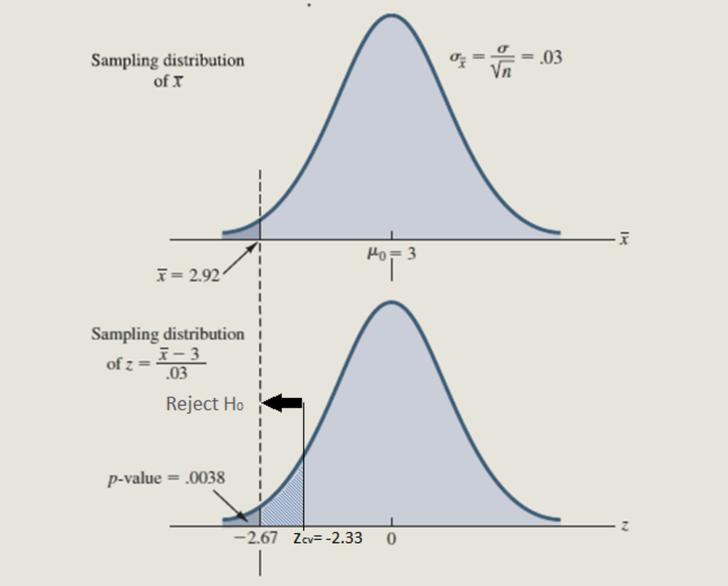
  5. Determine whether to reject H_0

     Because p-value= 0.0038≤ 0.01 we reject H_0

Example: Steps of Hypothesis Testing



Example: Steps of Hypothesis Testing



Example: Steps of Hypothesis Testing

- Hilltop Coffee

  Critical Value Approach

  4. Determine the critical value and rejection rule

     For α=0.01, the critical value is z_{cv}=-2.33

     Reject H_0 if z$\leq$-2.33

  5. Determine whether to reject H_0

     Because -2.67$\leq$ -2.33 we reject H_0

On the Agenda

Population Mean: One-Sample σ Known

- We know the distribution of $\bar{x}$ when σ for the population is known
- The standard deviation for the mean is $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$
- We called that the <u>σ known</u> case
- Under the null hypothesis we have a situation where $\mu = \mu_0$
- Hence, we use the test statistic (a.k.a. z-test)

$$z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

On the Agenda

Population Mean: One-Sample σ Unknown

- If we don't know the population standard deviation we can use s to estimate it
- We can estimate the standard deviation for the mean as $\frac{s}{\sqrt{n}}$
- we called that the σ unknown case
- In this case we use the t-table with $n-1$ degrees of freedom
- Under the null hypothesis we have a situation where $\mu = \mu_0$
- Hence, we use the test statistic (a.k.a. t-test)

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$

On the Agenda

Tests About Population Proportion

- We know the distribution of $\bar{p}$ to be normal for a large sample (CLT)
- Under the null hypothesis we have a situation where $p = p_0$
- Assuming $np_0 \geq 5$ and $n(1 - p_0) \geq 5$ the normality of $\bar{p}$ holds
- The standard deviation for the mean is $\sigma_{\bar{p}} = \sqrt{\frac{p_0(1-p_0)}{n}}$
- Hence, we use the test statistic (a.k.a. z-test)

$$z = \frac{\bar{p} - p_0}{\sigma_{\bar{p}}} = \frac{\bar{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$