Discrete Probability Distributions

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On the Agenda

1. Random Variables
2. Expected Value, Variance, and Standard Deviation
3. Binomial Probability Distribution

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Random Variables

- Random variable: Is a numerical description of the outcome of an experiment
- Discrete random variable: Is a random variable that may assume either a finite number of values or an infinite sequence of values
- Continuous random variable: Is a random variable that may assume any numerical value in an interval or collection of intervals

Example: Discrete Random Variables

- An accountant taking CPA examination
- The examination has four parts
- Random variable x =Number of parts of the CPA examination passed
- x may assume the finite number of values 0,1,2,3 or 4

Example: Discrete Random Variables

- Cars arriving at a toll booth
- Random variable x =number of cars arriving in one day
- x may assume the finite number of values $0, 1, 2, \cdots$
- We can count the customers arriving, but there is no finite upper limit on the number that might arrive

Random Variables

Experiment	Random Variable (x)	Possible values for the random variable	Type
Inspect a shipment of 50 radios	Number of defective radios	0,1,2, 3 …….50	Discrete with finite values
Operate a restaurant for one day	Number of customers	0,1,2,3 ……	Discrete with infinite values
Fill a soft drink can (max = 12.1 ounces)	Number of ounces	$0 \leq x \leq 12.1$	Continuous
Operate a bank	Time between customer arrivals in minutes	$x \geq 0$	Continuous

Discrete Probability Distributions

- The probability distribution for a random variable describes how probabilities are distributed over the values of the random variable

- We can describe a discrete probability distribution with a table, graph, or formula

Discrete Probability Distributions

- Two types of discrete probability distributions:
  - First type: uses the rules of assigning probabilities to experimental outcomes to determine probabilities for each value of the random variable
  - Second type: uses a special mathematical formula to compute the probabilities for each value of the random variable

Discrete Probability Distributions

- The probability distribution is defined by a probability function, denoted by $f(x)$, that provides the probability for each value of the random variable
- The required conditions for a discrete probability function are:

$$f(x) \geq 0$$

and

$$\sum f(x) = 1$$

Discrete Probability Distributions

- There are three methods for assigning probabilities to random variables:
  - Classical method
  - Subjective method
  - Relative frequency method
- The use of the relative frequency method to develop discrete probability distributions leads to what is called an <u>empirical discrete distribution</u>

Example: Discrete Probability Distributions

- DiCarlo Motors

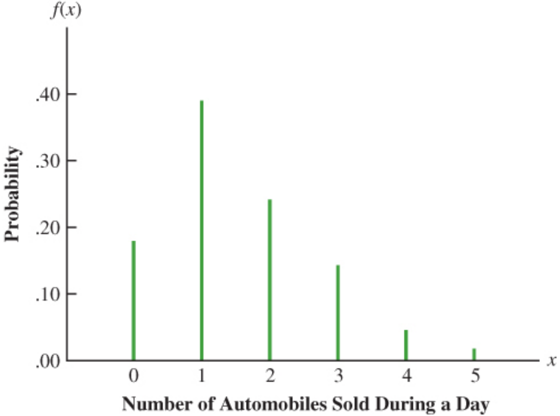
  Using past data on daily car sales for 300 days, a tabular representation of the probability distribution for sales was developed

Number of cars sold	Number of days	*x*	*f(x)*
0	54	0	.18
1	117	1	.39
2	72	2	.24
3	42	3	.14
4	12	4	.04
5	3	5	.01
Total	**300**		**1.00**

Example: Discrete Probability Distributions

- DiCarlo Motors

  Graphical representation of Probability Distribution.



Discrete Probability Distributions

- In addition to tables and graphs, a formula that gives the probability function, f(x), for every value of x is often used to describe the probability distributions

- Some of the discrete probability distributions specified by formulas are:

  - Discrete – uniform distribution

  - Binomial distribution

Discrete – Uniform Distribution

- The discrete uniform probability distribution is the simplest example of a discrete probability distribution given by a formula

- The discrete uniform probability function is

$$f(x) = \frac{1}{n}$$

where: $n =$ the number of values the random variable may assume

- The values of the random variable are equally likely

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Expected Value

- The expected value, or mean, of a random variable is a measure of its central location
- The expected value is a weighted average of the values the random variable may assume: The weights are the probabilities
- The expected value does not have to be a value the random variable can assume

$$E(x) = \mu = \sum xf(x)$$

Variance and Standard Deviation

- The variance summarizes the variability in the values of a random variable

- The variance is a weighted average of the squared deviations of a random variable from its mean. The weights are the probabilities.

$$Var(x) = \sigma^2 = \sum (x - \mu)^2 f(x)$$

- The standard deviation, σ, is defined as the positive square root of the variance

Example: Expected Value

- DiCarlo Motors

x	$f(x)$	$xf(x)$
0	.18	.00
1	.39	.39
2	.24	.48
3	.14	.42
4	.04	.16
5	.01	.05
	1.00	**1.50**

Example: Variance and Standard Deviation

- DiCarlo Motors

x	$x - \mu$	$(x - \mu)^2$	$f(x)$	$(x - \mu)^2 f(x)$
0			.18	
1			.39	
2			.24	
3			.14	
4			.04	
5			.01	
			1.00	

Example: Variance and Standard Deviation

- DiCarlo Motors

x	$x - \mu$	$(x - \mu)^2$	$f(x)$	$(x - \mu)^2 f(x)$
0	0 – 1.5 = - 1.5		.18	
1	1 – 1.5 = -.5		.39	
2	2 – 1.5 = .5		.24	
3	3 – 1.5 = 1.5		.14	
4	4 – 1.5 = 2.5		.04	
5	5 – 1.5 = 3.5		.01	
			1.00	

Example: Variance and Standard Deviation

- DiCarlo Motors

x	$x - \mu$	$(x - \mu)^2$	$f(x)$	$(x - \mu)^2 f(x)$
0	0 – 1.5 = - 1.5	2.25	.18	
1	1 – 1.5 = -.5	.25	.39	
2	2 – 1.5 = .5	.25	.24	
3	3 – 1.5 = 1.5	2.25	.14	
4	4 – 1.5 = 2.5	6.25	.04	
5	5 – 1.5 = 3.5	12.25	.01	
			1.00	

Example: Variance and Standard Deviation

- DiCarlo Motors

x	$x - \mu$	$(x - \mu)^2$	$f(x)$	$(x - \mu)^2 f(x)$
0	0 – 1.5 = - 1.5	2.25	.18	2.25 (.18)=.4050
1	1 – 1.5 = -.5	.25	.39	.0975
2	2 – 1.5 = .5	.25	.24	.0600
3	3 – 1.5 = 1.5	2.25	.14	.3150
4	4 – 1.5 = 2.5	6.25	.04	.2500
5	5 – 1.5 = 3.5	12.25	.01	.1225
			1.00	**1.2500**

Note: Variance of daily sales $= \sigma^2 = 1.25$. Hence, Standard deviation of daily sales$= \sigma = 1.\ 118$ cars!

On the Agenda

Binomial Probability Distribution

- Four Properties of a Binomial Experiment

  1. The experiment consists of a sequence of n identical trials
  2. Two outcomes, success and failure, are possible on each trial
  3. The probability of a success, denoted by p, and failure, denoted by $1 - p$, does not change from trial to trial
  4. The trials are independent

Binomial Probability Distribution

- Our interest is in the number of successes occurring in the n trials
- We let x denote the number of successes occurring in the n trials
- What would be the sample space?

Binomial Probability Distribution

- Binomial Probability Function

$$f(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

where

$x =$ the number of successes

$p =$ the probability of a success on one trial

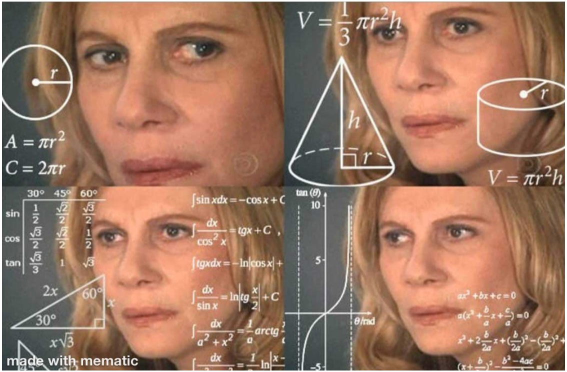
$n =$ number of trials

$f(x) =$ the probability of x successes in n trials

$n! = n \times (n-1) \times (n-2) \times \cdots 2 \times 1$

Binomial Probability Distribution

Looking at the Binomial Probability Function before Carlos explains it



Example: Binomial Probability Distribution

- Martin Clothing store:
- The store manager wants to determine the purchase decisions of next three customers who enter the clothing store
- On the basis of past experience, the store manager estimates the probability that any one customer will make a purchase is .30
- What is the probability that two of the next three customers will make a purchase?

Example: Binomial Probability Distribution

- Martin Clothing store:

  Use S to denote success (a purchase)

  Use F to denote failure (no purchase)

- We are interested in experimental outcomes involving two successes in the three trials
  - What is the probability of the first two customers buying and the third customer not buying?
    Denote by A=(S,S,F) this event, then

$$P(A) = p \times p \times (1 - p)$$

With a .30 probability of a customer buying on any one trial, then,

$$P(A) = 0.3 \times 0.3 \times 0.7 = 0.063$$

Example: Binomial Probability Distribution

- Martin Clothing store:
- Two other experimental outcomes result in two success and one failure.
- The probabilities for all three experimental outcomes involving two successes follow

Experimental Outcome	Probability
(S,S,F)	0.063
(S,F,S)	0.063
(F,S,S)	0.063

Example: Binomial Probability Distribution

- Martin Clothing store:

  Using the probability function

  Let: $p = 0.3$, $n = 3$, and $x = 2$

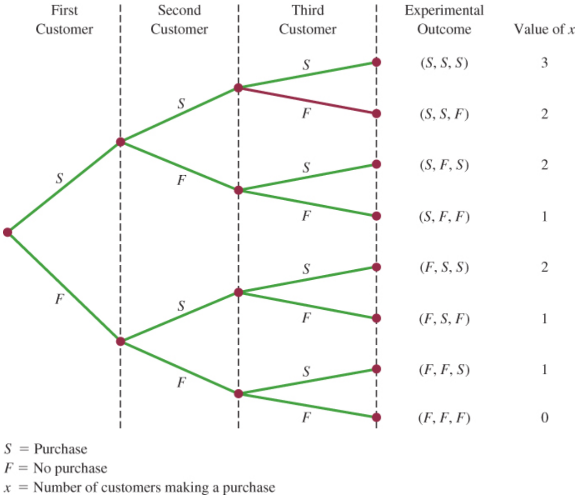
$$f(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

  Then

$$f(2) = \frac{3!}{2!(3-2)!} 0.3^2 (0.7)^1 = 0.189$$

Example: Binomial Probability Distribution

- Martin Clothing store:



Expected Value and Variance for Binomial Distribution

- Expected Value: $E(x) = \mu = np$
- Variance: $Var(x) = \sigma^2 = np(1-p)$
- Standard Deviation: $sigma = \sqrt{np(1-p)}$

Example: Martin Clothing store

- Expected Value: $E(x) = 3 \times 0.3 = 0.9$
- Variance: $Var(x) = 3 \times 0.3 \times 0.7 = 0.63$
- Standard Deviation: $\sigma = \sqrt{0.63} = 0.79$