Waste-Heat and Economic Growth in the Very Long Run

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Abstract

This paper presents a model of economic growth that incorporates production damages due to waste-heat. Our approach builds from the models developed to study how carbon emissions impact the economy; however, the direct waste-heat channel that we consider is distinct from the on-going greenhouse gas induced climate change. If energy use continues to rise exponentially with economic growth, then the direct heat also has the potential to elevate global temperatures. In the model, sustained economic growth continues in the long run only if something is done about this waste-heat.

Keywords: Climate Change; Economic Growth; Stefan-Boltzmann Law

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1 Introduction

We present a growth model in which economic activity affects the temperature because production generates waste-heat. In the model, increasing temperatures damage both the level of production and the growth rate. These damages can be offset by the accumulation of heat-saving (or abatement) technologies, which reduce the heat intensity of output. We use the model to simulate the economy's response to different amounts of the heat-saving technology and find that robust economic growth persists only as long as the waste-heat from production is sufficiently mitigated. In the very long run, the economy only grows as fast as the waste-heat saving technology allows. If the waste-heat damages are too high, then economic growth stops.

The growing literature on damages from climate change focuses almost exclusively on greenhouse gases and related pollutants (e.g. see the hundreds of papers discussed in Auffhammer (2018) and Stern (2008), and those listed in Dong et al. (2024) and Tol (2024)). Our approach builds from this literature, particularly from the dynamic integrated climate-economy (DICE) macro model approach (see Nordhaus (1992), Nordhaus (1994), and Hassler et al. (2016), among many more). However, rather than focusing on the carbon cycle, we incorporate (direct) waste-heat from production. We add this distinct mechanism into the model via the modified Stefan-Boltzmann equation presented in Murphy Jr. (2022).

The most heavily relied on sources for power emit large amounts of heat. For example, a typical coal plant releases two kWh of energy into the environment as waste-heat for every one kWh of electricity generated (Zevenhoven and Beyene (2011)). Power originating from nuclear or geothermal sources – some of it "clean" in terms of greenhouse gas emissions – are no better in terms of waste-heat. An additional four to ten percent of the remaining energy is then lost as heat during transmission. Wind, solar, and hydro power can be even less efficient for energy conversion; although, these renewable sources do not necessarily add as much heat to the Earth (e.g. sun light will hit the planet, regardless). Finally, up to one-third of resulting power is used to explicitly generate more heat for industrial processes or for buildings, etc.

Despite all this waste-heat, it is carbon emissions that drive the current concerns over increasing temperatures. At present, direct waste-heat has a much smaller impact. The contribution to the Earth's energy (im-)balance from waste-heat is approximately $0.1\frac{W}{m^2}$ (Murphy Jr. (2022) and Zevenhoven and Beyene (2011)), while similar estimates for the equivalent contribution from greenhouse gases are 5-10 times higher. However, with exponentially growing energy use accompanying world-wide economic growth, the waste-heat channel has the potential to increase the global temperature and become the main driver of climate change. Murphy Jr. (2022) illustrates the dramatic connection between energy consumption and the temperature, and we incorporate this channel into a growth model in order to quantify the economic impact.

2 A Model of Production, Waste-Heat Damages, and the Heat Intensity of Output

We use a discrete-time model to assess the relationship between production, waste-heat induced damages, and investments into improving the heat intensity of output. The damages effect both production levels and growth rates. A Stefan-Boltzmann equation governs how production and abatement relate to temperature changes.

Aggregate Output: A Cobb-Douglass function captures production Y at time t,

$$Y_t = (1 - D_t^Y)K_t^\alpha (Z_t N_t)^{1-\alpha} \tag{1}$$

where Z_t is total factor productivity (TFP), K_t and N_t stand for capital and labor, and parameter α is the capital intensity. The function D_t^Y governs the damages to the level of output.

TFP evolves as

$$Z_{t+1} = (1 - D_t^z)(1 + g_z)Z_t \tag{2}$$

where g_z is a constant exogenous component to TFP (or technology) growth and the function D_t^z governs the damages to that growth.

Capital accumulates according to

$$K_{t+1} = (1 - \delta)K_t + sY_t \tag{3}$$

where δ is the depreciation rate and s is a constant share of output reinvested in capital.

Damages: The damage function D_t is the pathway by which temperature affects output levels and growth. We use a standard DICE damage function (Nordhaus, 2008)

$$D_t = 1 - \frac{1}{1 + \psi(T_t - \overline{T})^2} \tag{4}$$

where T_t is the temperature and damages incur from positive deviations from the baseline temperature $\overline{T}$ scaled by constant ψ. Following Dietz and Stern (2015) and Moyer et al. (2014), the total damage is allocated to either the output level or to TFP growth. A share of damages f^z affects TFP growth

$$D_t^z = f^z D_t. \tag{5}$$

The damage to the level of output is

$$D_t^Y = 1 - \frac{1 - D_t}{1 - f^z D_t}. \tag{6}$$

Temperature and the Heat Intensity of Output: The Earth's equilibrium temperature can be approximated by all heat energy incurred from the sun plus heat created on Earth (e.g. from human activity), net of heat radiated back to space. We follow Murphy Jr. (2022) and use a modified Stefan-Boltzmann relation to track the Earth's energy balance. The basic

Stefan-Boltzmann equation is $E = \sigma T^4$, where E is energy emitted per surface area, σ is the Stefan-Boltzmann constant, and T is the temperature in degrees Kelvin (the magnitude of a Kelvin is equivalent to a degree Celsius). The following version of the equation incorporates the specific surface area and energy inputs for the Earth,

$$F_{\odot}(1-\gamma)\pi R_{\oplus}^2 + \frac{\overline{P}Y_t}{\Gamma_t} = 4\pi R_{\oplus}^2 \sigma T_t^4. \tag{7}$$

The first term measures the solar energy ($F_{\odot}$ is the solar flux at the top of the atmosphere) net of Earth's albedo (γ). The $\pi R_{\oplus}^2$ scales this to the projected area of Earth (solar rays only reach a circular portion of the spherical surface at a given moment). The second term on the left-hand side is the contribution of waste-heat emissions (from human activity). The key assumption is that producing Y requires heat-intensive energy, with constant $\overline{P}$ being the portion of production that exerts energy. See Murphy Jr. (2022) for more details.

We include heat-saving technologies as Γ_t, which captures the heat intensity of output. This variable is a stand in for all the ways in which the heat intensity of output could be reduced, including abatement, decoupling production from energy, or even creating the heat used for production more efficiently. As alluded to earlier, solar, wind, and hydro power could potentially weaken the relationship between production and waste-heat (although not eliminate it). Other investments captured by Γ could include more futuristic technologies like producing in space or as-of-yet undreamed of advances. The stock of Γ_t evolves according to

$$\Gamma_{t+1} = (1 + g_{\Gamma})\Gamma_t. \tag{8}$$

We ignore spillovers to TFP (see Donald (2024)) and differences across sectors (see Casey et al. (2024)). The main simulation results reported below consider different levels of the heat intensity of production over time by varying the parameter g_{Γ}, which governs the growth rate of Γ.

Rearranging Equation (7) and isolating temperature gives

$$T_t = \left(\frac{F_{\odot}(1-\gamma) + \overline{p}\frac{Y_t}{\Gamma_t}}{4\sigma} \right)^{1/4} \tag{9}$$

where $\overline{p} = \overline{P}/\pi R_{\oplus}^2$. Equation (9) encapsulates the key difference relative to a standard DICE model. Instead of climate change coming from carbon emissions, the temperature depends on the ratio of human activity (as captured by Y) relative to the heat intensity of production (Γ) and physical constants.

3 Damages at Different Levels of g_Γ

We use quantitative simulations to demonstrate the potentially large connection between production, waste-heat, and economic damages over long time horizons. The model is highly aggregated and relies on broad concepts of damages and the heat intensity of production; thus, the simulations should not be interpreted as forecasts.

3.1 Parameter Values

Table 1 lists the parameter values employed in the simulations, taking each period as a year.

Most of the parameters in Equation (9) come from well-known values in the Stefan-Boltzmann relation. The exceptions are $\bar{p}$ and g_Γ. We set the annual growth rate in Γ to $g_\Gamma = 0.004$ in order to match the annual difference in the growth rate of per-person (GDP) output (0.019) compared to the corresponding annual growth rate of energy use (0.015) from 1970-2022 (see Smil (2017) and Angus Madison). Although, this choice might underestimate the contributions from non-energy sources of heat, like frictions, etc. We set $\bar{p} = 5 \times 10^{-10}$ to match the temperature of 288 degrees Kelvin ($\overline{T} = 288$) at year $t = 2022$.

Following Dietz and Stern (2015), the damage function parameter $\psi = 0.0045$, and the share of damage allotted to growth $f^z = 0.05$. We set $\overline{T}$ to match the global temperature at

Table 1: Parameter Values

Parameter	Value	Description
Stefan-Boltzmann Relation		
$F_{\odot}$	1360 (Wm^{-2})	Solar flux
γ	0.293	Earth's albedo
σ	5.67×10^{-8} ($\mathrm{Wm}^{-2}\mathrm{K}^{-4}$)	Stefan-Boltzmann constant
$\bar{p}$	5.0025×10^{-10}	Current temperature of $K = 288$ at $t = 0$
g_{Γ}	0.004	Global GDP growth less energy growth
Damage Function		
ψ	0.0045	Dietz and Stern (2015)
$\overline{T}$	288	Initial temperature at $t = 0$
f^z	0.05	Dietz and Stern (2015)
Remaining Parameters		
α	0.4	Capital intensity of output
δ	0.1	Annual capital depreciation rate
s	0.23	Average capital accumulation
g_z	0.019	World GDP growth
N	constant	UN Projections

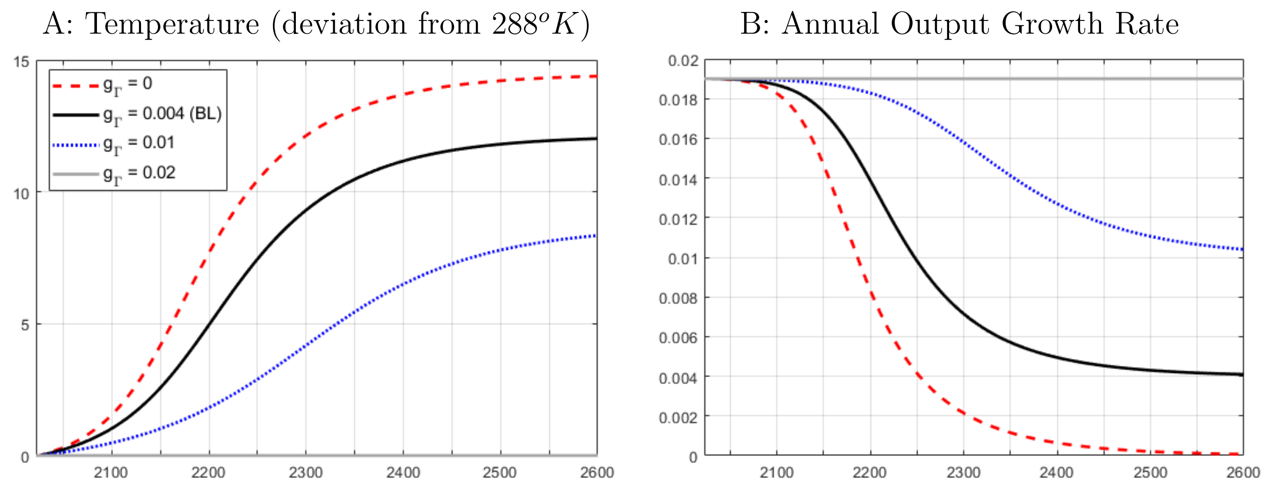
year $t = 2022$.

We use standard parameter values for production. Capital intensity in the production function $\alpha = 0.4$, the capital depreciation rate $\delta = 0.1$, the investment rate $s = 0.23$ in line with average capital accumulation, and the growth in TFP, $g_z = 0.019$, is set to match World GDP growth from 1970-2022. We keep the population N constant, which is close to global projections after 2100 (UN World Population Prospects (2024); see Kelly and Kolstad (2001) and Casey and Galor (2017) for more on this topic).

3.2 Simulation Results

We simulate the model economy until 2600 with different levels of the heat intensity of output (by changing g_{Γ}). The left panel of Figure 1 plots the resulting change in temperature ($T - \overline{T}$) in degrees Kelvin over time. The right panel plots the annual growth rate for output per person. Note, none of the simulations include the effects from greenhouse gases. Figure 1 captures our main findings.

Figure 1: The Impact of g_Γ on Temperature and per-Capita Output Growth



The solid black series is the baseline parameterization (BL), with the other curves indicating alternative growth rates (g_Γ). Looking first at the left panel, temperature increases are modest through the end of the century, reaching only $1-2^oK$ above the current temperature by 2100. As time passes, however, the temperature rapidly increases until the economy settles into a steady state around 2600. Looking across the series, it is clear that higher growth in Γ moderates the temperature (Panel A) and the resulting reduction in damages leads to higher long run output growth (Panel B).

We stress three observations from the simulations. First, if the growth in Γ is less than the growth rate of technology ($g_\Gamma < g_z = 0.019$), then heat emissions from production can lead to substantial increases in the Earth's temperature. However, if $g_\Gamma > g_z$ (e.g., $g_\Gamma = 0.02$ in Figure 1), then all heat damages are prevented, the temperature does not increase, and there are no economic damages from waste-heat.

Second, despite the reduction in output, positive long-run economic growth can still be maintained, even in the face of rising temperatures. The damages from higher temperatures reduce economic growth, but output growth can continue indefinitely as long as the heat intensity of output (Γ) continues to improve. Table 2 reports output losses relative to an economy with 'no damages', at 100 year intervals. Scenarios where the growth of Γ exceeds TFP growth (i.e. $g_\Gamma > g_z$) correspond to this 'no damages' case. In each scenario, the economic damages in 2100 are small. Even without substantial investments into improving the heat intensity of production, the damages are less than a year's worth of growth. By 2500, though, the damages become extreme.

Table 2: Losses in Output Per Person (%)

	Growth rate in Γ		
year	$g_\Gamma = 0$	$g_\Gamma = 0.004$	$g_\Gamma = 0.01$
2100	1.50	0.67	0.02
2200	39.26	18.97	3.05
2300	85.55	66.58	18.81
2400	97.51	90.92	49.66
2500	99.61	97.82	75.35
2500	99.94	99.50	89.15

Third, the flattening of output growth rates toward the end of the model simulations suggest that long run growth is eventually governed by the growth rate for Γ. When Γ grows slower than technology ($g_\Gamma < g_z$), growth in overall output converges to g_Γ. In the long run, output growth is capped by the technology accumulated to extinguish the waste-heat because high temperatures reduce growth. With a positive g_Γ, economic growth can persist indefinitely – albeit at a lower rate.

Adding the waste-heat channel into a DICE-style model can markedly impact the trajectory of economic growth. We note, however, that even without reductions in the heat intensity of output, the economy does not necessarily collapse. Rather, the economy eventually stagnates, with damages from high temperatures completely eliminating the gains from technological progress and output reduced by several magnitudes.

To further quantify the results, we performed a series of basic welfare calculations based on the notion of compensating variation in consumption (output minus investment) using a standard utility set-up. For brevity, we do not provide the full details here, but they are available upon request. As shown in Table 2, the output (and therefore consumption) losses can be large – in the distant future. Thus, our findings suggest that the welfare effects can be substantial (30% or more) if the discount rate is small. However, with typical discount rates, the welfare differences across g_Γ values are relatively minor because the damages occur far into the future. This general pattern holds true for time-invariant discounting and declining discount rates; although, the exact magnitudes differ. See Groom et al. (2005) and Arrow et al. (2013) for more on declining discount rates in climate change models.

A few caveats for this paper's dramatic results are in order. First, the model simulations assume constant saving and consumption choices, in addition to exogenous technology growth. Thus, the economy might be unrealistically unstable because we do not account for changing behaviors. For example, if the decline in heat intensity is small enough to stop growth, then, in a fuller model, one might expect the return to heat saving innovations to become very large, inducing further investments that allow growth to continue. Second, we have not included tipping points or cascade effects. Nor have we allowed for technological breakthroughs in energy production. Each of these factors could fundamentally alter the future relationship between economic growth and the environment.

4 Conclusion

Nearly all human activity generates heat, either in and of itself or as a by-product of performing work. Energy used explicitly for indoor temperature control or as an input to manufacturing are obvious examples. Going forward, however, the waste-heat from electricity for super computers and transportation might become a greater concern, even if the power comes from what we now consider to be clean or renewable sources. Our model suggests that large investments will be necessary to manage the waste-heat, if robust economic growth is to be maintained.

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