PRODUCTION AND OPERATIONS MANAGEMENT
Vol. 00, No. 0, Xxxxx 0000, pp. 000–000
ISSN 1059-1478 | EISSN 1937-5956 | 00 | 0000 | 0001

POMS
DOI 10.1287/xxxx.0000.0000

Managing material shortages in project supply chains: inventories, time buffers and supplier flexibility

Panos Kouvelis
Olin Business School, Washington University in St. Louis, St. Louis, Missouri, USA. kouvelis@wustl.edu

Xingxing Chen
Robins School of Business, University of Richmond, Richmond, Virginia, USA. chen@richmond.edu

Yu Xia
Mason School of Business, The College of William & Mary, Williamsburg, Virginia, USA. amy.xia@mason.wm.edu

Abstract: We consider a two-stage project supply chain with a downstream project firm producing an engineer-to-order (ETO) complex product or a make-to-order (MTO), low-volume, customized industrial product as a project, and an upstream contract supplier supplying a key material to the project. The project faces two uncertainties: project activity time uncertainty and material consumption uncertainty, which may be positively or negatively correlated. In anticipation of these uncertainties, the project firm has to carefully decide its promised project due date to its project customer, against which harsh penalties will be assessed, and his material order quantity to commit to the contract supplier in advance. In most practical settings, project firms order from contracted suppliers via a flexible wholesale price contract consisting of a discounted advance order price and a risk-premium adjusted expedite order price. The discounted advance order price encourages the project firm to take more inventory risk in the supply chain, and the expedite order price incentivizes the supplier to bear more inventory risk by carrying safety stock in excess of the project firm's advance material order. We formulate an optimization model that solves the project firm's project due date and material order problem, which takes into account the supplier's strategic reaction to the project firm's material order under the flexible wholesale price contract. We show that for MTO projects, risk-sharing with suppliers on project materials is less important to the project firm, with the project firm assuming ownership of all material inventory in the channel and setting a deliberate project due date being the key. On the other hand, for ETO projects, risk-sharing with contracted suppliers assumes critical importance. Project firms managing ETO projects should fully exploit the flexibility in the material supply contract to optimally drive the supplier's safety stock level and set the project due date reflecting the shared risk in the supply chain.

Key words: project supply chains, wholesale price contracts, project due date, supply flexibility, flexible contracts, time buffers.

History: August 25, 2023.

1 Introduction

Project supply chains are chains that provide unique products or deliver unique solutions for specific customers, and as a result are configured-to-order or even designed and engineered-to-order (see similar definition in Seifert and Markoff (2017)). The immediate examples for project supply chains include infrastructure projects (buildings, highways, bridges, dams, etc.), airplanes, power stations, defense equipment, manufacturing plants, and telecommunication networks. Major companies on the Fortune 500 list, such as Boeing, General Dynamics, Raytheon Technologies, and General Electrics, have a significant part of their revenue derived through project supply chains.

In the last 10 years, we have seen an increase in the importance of project supply chains, with companies that were previously running traditional product chains shifting towards managing complex project chains. As documented in Dell (2021), the company transitioned from a public-to-private company in an effort to pursue a transformation away from a pure PC-products business and into a solution strategy, which encompasses selling end-to-end IT solutions, software, and services. In our own consulting experiences, we have seen companies in the industrial automation segment adopting similar strategies. Emerson[1], with its industrial automation division (around $12 Billion in revenues), is supporting its customers in major infrastructure development industries (oil and gas, power, metals and mining, refining, etc.) via orchestrating projects that install a complex portfolio of customized products and provide critical services (maintenance, monitoring, etc.). In a similar fashion, Belden[2], a $2 Billion industrial automation company of legacy connectivity products (wires, connection, etc.), is transforming itself into an “Enhanced Solutions Delivery” business model for customers with needs for smart building and broadband/5G infrastructure applications. Facing the trend of “Industry 4.0” and the challenges from IT/OT (Operation Technology) convergence, Belden has launched a digital transformation to provide full-suite solutions to its customers. Since 2019, the company has shifted its focus from product sales to solution sales by initiating projects that encompass product installation, software development, and customer support services tailored to meet customer needs (Cespedes and Klopfenstein (2023)). While the importance of project supply chains is increasing, the topic of these chains is almost forgotten and is heavily understudied in the supply chain literature. Our paper takes the first step in developing important concepts in the risk-planning and risk-management of these chains. We offer fresh insights via a stylized model on the roles and uses of inventory buffers, time buffers, and backup suppliers’ processing flexibility for the effective management of project supply chains.

Despite the risk-mitigating efforts inserted in managing projects, all projects are still subject to project delays, which negatively impact the financial performance and the future operation of all parties within a project supply chain. The 9th Global Project Management Survey (2017) reports that approximately 50% of projects are completed on time. Calvo et al. (2019) estimate that 42% of public projects in the United States were behind schedule. Unpredictable variability in activity times and failure to compensate adequately for activity time variations is a prominent cause of project delay (Hughes (1986)). In addition to activity time variations, material shortages for key project activities add another layer to project delays. For instance, the signature Miami bridge construction project has been severely delayed due to the shortages of gantry

[1] Discussion with Ryan Meier, Director, Digital Supply Chain Operations, and Eric Carlson, VP-Operations and Supply Chain, Emerson.

[2] Discussion with Roel Vestjens, President & CEO, Belden.

and concrete (Lauren (2023)). Supply chain bottlenecks and shortages that originated during the pandemic, although improved, are still reported as being the primary cause of delays for various construction projects (Simpson (2023)). In several construction project studies, the shortage of construction materials is identified as a key factor for contractors' performance delays (Majid and McCaffer (1998), Kazaz et al. (2012), Assaf and Al-Hejji (2006), Rahman et al. (2017)).

In other business practices, project delays are also frequently encountered and carry significant consequences. Bombardier, one of the world's largest manufacturers of street cars, experienced multiple rounds of delivery delays due to material shortages between 2015 and 2017 (Spurr et al. (2017)). Another famous example of project delays that we have witnessed in the last couple of decades is Boeing's 787 Dreamliner project. Since its launch in 2004, a series of delays and mishaps have plagued the aircraft, and one significant factor out of the many delay factors is the industry-wide shortage of fasteners (Sanders and Cameron (2011)). More recently, Airbus struggled to keep up with its scheduled deliveries of airplanes with delays attributed to availability of components and quality lapses (Katz (2021)). Wind-turbine makers for onshore projects also reported project completion problems due to material shortages and other supply chain bottlenecks (Hiller (2021)). While some factors that cause project delays are uncontrollable (e.g., war, pandemics, and natural disasters), material shortages can be better managed in project supply chains with improved planning and collaboration strategies.

We seek to study the integrated management of project risks in a project supply chain environment that tackles both project activity time uncertainty and material consumption uncertainty. To frame our research question, we use a characteristic project supply chain environment consisting of a project firm that operates under both types of uncertainties. Nooter/Eriksen (N/E)[3], a leading supplier of heat recovery steam generators, produces engineer-to-order "boilers" that are handled as one-at-a-time projects. These products are mostly delivered to power plants, utilities, and other industrial customers at price tags in the $15-50 million range. For N/E, setting a project due date that fully reflects product complexity and material supply uncertainty is the key to customer satisfaction and profitability. According to the company, material shortages account for over 50% of their project delays. To mitigate material shortages and ensure on-time deliveries of their products, a common practice by N/E is to use flexible contracts with key suppliers (e.g., the supplier accepts advance purchase orders with a price discount and later at-once purchase orders with a price premium) that incentivize suppliers to carry safety stocks for expensive components, and/or use dual sourcing with a backup supplier for unforeseeable material shortages. We observe a similar risk management practice at China Communications Construction Company (CCCC), a state-owned publicly traded company delivering engineer-to-order products (projects) including highways, bridges, tunnels, and ports.

Working with customers in a variety of industries, Belden now sells custom solutions that incorporate products in low but hard-to-predict volumes along with the installation and maintenance of these products.

[3] Discussions with Matthew Burns, Executive Vice President, Nooter/Eriksen.

Belden's new pricing model charges a one-time fee based on the value created by a solution, which necessitates a make-to-order (MTO) project supply chain (either configure-to-order (CTO) or assemble-to-order (ATO)) with significant lead times for in-house planning and testing (Cespedes and Klopfenstein (2023)). Similarly, Emerson delivers make-to-order products (e.g., valves, measurement systems, fluid controls) to industrial customers in diverse industries. Some of these products may be engineer-to-order when new applications are developed, but most of them are make-to-order with mature-stage technologies that require less expensive components. Both Emerson and Belden are subject to contractual penalties for late deliveries of products beyond negotiated due dates. The main challenges in meeting the due dates include the project time uncertainty and unplanned material shortages. Executives from both companies have emphasized the increased importance of material shortages in the last few years, and even before the pandemic.

In this research, we investigate how a project firm should account for and mitigate both project activity time and material consumption uncertainties when determining the project due date and material planning decisions. Product design errors, process/product re-engineering, and failed/repeated processes frequently impact both of these two uncertainties. For example, if the process/product re-engineering activity (that may take a considerable amount of time) results in a more efficient usage of a particular material in subsequent production activities, or when a lean practice and material substitution lead to increased/decreased activity times, we observe correlated project activity time and material consumption uncertainties that must be taken into account when the project firm decides its project due date and material order decisions.

In our stylized model, we use a general probability distribution to capture the correlation between project activity time and material consumption uncertainties. We address the project firm's project due date and material order decisions in face of these uncertainties. The material order decision is embedded in a two-stage material supply chain under a flexible wholesale contract. The flexible contract allows the project firm and the contract supplier to determine material risk allocation in the supply chain. Under this contract, the project firm first decides its advance order quantity to the supplier, which is followed by the supplier's production decision that must meet the project firm's advance order quantity. Any excess production quantity are held by the supplier as safety stock, which can be sold to the project firm later at a predetermined and higher price if material shortage occurs. If the contract supplier is not able to cover the project firm's material shortage, a backup supplier is used to cover any shortage not covered by the contract supplier, at a must higher cost with a much longer delay. Taking into account project activity time variation and the possible delay due to a material shortage, the project firm deliberately decides its promised project due date to its customer, against which harsh penalties will be assessed. To manage the joint risks in the project supply chain, the project firm employs an integrated risk management strategy consisting of inventory buffers (e.g., order extra materials), incentives for supplier responsiveness (e.g., safety stock at the contracted supplier), contingent orders from a backup supplier, and built-in time buffers for the project (e.g., inserting slack time in the project due date). We aim to understand: (1) what is the most effective usage of these buffers, and (2) how do these buffering activities impact one another in their most effective usage by the project firm.

In the interpretation of our model results, we find that (1) for projects using standard, commodity type materials of low cost (e.g., MTO projects, either CTO or ATO), the project firm should bear all material consumption risk and set a project due date jointly with its material decision; (2) for complex projects using non-commodity specialty materials of high cost (e.g., ETO projects), the project firm should fully leverage its relationship with the contract supplier and the flexibility in the supply contract, to effectively share risks with the supplier and set a project due date according to the safety stock carried by the supplier. From a risk management perspective, our results suggest that MTO chains should focus on building the project firm's own inventory buffer, use market suppliers as a backup for material shortages, and set an appropriate project due date that is coupled with its own material order decision. For the ETO complex project chains, our results suggest that the project firm should exploit its contractual relationship with the material supplier and incentivize the supplier to carry safety stock of needed materials, use a backup market supplier when needed, and set the project due date reflecting the shared risk with the supplier.

The rest of our paper is organized as follows. In Section 2, we review relevant literature. In Section 3, we formulate the project firm and the contract supplier's problems in face of the project activity time and material consumption uncertainties under the flexible wholesale price contract. In Section 4, we provide the analytical results for the main model formulated in Section 3. Numerical study of our project supply chain results are discussed in Section 5, and an extension of our main model is provided in Section 6. In Section 7, we conclude with a summary of our main managerial insights for managing project supply chains in ETO and MTO environments.

2 Literature Review

Our work relates to three streams of literature: (1) project due date management, (2) supply chain management under wholesale price contracts, and (3) project supply chains.

The stream of research on project due date management peripherally relates to our work, but most of the works in this stream study the due date problem without jointly considering the project activity time variation and material shortage risks. Early works under project scheduling framework (e.g., Baker and Scudder 1990) study the project due date problem in the context of sequencing project activities with deterministic activity times. More recent works consider stochastic project activity times (e.g., Zhu et al. 2007, Xia et al. 2008). Most of these works only consider the time aspect of a project when setting a project due date. For more works under the project scheduling framework, we refer the readers to Baker and Trietsch (2018). Another class of the due date management problem, known as the lead time quotation problem, studies how to quote reliable lead times to customers who are sensitive to the quoted lead times. At its core, this problem investigates how to set due date (e.g., the quoted lead time) that trades off between time-sensitive demand

and the tardiness cost beyond the lead time. For a comprehensive review of this problem, we refer readers to Keskinocak and Tayur (2004). Recent works by Savaşaneril et al. (2010) and Chen and Moinzadeh (2018) study the lead time quotation problem in conjunction with base-stock inventory control policies. Our work studies how to set project due date that is subject to both project activity time variation and a possible material shortage delay in project supply chains. We explore the risk management impact of material shortages and investigate the use of time and inventory buffers as risk-mitigating strategies for managing projects.

Another research stream also peripherally relating to our work is supply chain management using wholesale price contracts, including both single and two-price (flexible) wholesale contracts. The common structure of a supply chain studied in this stream is a two-stage supply chain with a supplier (or manufacturer) supplying a downstream "newsvendor" facing uncertain demand. Lariviere and Porteus (2001) first study the single price wholesale contract in the standard supply chain setup. Cachon (2004) extends the analysis to study a flexible wholesale price contract offering two ordering opportunities under the same supply chain structure. Dong and Zhu (2007) build upon Cachon (2004) and characterize Pareto efficient contracts among all flexible wholesale price contracts. Extending these classical works, the flexible wholesale price contracts have been further investigated and applied to various settings. Xie et al. (2010) study earlier order commitment on a decentralized supply chain with one manufacturer and multiple retailers. Wang et al. (2014) examine how the principles of push and pull contracts can be extended to a three-tier supply chain consisting of a supplier, a contract manufacturer (CM), and an original equipment manufacturer (OEM). To evaluate the effectiveness of these contracts, the study compares two supply chain structures: one involving OEM contracts with both the CM and the supplier, and the other involving OEM contracts with the CM only. Davis et al. (2014) apply a behavioral model to study the flexible contract in order to compare and verify supply chain efficiency. Through the analysis of the flexible contract scheme, Guan et al. (2015) obtain equilibrium strategies of the two suppliers and one buyer supply chain under a flexible contract. Gou et al. (2016) investigate how an outside market can influence local supplier–retailer contracts by taking into account the supplier's production capacity and the outside market barriers. Tang and Girotra (2017) analyze how to use the discount in advance purchases to incentivize information sharing for a retailer in a supply chain with a dual-sourcing wholesaler. In a retail supply chain, Hou and Lu (2022) study the allocation of inventory risks of flexible contracts. Wei and Huang (2022) investigate the effective use of an advance purchase discount to manage stochastic demand in a supply chain subject to carbon emission tax regulations and in the presence of green technology investments.

Our work focuses on a two-stage project supply chain. The downstream project firm in our two-stage chain model has an objective function that accounts for project-related costs (e.g., overhead) and contractual penalties due to late completion, in addition to the usual material ordering costs. While on the surface both the "newsvendor retailer" chain and our project chain can be analyzed as a two-stage chain, their detailed operations, objective functions, and subsequent mathematical analysis are substantially different. We seek to understand how risk management of material shortages within project supply chains can be effectively handled. Such issues have not been studied in the above-mentioned "newsvendor" supply chain literature.

The stream of research closest to our work is the study of project supply chains. As we have emphasized before and will become apparent from the briefness of the related literature, the topic is heavily understudied. Kwon et al. (2010) study delayed payment contracts between a manufacturer (project manager) and n independent suppliers (contractors) in the context of parallel project tasks to investigate suppliers' effort levels and the manufacturer's net profit under the payment scheme. Chen and Lee (2017) investigate the material delivery problem between an upstream supplier and a downstream manufacturer in a two-stage project supply chain, and they focus on designing time-based incentive contracts that coordinate the two-stage chain from a project time perspective. Our work employs a project supply chain structure similar to Chen and Lee (2017). In contrast with their work, we do not study contract designs for time coordination. Instead, we focus on optimizing the project due date decision together with the material planning decisions under a flexible supply contract in the presence of both project activity time and material shortage risks. Our work is among the first to explicitly model project supply chains with both time and inventory considerations, and we offer fresh risk management insights on managing project supply chains.

3 Model

We study a two-stage project supply chain with an upstream material supplier supplying a key material to a downstream project firm, in which the project firm executes a designated project subject to project activity time and material consumption uncertainties. The project revenue is fixed and denoted by V. Due to project uncertainties, the project firm needs to make careful planning decisions on the project due date and material order decisions, against which mismatch costs will be incurred and is detailed below.

Material order decision. We assume the project is subject to uncertain material consumption $\mathbf{x}_1$[4], which has a known distribution $f_1(x)$. The project firm sources the project material from a contracted supplier using a flexible wholesale price contract, and from a backup supplier if needed. The contracted supplier offers a flexible wholesale price contract that allows two ordering opportunities: an advance order opportunity at a discounted unit price w and a later expedite order opportunity at a premium unit price w_1, satisfying $w \leq w_1$. Under this contract, the project firm first places an advance order quantity Q_m to the supplier well before the project starts. The supplier then decides a production quantity Q_s subject to $Q_s \geq Q_m$. The supplier delivers Q_m to the project firm by the project's starting time, and any excess (e.g., $Q_s - Q_m$) serves as safety stock held by the supplier that can be used by the project firm against material shortages through expedited orders. After the project's material consumption level gets realized in the project's execution phase (we denote the realization by x_1) and if it exceeds the project firm's own inventory Q_m, the firm encounters a material

[4] We use bold letters to denote random variables in order to differentiate from the realization of the variables.

shortage $x_1 - Q_m$, which must be fulfilled. The project firm then sends an expedited order to the contracted supplier to cover as much shortage as it can, up to the supplier's reserved safety stock $Q_s - Q_m$. If the supplier cannot fully cover the shortage (e.g., $x_1 - Q_m > Q_s - Q_m$), the project firm has to resort to a backup supplier to cover the remaining shortage, i.e., $(x_1 - Q_s)^+$, at a higher expected spot price w_2. Expedited orders fulfilled by the contracted or backup suppliers can incur significant supply delays to the project.

Effective total supply delay. The supply delay in expediting from the contracted supplier (who holds readily available inventory for expedited shipping) is in general much shorter than that from the backup supplier (who may not have readily available inventory). For tractability considerations, we consider a linear delay function for the supply lead-time in receiving expedited inventory from the contracted or backup supplier: (1) the supply delay from the contracted supplier is modeled by $a_1 \cdot \min((Q_s - Q_m), (\mathbf{x}_1 - Q_m)^+)$, where $\min((Q_s - Q_m), (\mathbf{x}_1 - Q_m)^+)$ is the expedited quantity, and (2) the supply delay from the backup supplier is $a_2 \cdot (\mathbf{x}_1 - Q_s)^+$, where $(\mathbf{x}_1 - Q_s)^+$ is the expedited quantity. Both a_1 and a_2 are known parameters with $a_1 < a_2$. In our main model below, we consider the effective total supply delay to be the maximum of the two delays (e.g., the two delays overlap with each other). To account for the difference of the responsiveness of the suppliers, we further impose $0 = a_1 < a_2$, i.e., the delay from the contracted supplier is negligible. As a result, the effective total supply delay caused by material shortages is equal to the supply delay from the backup supplier and modeled by $a_2 \cdot (\mathbf{x}_1 - Q_s)^+$. In Section 6, we extend our work to model the effective total supply delay by the summation of the two delays (e.g., the two delays do not overlap with each other). We derive structurally similar results for this model variant in the later section.

Project due date decision. We assume the project completion time is equal to the sum of the uncertain project activity time $\mathbf{x}_2$, which has a known probability density function $f_2(\cdot)$, and the effective total supply delay $a_2 \cdot (\mathbf{x}_1 - Q_s)^+$. The uncertain activity time $\mathbf{x}_2$ captures the inherent unpredictability of activity duration in a project, which is independent of the material planning decision of the project. The effective total supply delay, when it materializes due to material shortages not planned for by the project firm, will extend the project's overall completion time. Under the maximum delay model described above, material shortages not planed for by the project firm (in this case not planed for by the two-stage supply chain) extends the project completion time by $a_2 \cdot (\mathbf{x}_1 - Q_s)^+$, where Q_s is the total material quantity in the two-stage supply chain. We note that $a_2 \cdot (\mathbf{x}_1 - Q_s)^+$ is endogenous to the production decision (Q_s) of the contracted supplier, and Q_s is the best response of the supplier under the incentives provided in the flexible wholesale price contract (w, w_1), subject to meeting the project firm's advance order quantity Q_m. In face of the unpredictable project completion time $\mathbf{x}_2 + a_2 \cdot (\mathbf{x}_1 - Q_s^*)^+$, the project firm decides a project due date T_m that will most likely deviate from the realization of the random project completion time. Following the project due date literature (Baker and Trietsch (2018)), we impose a linear cost function to capture the mismatch cost between the realized project completion time and the set project due date. Specifically, the mismatch cost is modeled by the sum of project overhead cost and project delay cost. The project overhead cost is modeled by $b_1 T_m$ and the project delay cost is modeled by $b_2[\mathbf{x}_2 + a_2 \cdot (\mathbf{x}_1 - Q_s^*)^+ - T_m]^+$, where b_1 is the unit time overhead cost, b_2 is the unit time delay cost, and $[\mathbf{x}_2 + a_2 \cdot (\mathbf{x}_1 - Q_s^*)^+ - T_m]^+$ is the project delay time.

Based on the above discussion, we can express the project firm's expected material cost $C_q(Q_m)$ and expected time cost $C_t(Q_m, T_m)$ by:

$$C_q(Q_m) = \mathbb{E}_{\mathbf{x}_1}\left\{wQ_m + w_1 \cdot \min(Q_s^* - Q_m, (\mathbf{x}_1 - Q_m)^+) + w_2 \cdot (\mathbf{x}_1 - Q_s^*)^+\right\} \tag{1a}$$

$$C_t(Q_m, T_m) = \mathbb{E}_{\mathbf{x}_1,\mathbf{x}_2}\left\{b_1 T_m + b_2[\mathbf{x}_2 + a_2 \cdot (\mathbf{x}_1 - Q_s^*)^+ - T_m]^+\right\} \tag{1b}$$

where Q_s^* is the supplier's optimal production quantity subject to meeting Q_m and is formally defined shortly after. The expected profit of the project firm is therefore expressed by

$$\pi_m(Q_m, T_m) = V - C_q(Q_m) - C_t(Q_m, T_m) \tag{2}$$

For the contracted supplier, assuming her marginal production cost of material is constant and denoted by c, her expected profit from supplying the project material to the project firm is expressed by

$$\pi_s(Q_s) = \mathbb{E}_{\mathbf{x}_1}\ \left\{wQ_m + w_1 \cdot \min(Q_s - Q_m, (\mathbf{x}_1 - Q_m)^+) - cQ_s\right\} \tag{3}$$

where $Q_s \geq Q_m$. We denote the supplier's optimal production by $Q_s^* \triangleq \arg\max_{\{Q_s:Q_s \geq Q_m\}} \pi_s(Q_s)$.

Additional notations. We assume that $\mathbf{x}_1$ and $\mathbf{x}_2$ are arbitrarily correlated in our work. As such, we denote their joint probability density function by $f(x_1, x_2)$, whose support is $\mathbb{R}_+^2 = (0, +\infty) \times (0, +\infty)$. For presentation convenience, we write $\int_0^{+\infty}\int_0^{+\infty} g(\cdot) f(x_1, x_2) dx_1 dx_2$ as $\iint g(\cdot) f(x_1, x_2) dx_1 dx_2$ for any function $g(\cdot)$. To further ease the presentation of our analytical results, we introduce the Heaviside function as below (for more information about this function, please refer to Appendix EC.1):

$$u(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

By using $u(x)$, we define five functions $H_i(\cdot) : i \in \{1, 2, ..., 5\}$ that will be applied to characterize our analytical results. The detailed definitions of the $H_i(\cdot)$ functions are provided in Table 1. Intuitively, each of the H_i functions represents the probability of $(\mathbf{x}_1, \mathbf{x}_2)$ falling in a certain area, and we provide an illustration of these areas by Figure EC.1 in Appendix EC.2.

4 Analysis

In this section, we analyze the problems faced by the project firm and the contracted supplier embedded in a project supply chain as formulated in Section 3. In the analysis below, we first derive the supplier's optimal production decision (Q_s^*) contingent on the advance order quantity Q_m of the project firm (Section 4.1). We then derive the project firm's optimal advance order (Q_m) and project due date (T_m) decisions jointly (Section 4.2). While our project supply chain model cannot be derived from a newsvendor supply chain, we discuss in Section 4.3 how the newsvendor supply chain can be treated as a special case of our project supply chain model and highlight the difference between the analytical results for these chains.

Table 1 Model parameters & defined functions.

Parameters:	
w	The per unit advance order cost from the contracted supplier (variable cost)
w_1	The per unit expedited order cost from the contracted supplier (variable cost)
w_2	The per unit expedited order cost from a backup supplier (variable cost)
a_1	The lead time for expedited orders from the contracted supplier (variable time)
a_2	The lead time for expedited orders from a backup supplier (variable time)
b_1	The per unit time overhead cost of a project (variable cost)
b_2	The per unit time delay penalty of a project (variable cost)
c	The production cost for the contracted supplier (variable cost)
$\mathbf{x}_1$	A random variable for the material consumption of a project
$\mathbf{x}_2$	A random variable for the activity time of a project
Functions:	
$u(x)$	Heaviside function (univariate step function)
$f(x_1,x_2)$	The joint probability density function of $(\mathbf{x}_1,\mathbf{x}_2)$
$f_i(x)$	The marginal probability density function of $\mathbf{x}_i$, $i \in \{1,2\}$
Defined $H_i(\cdot)$ Functions:	
$H_1(q)$	$=\iint[u(x_1-q)]f(x_1,x_2)dx_1dx_2$
$H_2(q,t)$	$=\iint[u(x_1-q)\cdot u(x_2+a_2(x_1-q)-t)]f(x_1,x_2)dx_1dx_2$
$H_3(q,t)$	$=\iint[(1-u(x_1-q))\cdot u(x_2-t)]f(x_1,x_2)dx_1dx_2$
$H_4(q,t)$	$=\iint[u(x_1-q)\cdot(1-u(x_1-\tilde{q}_s))\cdot u(x_2+a_1(x_1-q)-t)]f(x_1,x_2)dx_1dx_2$
$H_5(q,t)$	$=\iint[u(x_1-\tilde{q}_s)\cdot u(x_2+a_1(x_1-q)+(a_2-a_1)(x_1-\tilde{q}_s)-t)]f(x_1,x_2)dx_1dx_2$

4.1 The Contracted Supplier's Optimal Production

Given the project firm's advance order quantity Q_m, the supplier chooses her production quantity Q_s to maximize $\pi_s(Q_s)$, subject to $Q_s \geq Q_m$. The supplier's problem is a classic newsvendor problem subject to a production constraint. Her uncertain demand is the unknown expedited order coming from the project firm due to material shortages in the project's execution phase. To decide her production decision in face of this uncertainty, she assesses a marginal production cost c as the overage cost and uses the profit margin $w_1 - c$ (based on the expedited order price) as the underage cost. Under a demand distribution characterized by $f(x_1,x_2)$, the supplier chooses her production quantity $\tilde{q}_s$ that solves the newsvendor fractile by $H_1(Q_s) = \frac{c}{w_1}$, which is then compared with her production constraint $Q_s \geq Q_m$. We characterize the supplier's optimal production decision Q_s^* formally in the following proposition.

PROPOSITION 1. *$\pi_s(Q_s)$ is concave. Given an advance order Q_m from the project firm, the supplier's optimal production quantity is decided by $Q_s^* = \max\{Q_m, \tilde{q}_s\}$, where $\tilde{q}_s$ is the unique solution to the following equation*

$$w_1 H_1(\tilde{q}_s) - c = 0 \tag{4}$$

We refer to $\tilde{q}_s$ as the *supplier's unconstrained optimal production quantity*. Since the supplier's production quantity has to meet the project firm's order quantity Q_m, the optimal production is set to $\max\{Q_m, \tilde{q}_s\}$. This indicates that, when $Q_m \geq \tilde{q}_s$, the supplier will follow the project firm's order quantity. Following the standard supply chain literature terminology (see Cachon (2004)), the resulting chain is a "push" supply chain. When $Q_m < \tilde{q}_s$, the supplier will carry excess stock $(\tilde{q}_s - Q_m)$ as safety stock for the project firm, and by the standard terminology the resulting chain is a "push-pull" one. A higher order price w_1 leads to a higher $\tilde{q}_s$, and it serves as the main incentive for the supplier to carry more safety stock in the supply chain.

4.2 The Project Firm's Optimal Solution

The project firm seeks to find (Q_m, T_m) that maximizes his expected profit $\pi_m(Q_m, T_m)$. In order to find the optimal decision, we divide the decision space of (Q_m, T_m) into two regions (based on the supplier's production decision): $A = \{(Q_m, T_m) : Q_m \geq \tilde{q}_s, T_m \geq 0\}$ and $B = \{(Q_m, T_m) : 0 \leq Q_m \leq \tilde{q}_s, T_m \geq 0\}$. Next we first derive the project firm's optimal decisions in region A and B separately, and then compare the optimal profit across the two regions to derive the project firm's overall optimal decision. As the analysis will reveal, these two regions are not only distinct mathematically, but they also imply different project execution strategy for the project supply chain. In region A, the project supply chain is a "Push" one, and in region B it is a "Push-Pull" one.

4.2.1 The optimal solution in region A (Push region).

We denote the project firm's expected profit in region A by $\pi_m^A(Q_m, T_m)$, and the optimal solution in this region by $(Q_m^A, T_m^A) = \arg\max_{(Q_m, T_m) \in A} \pi_m^A(Q_m, T_m)$. By Proposition 1, when $(Q_m, T_m) \in A$, we have $Q_s^* = Q_m$. Therefore, by setting $Q_s^* = Q_m$ in $\pi_m(Q_m, T_m)$, we obtain $\pi_m^A(Q_m, T_m)$ as follows:

$$\pi_m^A(Q_m, T_m) = \mathbb{E}_{\mathbf{x}_1, \mathbf{x}_2} \{V - wQ_m - w_2(\mathbf{x}_1 - Q_m)^+ - b_1 T_m - b_2[\mathbf{x}_2 + a_2(\mathbf{x}_1 - Q_m)^+ - T_m]^+\}$$

Our next proposition characterizes the project firm's optimal solution (Q_m^A, T_m^A) in region A. To facilitate presentation of our result, we define $\tilde{t}_s$ as the unique solution to the following equation:

$$b_2 H_2(\tilde{q}_s, \tilde{t}_s) + b_2 H_3(\tilde{q}_s, \tilde{t}_s) - b_1 = 0 \tag{5}$$

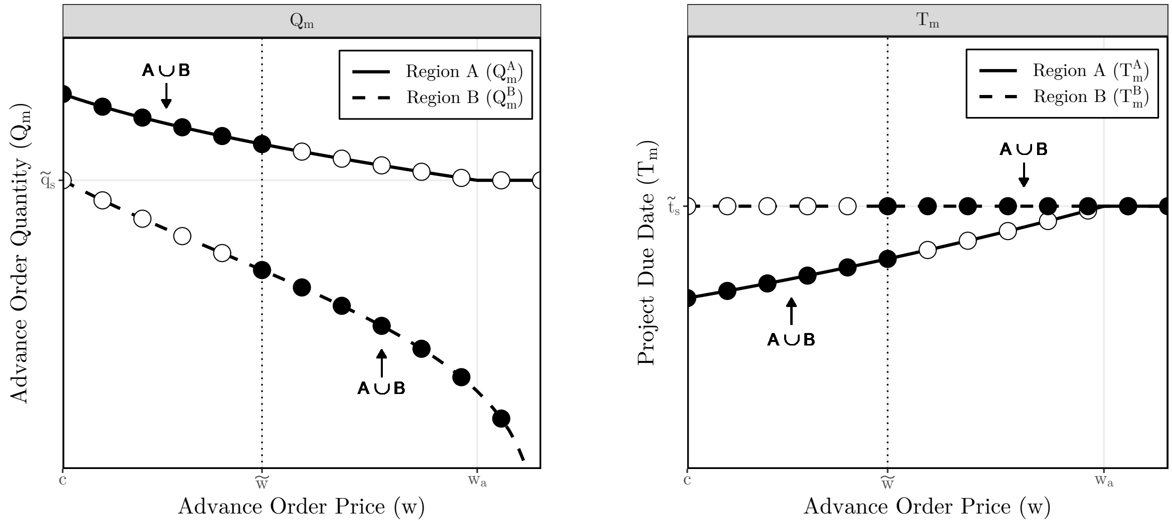
where $\tilde{q}_s$ is defined in Proposition 1. As will be revealed shortly, $\tilde{t}_s$ is the project firm's optimal project due date decision that couples with an advance order decision $\tilde{q}_s$. Both $\tilde{q}_s$ and $\tilde{t}_s$ vary with w_1, but not with w.

PROPOSITION 2. *$\pi_m^A(Q_m, T_m)$ is concave in (Q_m, T_m). Let $w_a = w_2 H_1(\tilde{q}_s) + a_2 b_2 H_2(\tilde{q}_s, \tilde{t}_s)$. For any contract (w, w_1): if $w \geq w_a$, the optimal decision of the project firm in region A is $(Q_m^A, T_m^A) = (\tilde{q}_s, \tilde{t}_s)$; otherwise, $(Q_m^A, T_m^A) = (q^A, t^A)$, where (q^A, t^A) is the unique solution to the following system of equations:*

$$\begin{cases} w_2 H_1(q^A) + a_2 b_2 H_2(q^A, t^A) - w = 0 \\ b_2 H_2(q^A, t^A) + b_2 H_3(q^A, t^A) - b_1 = 0 \end{cases} \tag{6}$$

and they satisfy $q^A > \tilde{q}_s$, $t^A < \tilde{t}_s$. In addition, w_a is decreasing in w_1.

Figure 1 The project firm's optimal solutions for Region A, B and A ∪ B.



(a) Optimal Advance Order Quantity (b) Optimal Project Due Date

We first note that (q^A, t^A) is an interior solution of region A, and $(\tilde{q}_s, \tilde{t}_s)$ is a boundary solution of region A. Proposition 2 states that for every w_1, there exists a w_a such that when $w < w_a$, the project firm should order q^A that is greater than $\tilde{q}_s$ ($q^A > \tilde{q}_s$), and set the project due date at t^A that is smaller than $\tilde{t}_s$ ($t^A < \tilde{t}_s$). That is, for $w < w_a$, the project firm pushes the supply chain with an inventory level that is greater than the supplier's unconstrained production quantity, and hence the project firm bears all inventory risks in the chain. In return he enjoys the benefit of a shorter project due date. On the other hand, when $w \geq w_a$, the project firm would just order $\tilde{q}_s$, with the coupling optimal project due date set at $\tilde{t}_s$. In other words, the project firm adopts the ordering decision on the boundary ($Q_m^A = \tilde{q}_s$) when $w \geq w_a$. Since w_a is decreasing in w_1, the project firm is more inclined to operating at the boundary decision $\tilde{q}_s$ (which will eventually result in a *push-pull* mode) for a wider range of w (e.g., $w > w_a$), as the expedite price w_1 increases.

The optimal solution (Q_m^A, T_m^A) is illustrated in Figure 1 by the solid lines[5]. In Figure (1a), we illustrate the optimal advance order (Q_m^A) in region A, and in Figure (1b), we illustrate the optimal project due date(T_m^A) in region A. We show these two optimal decisions as a function of advance order price (w), by fixing the expedite order price (w_1) at some level. As is illustrated in Figure (1a), the optimal advance order Q_m^A is decreasing in w until w reaches w_a, after which Q_m^A stays on the boundary $\tilde{q}_s$ due to the constraint $Q_m \geq \tilde{q}_s$. Similarly in Figure (1b), we see that the optimal project due date T_m^A increases with w until w reaches w_a, after which T_m^A stays put at $\tilde{t}_s$. The substituting relationship between Q_m^A and T_m^A as w increases is quite intriguing, as it sheds light how the project firm should leverage its control over inventory and time buffers to mitigate increasing material cost.

[5] The *benchmark wholesale price* $\tilde{w}$ in Figure 1 will be formally defined in Section 4.2.3 (see Theorem 1).

Next, we formally characterize the relationship between Q_m^A and T_m^A when the cost parameters in the studied model change. We focus on studying the impact of four parameters (w, w_1, b_1, b_2) and summarize the results below.

PROPOSITION 3. *The impact of* w, w_1, b_1, b_2 *on* (Q_m^A, T_m^A) *are characterized as follows:*
(1) $\frac{dq^A}{dw} < 0$, $\frac{dt^A}{dw} > 0$, $\frac{d\tilde{q}_s}{dw} = 0$, $\frac{d\tilde{t}_s}{dw} = 0$. *As a result,* $\frac{dQ_m^A}{dw} \leq 0$, $\frac{dT_m^A}{dw} \geq 0$.
(2) $\frac{dq^A}{dw_1} = 0$, $\frac{dt^A}{dw_1} = 0$, $\frac{d\tilde{q}_s}{dw_1} > 0$, $\frac{d\tilde{t}_s}{dw_1} < 0$. *As a result,* $\frac{dQ_m^A}{dw_1} \geq 0$, $\frac{dT_m^A}{dw_1} \leq 0$.
(3) $\frac{dq^A}{db_1} > 0$, $\frac{dt^A}{db_1} < 0$, $\frac{d\tilde{q}_s}{db_1} = 0$, $\frac{d\tilde{t}_s}{db_1} < 0$. *As a result,* $\frac{dQ_m^A}{db_1} \geq 0$, $\frac{dT_m^A}{db_1} \leq 0$.
(4) $\frac{dq^A}{db_2} > 0$, $\frac{dt^A}{db_2} > 0$, $\frac{d\tilde{q}_s}{db_2} = 0$, $\frac{d\tilde{t}_s}{db_2} > 0$. *As a result,* $\frac{dQ_m^A}{db_2} \geq 0$, $\frac{dT_m^A}{db_2} \geq 0$.

A general observation is that as w, w_1, or b_1 increases, the optimal advance order (Q_m^A) and project due date (T_m^A) decisions of the project firm move in opposite directions. This implies that the two decisions substitute for each other when any of these parameters changes. More specifically, as the advance order price w increases, the project firm should decrease its advance order quantity Q_m^A and increase the project duration T_m^A. As the project's overhead cost b_1 increases, the firm should shorten the project duration T_m^A and increase the material order quantity Q_m^A. As the expedited order price w_1 increases, since the project firm's decision Q_m^A can be viewed as $Q_m^A = \max\{q^A, \tilde{q}_s\}$, where q^A is not affected by w_1 and $\tilde{q}_s$ increases with w_1, the decision Q_m^A increases. In the meantime, the project firm's due date decision T_m^A decreases.

An interesting observation is that as the project delay parameter b_2 increases, the optimal advance order (Q_m^A) and project due date (T_m^A) decisions of the project firm move in the same direction. This implies that the two decisions complement each other as b_2 changes. More specifically, as the project delay cost rate b_2 increases, the project firm should increase both his project due date and material order decisions, as an effective risk-mitigating strategy for the increased delay penalty cost. As b_2 decreases, the project firm should decrease both decisions. This is in contrast to the observations from the other three parameters.

Our results for the "push" project supply chain imply clear roles for the inventory buffer (the project firm's own inventory) and time buffer (available slack time in project completion). As the advance order price (w) increases, the project firm favors lower buffer inventory and more slack time in project execution. For increases in project overhead cost (b_1), the project firm favors less available time slack but higher inventory. However, for increased penalties (b_2) in project completion failures, both buffers are needed in higher amount to effectively manage project risks.

4.2.2 The optimal solution in region B (Push-Pull region).

We denote the project firm's expected profit in region B by $\pi_m^B(Q_m, T_m)$, and the optimal solution in this region by $(Q_m^B, T_m^B) = \arg\max_{(Q_m,T_m)\in B} \pi_m^B(Q_m, T_m)$. By Proposition 1, when $(Q_m, T_m) \in B$, we have $Q_s^* = \tilde{q}_s$. Therefore, by setting $Q_s^* = \tilde{q}_s$ in $\pi_m(Q_m, T_m)$, we obtain $\pi_m^B(Q_m, T_m)$ as follows:

$$\pi_m^B(Q_m, T_m) = \mathbb{E}_{\mathbf{x}_1,\mathbf{x}_2}\ \{V - wQ_m - w_1 \cdot \min(\tilde{q}_s - Q_m, (\mathbf{x}_1 - Q_m)^+) - w_2(\mathbf{x}_1 - \tilde{q}_s)^+$$
$$-b_1 T_m - b_2[\mathbf{x}_2 + a_2(\mathbf{x}_1 - \tilde{q}_s)^+ - T_m]^+\}$$

Looking at the above $\pi_m^B(Q_m, T_m)$, it is clear that the optimization of Q_m and T_m can be decoupled, since the T_m decision depends on $\tilde{q}_s$ rather than Q_m. Below we summarize the project firm's optimal decision in region B.

PROPOSITION 4. *$\pi_m^B(Q_m, T_m)$ is concave in (Q_m, T_m). For any $w \in [c, w_1]$, the optimal decision of the project firm in region B is $(Q_m^B, T_m^B) = (q^B, t^B)$, where (q^B, t^B) is the unique solution to the following equations:*

$$\begin{cases} w_1 H_1(q^B) - w = 0 \\ b_2 H_2(\tilde{q}_s, t^B) + b_2 H_3(\tilde{q}_s, t^B) - b_1 = 0 \end{cases} \tag{7}$$

We first note that the optimal solution of region B satisfies $q^B \leq \tilde{q}_s$ and $t^B = \tilde{t}_s$ (by the definition of $\tilde{t}_s$, as long as $w \geq c$), with the inequality becoming binding if $w = c$. Since $Q_s^* = \tilde{q}_s$ in this region, we have that $Q_m^B \leq Q_s^*$. By the supply chain terminology, the project supply chain is a *push-pull* one and the inventory risk is shared between the two firms in the chain. Second, we note that (Q^B, T^B) can always be solved from the first order condition of $\pi_m^B(Q_m, T_m)$ through Eqn (7), and the two equations in (7) are independent.

Behind the above mathematical result, there is a powerful cross-functional managerial insight for managing project executions within project supply chains. In general, project firms are functionally organized with project managers dealing with project execution time and procurement managers placing material orders. According to our result, in a push-pull chain the project and procurement managers are able to work independently to optimize their project due date and material ordering decisions that align with each other's interests. In other words, the push-pull chain does not require compulsory internal collaboration between different functional areas of a firm, and coordination is naturally achieved. However, the push-pull chain calls for a deeper external collaboration between the project firm and the supplier, since the project firm's project due date decision t^B is heavily dependent on the supplier's safety stock set through her production decision $\tilde{q}_s$. As can be seen through the cost function $C_t(Q_m, T_m)$, a supplier that carries a higher safety stock by setting a larger $\tilde{q}_s$ can help the project firm reduce the time buffer needed towards minimizing $C_t(Q_m, T_m)$, which will result in a reduced optimal cost of $C_t(Q_m, T_m)$ for the project firm.

The optimal solution (Q_m^B, T_m^B) is illustrated in Figure 1 by the dashed lines. In Figure (1a), the optimal advance order Q_m^B is decreasing in w, starting with $Q_m^B = \tilde{q}_s$ when $w = c$. In Figure (1b), we see that the optimal project due date T_m^B does not change with w. This is because w only impact the risk allocation of materials between the supplier and the project firm, and does not impact the project delay risk that is due to material shortages within the chain. The project delay risk due to material shortages in the chain depends on the total material supply of the chain, which is $\tilde{q}_s$ in this case and is independent of w.

Below we characterize the impact of the cost parameters (w, w_1, b_1, b_2) on (Q_m^B, T_m^B), in a similar fashion to what we have shown for (Q_m^A, T_m^A). Since in region B we have $(Q_m^B, T_m^B) = (q^B, t^B)$ by Proposition 4, we present the impact of (w, w_1, b_1, b_2) on (Q_m^B, T_m^B) directly.

Figure 2 Illustration of the non-concavity of $\pi_m(Q_m, T_m)$

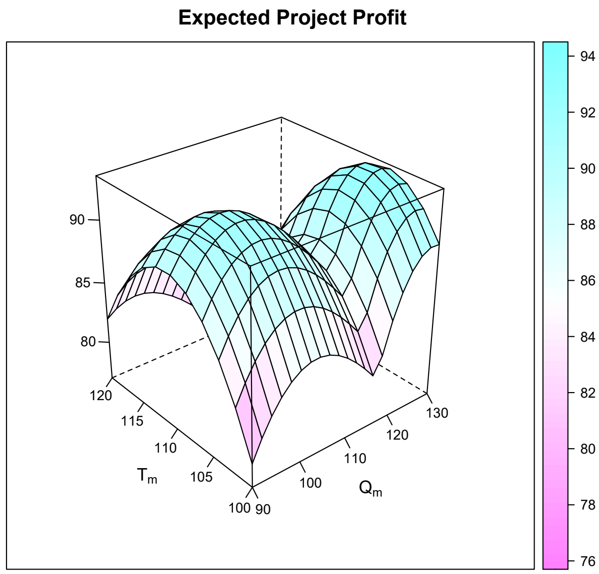


Figure parameters: $c = 1$, $w = 1.5$, $w_1 = 4$, $w_2 = 5$, $b_1 = 4$, $b_2 = 10$, $a_2 = 1$. $(\mathbf{x}_1, \mathbf{x}_2)$ follows a bivariate Gaussian distribution with $\mu = (100, 100)$, $\sigma = (20, 20)$, and $\rho = 0$.

PROPOSITION 5. *The impact of w, w_1, b_1, b_2 on on (Q_m^B, T_m^B) are characterized as follows:*
(1) $\frac{dQ_m^B}{dw} < 0$, $\frac{dT_m^B}{dw} = 0$; *(2)* $\frac{dQ_m^B}{dw_1} > 0$, $\frac{dT_m^B}{dw_1} < 0$; *(3)* $\frac{dQ_m^B}{db_1} = 0$, $\frac{dT_m^B}{db_1} < 0$; *(4)* $\frac{dQ_m^B}{db_2} = 0$, $\frac{dT_m^B}{db_2} > 0$.

We first note that w only impacts Q_m^B, b_1 and b_2 only impact T_m^B, and w_1 impact both Q_m^B and T_m^B. The impact of w, b_1, b_2 on (Q_m^B, T_m^B) are quite intuitive. A higher advance order price w decreases the advance order Q_m^B, a higher project overhead cost b_1 reduces the planned project due date T_m^B, and a higher project penalty b_2 increases the due date T_m^B. The impact of w_1 is relatively more involved. First, a higher w_1 increases the project firm's advance order quantity Q_m^B, and it also increases the supplier's unconstrained production quantity $\tilde{q}_s$. Since the project firm's project due date t^B is set to satisfy Eqn (7), we can easily verify that a smaller t^B is needed when $\tilde{q}_s$ is increased. In other words, the increased safety stock held by the supplier partially substitutes for the time buffer held by the project firm.

4.2.3 The unified optimal solution in region A ∪ B.

Although we have shown that $\pi_m^A(Q_m, T_m)$ and $\pi_m^B(Q_m, T_m)$ are both concave functions in region A and region B, respectively, $\pi_m(Q_m, T_m)$ is in general not a concave function in the unified region A ∪ B. In Figure 2, we illustrate the non-concavity of $\pi_m(Q_m, T_m)$ using a specific example, the parameters of which are provided underneath the figure. It is obvious that $\pi_m(Q_m, T_m)$ is bimodal in the shown example.

In order to find the project firm's optimal decision in A ∪ B, we are going to compare the firm's optimal profits in region A and B. We let $\Delta(w, w_1) = \pi_m^A(Q_m^A, T_m^A) - \pi_m^B(Q_m^B, T_m^B)$ be the difference of optimal profits obtained by the locally optimal decisions of region A and B. In our following analysis, we will treat Δ as a function of (w, w_1), where (w, w_1) can have both a direct effect (through the price effect in the $C_q(Q_m)$ term) and an indirect effect (through their impact on the ordering and production decisions of the firms) on π_m^A and π_m^B, and hence on Δ. It is worthwhile to point out that when $Q_m = \tilde{q}_s$ (e.g., the project firm's advance order quantity is set on the boundary line between A and B), we have $\pi_m^B(\tilde{q}_s, T_m) = \pi_m^A(\tilde{q}_s, T_m)$ for all T_m. This continuity result is useful when we compare the optimal profits of region A and B. In our main theorem below, we show that $\Delta(w, w_1)$ possesses a monotone property, and we characterize the project firm's optimal decision in the unified region $A \cup B$ based on this property.

THEOREM 1. *$\Delta(w, w_1)$ is decreasing in w. The optimal solution (Q_m^*, T_m^*) of the project firm can be characterized as follows: for any w_1, there exists some unique $\widetilde{w} \in (c, w_a)$ such that $\Delta(\tilde{w}, w_1) = 0$, and as a result, we have*

(1) for contracts in $\{(w, w_1) : w < \widetilde{w}\}$, $(Q_m^*, T_m^*) = (q^A, t^A)$;
(2) for contracts in $\{(w, w_1) : w = \widetilde{w}\}$, $(Q_m^*, T_m^*) = (q^A, t^A)$ or $(Q_m^*, T_m^*) = (q^B, t^B)$;
(3) for contracts in $\{(w, w_1) : w > \widetilde{w}\}$, $(Q_m^*, T_m^*) = (q^B, t^B)$.

We note that when $w = \widetilde{w}$, there exist two optimal solutions; one is an interior point in region A, and the other is an interior solution in region B. Both solutions generate the same optimal profit for the project firm. We refer to $\widetilde{w}$ as the "benchmark price", which satisfies $\Delta(\widetilde{w}, w_1) = 0$. We would like to point out that $\widetilde{w}$ is effectively a function of w_1 (e.g., $\widetilde{w}$ is implicitly determined by w_1 through $\Delta(\widetilde{w}, w_1) = 0$). Moreover, in the detailed expression for $\Delta(w, w_1) = 0$, (q^A, t^A) and (q^B) are also present and they are all impacted by w (see Eqn (6) - (7)). Therefore, to solve $\widetilde{w}$, one needs to jointly solve $\Delta(w, w_1) = 0$ with Eqn (6) - (7). Since the detailed expression for $\Delta(w, w_1) = 0$ is rather lengthy, we refer the readers to Appendix EC.3 within the proof of Theorem 1 for the full specification.

The optimal solution (Q_m^*, T_m^*) is illustrated in Figure 1 by the dark circled segments of the solid and dashed lines, where the two lines represent the optimal solution of region A and B, respectively. In Figure (1a), we see that $Q_m^* = Q_m^A$ when $w \leq \widetilde{w}$, indicating that the project firm should push the project chain, and $Q_m^* = Q_m^B$ when $w \geq \widetilde{w}$, indicating the project firm should push-pull (risk-sharing with supplier). Similarly, in Figure (1b), we see that $T_m^* = T_m^A$ when $w \leq \widetilde{w}$, and $T_m^* = T_m^B$ when $w \geq \widetilde{w}$. The optimal project due date decision aligns with the optimal material order decision, based on the push or push-pull strategy that should be adopted by the project firm and is determined by the relationship between w and $\widetilde{w}$.

According to Theorem 1, when the advance order price w is low relative to the benchmark price $\widetilde{w}$, the project firm prefers to order more than the supplier's unconstrained production quantity in the advance order and use his own inventory in dealing with material risks. As a result, the project firm runs a push supply chain. When the advance order price w is high relative to the benchmark price $\widetilde{w}$, the project firm leverages the flexibility in the supply contract and risk shares with the supplier. The project firm in this case prefers to have the supplier carry safety stock that will help him mitigate material shortage risks. In both cases, the corresponding project due date will be set according to the total material supply level in the supply chain, which is equal to the project firm's own order quantity in the former case and equal to the supplier's unconstrained production quantity in the latter.

To derive further insights on project supply chain management of ETO vs. MTO projects, we embellish into differences between the management of these project types. ETO projects address unique nature engineering challenges and rely on new technologies or stretch the application of existing technologies. As a result, ETO projects commonly require high quality, specially engineered expensive materials sourced from specialized suppliers. On the other hand, MTO projects for most cases use standard technologies and existing, often commoditized, materials sourced from an ample, competitive, and less expensive supply base. Within our modeling environment, we translate the above observation into material-related cost and price parameters, i.e., the parameter vector (c, w, w_1, w_2), with higher magnitude values of a vector standing for ETO projects and lower values for MTO projects.

We proceed to obtain further analytical results in support of richer managerial insights for managing these two types of projects as follows. Project firm 1 manages a project supply chain for a project with a parameter vector (c, w, w_1, w_2). Project firm 2 manages a project supply chain with a parameter vector that is β times that of the above, i.e., $(\beta c, \beta w, \beta w_1, \beta w_2)$, with $\beta > 1$ representing a more expensive material and $\beta < 1$ a less expensive one. We keep the relative ratio between the components of the vector constant in order to focus on the average effect of an increase or decrease in material costs. We refer to the project of firm 1 as the "benchmark project". Project firm 1 may be managing the project supply chain optimally either as a "push" chain or as a "push-pull" chain (according to Theorem 1). We next characterize the optimal supply chain management strategy for project firm 2, with its material cost parameter vector β-scaled that of project firm 1 and the optimal strategy contingent on the β magnitude.

THEOREM 2. *There exists some $\bar{\beta} \in [0, +\infty]$ such that*

1. *if $\beta > \bar{\beta}$, project firm 2 should operate a "push-pull" supply chain;*
2. *if $\beta < \bar{\beta}$, project firm 2 should operate a "push" supply chain;*
3. *if $\beta = \bar{\beta}$, project firm 2 is indifferent between operating a "push-pull" or "push" supply chain.*

In particular, if the optimal decision of project firm 1 is to operate a "push-pull" supply chain, then $\bar{\beta} \leq 1$; if the optimal decision of project firm 1 is to operate a "push" supply chain, then $\bar{\beta} \geq 1$.

According to the above result, if project firm 1 managing the "benchmark project" should optimally use a "push-pull" chain, project firm 2 with a more expensive material vector (i.e., $\beta > 1$) should also use a "push-pull" chain. If project firm 1 optimally uses a "push" chain, then project firm 2 with a less expensive material vector (i.e., $\beta < 1$) should also use a "push" chain. Allowing for some loose interpretation of the result, and assuming we can identify a "benchmark project" for an application environment that could be optimally managed in either a "push" or "push-pull" approach (i.e., $\beta = \bar{\beta}$ for it), then ETO projects for that application environment with $\beta > \bar{\beta}$ should be optimally managed in a "push-pull" fashion. Similarly, MTO projects for that application environment with $\beta < \bar{\beta}$ should be managed in a "push" fashion.

Our results provide the following managerial implications for managing project supply chains in practice. For complex ETO projects delivering unique and highly customized products using exotic and expensive materials, project firms should leverage the flexible contract in their relationship with the contracted supplier and rely on the safety stock of the supplier when determining its project due date. When N/E produces expensive (priced at $50 million) boilers for power plants, they sign flexible contracts with key suppliers and expect them to carry safety stock for unexpected material requirement. MTO projects use mostly push strategies in creating CTO/ATO products that are built on mature technologies via standardized modules and commodity type materials. For example, Emerson's flow control division (the Fisher brand), which installs valves and measurement systems for industrial customers, runs mostly push supply chains. The use of standardized materials of low prices reduces the inventory risk for a project firm, and their typically lower costs make it affordable for the project firm to bear all inventory risk in the supply chain.

Taking a supplier's perspective, the material's prices are in general cheaper across all stages of the material supply chain, or a higher discount of the advance order price may be accepted by a supplier, when the material technology is mature and there are more qualified suppliers in the material supply market, as evidenced in the MTO examples. As a result, we anticipate that a "push" supply chain will be adopted as a stackelberg game equilibrium between the project firm and the contract supplier. On the other hand, when the material technology is niche and there is limited availability of qualified suppliers, the material's prices are most likely more expensive across the supply chain and a supplier is less likely to accept an aggressive discount for the advance order price. The project firm should then share the material inventory risk with the supplier via the flexible wholesale contract. The higher project material cost relative to project time cost is driving the project firm to enable material risk-sharing with the supplier. The extra material safety stock carried by a contract supplier is extremely valuable to a project firm when the material cost is high. N/E and Emerson, for their selected ETO projects, use a carefully selected set of "preferred suppliers" in managing the supply of certain expensive materials in their supply chains. These suppliers are rewarded over time for their product availability with more orders allocated to them under some premium-priced flexible contracts.

4.3 Special Case: Newsvendor Supply Chain.

The project supply chain problem formulated in Section 3 (see equations (1) - (3)) has to deal with the unique aspect of the project firm's objective function that incorporates project overhead costs and delay penalties while accounting for uncertain project time and material consumption. It also accounts for the flexible supply contract and the intricacies of logistical delays in expediting materials from the contracted or the backup supplier. These unique attributes of the problem are not captured in, and do not follow from, the newsvendor with flexible wholesale contract model and its related literature (Cachon (2004), Dong and Zhu (2007)).

However, some newsvendor with flexible wholesale contract model (referred to as the newsvendor supply chain) can be viewed as a special case of our project supply chain model under the following interpretation and assumptions. The downstream firm of a newsvendor supply chain, a newsvendor, faces uncertain product demand that has the same distribution as $\mathbf{x}_1$ in our model. The newsvendor orders product inventories from an upstream supplier under a flexible wholesale price contract (w, w_1), and he fulfills the product demand either through his own inventories (that are ordered in advance at w) or through an expedited order to the supplier for her safety stock (at w_1). In this newsvendor variant, we impose the further restriction that any product shortages not covered by the above execution of the contract are always covered by a backup supplier at a unit price w_2. WLOG, we assume $w_1 \leq w_2$. We note that such a further restriction on meeting demand shortages via a backup source has also been considered in prior supply chain works; see Lee et al. (2000), Xie et al. (2010), and references therein. The retail price of the product is fixed and given by p.

Now let us see the reduction of the project supply chain to the newsvendor supply chain. We use a superscript ϕ to indicate profit functions and decisions in a newsvendor chain and use the same subscripts $\{m, s\}$ to indicate the downstream and upstream firms in the chain. Based on the above description, the expected profit of the firms in the newsvendor supply chain can be formulated as

$$\begin{aligned}\pi_m^\phi(Q_m) &= \mathbb{E}_{\mathbf{x}_1}\left\{p\cdot\mathbf{x}_1 - wQ_m - w_1\cdot\min\{Q_s^\phi - Q_m, (\mathbf{x}_1 - Q_m)^+\} - w_2\cdot(\mathbf{x}_1 - Q_s^\phi)^+\right\}\\ \pi_s^\phi(Q_s) &= \mathbb{E}_{\mathbf{x}_1}\left\{wQ_m + w_1\cdot\min(Q_s - Q_m, (\mathbf{x}_1 - Q_m)^+) - cQ_s\right\}\end{aligned}$$

where $Q_s^\phi = \arg\max_{Q_s \geq Q_m}\pi_s^\phi(Q_s)$. We note that the supplier (upstream firm) faces a problem that is identical to the one faced by the supplier in our project supply chain model. As a result, $Q_s^\phi = Q_s^*$. The newsvendor (downstream firm) faces a problem that can be viewed as a special case of the problem faced by a project firm, by setting $V = p\cdot\mathbb{E}(\mathbf{x}_1)$ and $b_1, b_2 \to 0$ (in the extreme, $b_1 = b_2 = 0$). As such, the newsvendor firm can be regarded as a project firm who makes his material order decision when his project time related costs are negligible. Since this is a special case of our studied model (by setting $b_1 = b_2 = 0$), Theorem 1 can be applied and we present an updated result on the newsvendor's optimal decision, which constitutes a contribution of our work to the newsvendor literature.

COROLLARY 1. *The optimal advance order Q_m^ϕ of the newsvendor can be characterized as follows: for any w_1, there exists some unique $\widetilde{w}^\phi$ such that*

(1) for contracts in $\{(w, w_1) : w < \widetilde{w}^\phi\}$, $Q_m^\phi = q^{\phi,A}$;

(2) for contracts in $\{(w, w_1) : w > \widetilde{w}^\phi\}$, $Q_m^\phi = q^{\phi,B}$;

(3) for contracts in $\{(w, w_1) : w = \widetilde{w}^\phi\}$, $Q_m^\phi = q^{\phi,A}$ or $Q^\phi = q^{\phi,B}$.

$q^{\phi,A}$ and $q^{\phi,B}$ satisfy the following equations. Moreover, $q^{\phi,A} < q^A$ and $q^{\phi,B} = q^B$.

$$w_2 H_1(q^{\phi,A}) - w = 0 \tag{8}$$

$$w_1 H_1(q^{\phi,B}) - w = 0 \tag{9}$$

Given the optimal decisions of the newsvendor supply chain and the project supply chain, the first interesting comparison that we would like to make is how the push vs. pull-pull (risk-sharing) contract space changes across the two chains. This comparison is done by comparing $\widetilde{w}$ and $\widetilde{w}^{\phi}$ in the two respective models, and we summarize our finding with the following proposition.

PROPOSITION 6. *$\widetilde{w}^{\phi} < \widetilde{w}$.*

Proposition 6 implies that the project firm pushes the supply chain in more contract regions than the newsvendor firm. For any fixed w_1: if $w < \widetilde{w}^{\phi}$, both the project firm and the newsvendor would push the supply chain; if $\widetilde{w}^{\phi} < w < \widetilde{w}$, the project firm would push the supply chain while the newsvendor would push-pull; if $w > \widetilde{w}$, both firms would push-pull the supply chain. Figure 3 illustrates the push and push-pull regions, with the highlighted area depicting the region in which the newsvendor would push-pull but the project firm would push the supply chain.

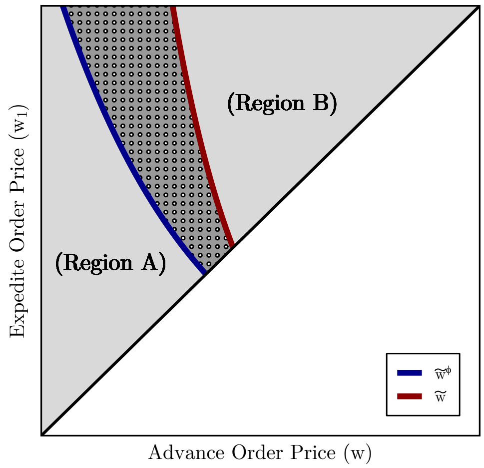
Next, we compare the material order and production decisions between the project and newsvendor supply chains. As will be revealed shortly, the project firm's advance order quantity is always greater than or equal to that of the newsvendor firm, and so is the contracted supplier's production quantity. The next proposition summarizes the result.

PROPOSITION 7. *The project firm's order is no less than the newsvendor, and so is the supplier's production quantity. Specifically,*

(1) for contracts in $\{(w, w_1) : w < \widetilde{w}\}$, $Q_m^* > Q_m^{\phi}$ and $Q_s^* > Q_s^{\phi}$;
(2) for contracts in $\{(w, w_1) : w > \widetilde{w}\}$, $Q_m^* = Q_m^{\phi}$ and $Q_s^* = Q_s^{\phi}$;
(3) for contracts in $\{(w, w_1) : w = \widetilde{w}\}$, $Q_m^* \geq Q_m^{\phi}$ and $Q_s^* \geq Q_s^{\phi}$.

In the contract region where the project firm strictly pushes the supply chain (case 1), the project firm orders strictly more than the newsvendor, and the supplier follows the advance order. In the contract region where the project firm strictly push-pulls (case 2), the project supply chain behaves identically to the newsvendor chain. When the project firm is indifferent between push and push-pull (case 3), he orders strictly more than the newsvendor if he pushes the chain, and orders the same if he push-pulls. The proposition also implies that material inventories are more valuable to the project firm than to the newsvendor. As a result, the project firm always carries more inventories, and so does the project supply chain.

Figure 3 Illustration of the relationship between $\widetilde{w}$ and $\widetilde{w}^{\phi}$.



5 Numerical Study

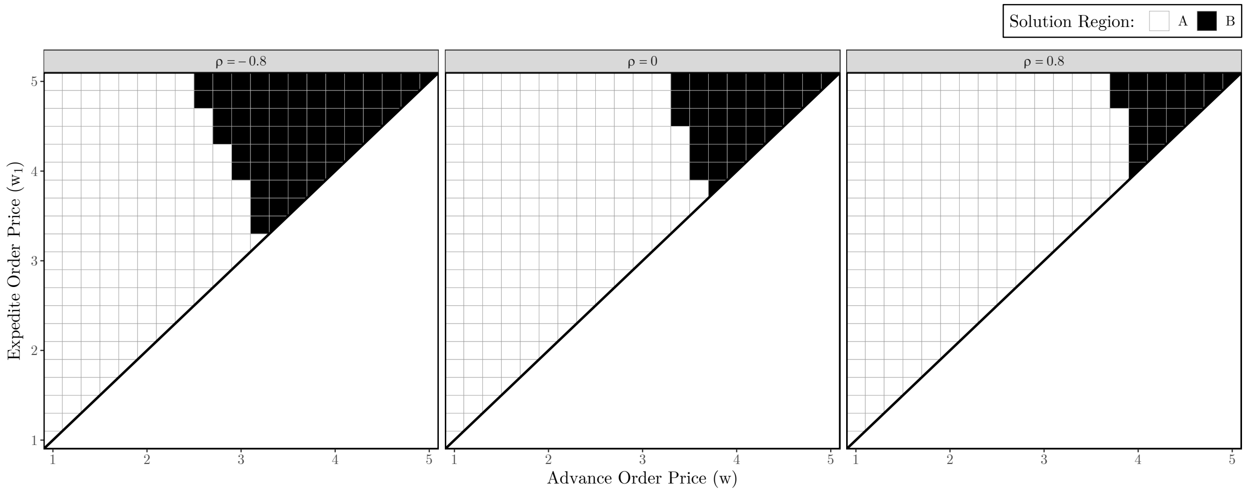
In this section, we conduct numerical experiments to illustrate how the project firm responds to operational changes in the operating environment. We highlight the firm's response to each operational change by comparing the contract regions in which push or push-pull strategies are optimal, and by comparing the shifts in the optimal quantities (e.g., Q_m^*, T_m^*). Our numerical experiments are conducted by varying the following parameters that represent unique operational changes of an environment: (1) the correlation between project time and material consumption uncertainties (ρ); (2) the responsiveness of the back supplier (a_2); (3) the project delay penalty (b_2), and (4) the project overhead cost (b_1).

5.1 The Impact of the Correlation between the Project Uncertainties (ρ).

We first investigate how the correlation between the project time and material uncertainties impact the project firm's optimal decisions and profit. We consider a bivariate Gaussian distribution of $(\mathbf{x}_1, \mathbf{x}_2)$ with mean $\mu = (100, 100)$ and standard deviation $\sigma = (20, 20)$, and we vary the correlation $\rho \in \{-0.8, 0, 0.8\}$. We fix the other parameters at $c = 1$, $w_2 = 5$, $b_1 = 4$, $b_2 = 10$, $a_2 = 5$, and $V = 1200$.

First, our numerical results show that as ρ increases, the project firm pushes the supply chain more often over a wider range of contracts. This is illustrated by Figure 4, in which the white area (region A) represents the contract regions where the project supply chain operates in a *push* mode. Figure 4 shows the area of region A enlarges as ρ increases from -0.8 to 0.8 (from left to right). The result indicates that when the two project uncertainties are more positively correlated, the project firm is going to push the supply chain under more contracts. We also observe that the supply chain operates in a *push-pull* mode (region B) when w is high, regardless of the ρ and w_1 values. However, the area of region B enlarges when ρ get smaller. This indicates that when the two project uncertainties are more negatively correlated, the project firm is more inclined to share material risks with the contract supplier in the supply chain.

Figure 4 The impact of ρ on the risk-sharing strategy of the supply chain.



We next investigate the impact of ρ on the project firm's optimal decisions and profit. We seek to understand how the firm is going to adjust his material order and project due date quantities as ρ changes, and how the firm's profit is impacted by ρ. We present the numerical results on the project firm's optimal decisions and profit in Table 2. In this table, each row gives the optimal decisions (Q_m^* and T_m^*) and the associated profit for a given wholesale price contract (indicated by the w, w_1 columns). The "OPT Region" columns indicate whether the project supply chain operates in a *push* mode (indicated by "A") or in a *push-pull* mode (indicated by "B").

We observe that as ρ increases, for all contracts, the optimal advance order Q_m^* increases, the optimal project due date T_m^* decreases, and the optimal profit decreases. This shows that inventory buffers and time buffers substitute for each other as ρ changes. Besides, we observe the changes in Q_m^* and T_m^* are mild when the firm's decisions remain in one region and much wilder when the firm's decisions switch regions. For example, when $(w, w_1) = (2.5, 3.0)$, the project firm's optimal decisions remain in region A as ρ changes from 0 to 0.8. The firm's Q_m^* decision increases slightly from 132.01 to 133.79, and T_m^* decision decreases slightly from 106.32 to 105.09. For another example, when $(w, w_1) = (3.5, 4.5)$, the project firm's optimal decisions switch from region A to region B as ρ changes from 0 to 0.8. The firm's Q_m^* decision increases significantly from 84.71 to 130.46, and T_m^* decision decreases from 111.67 to 105.13. Finally, when $(w, w_1) = (4.0, 4.5)$, the project firm's optimal decisions remain in region B as ρ changes from 0 to 0.8. The firm's Q_m^* decision barely changes, but the T_m^* decision changes from 111.67 to 106.41.

In the final example above, we observe that although the material order decision Q_m^* barely changes, the project due date decision T_m^* changes (decreases) much more significantly. This is very surprising as one would think a high positive correlation (ρ) would increase both decisions. We provide a brief explanation on why T_m^* may actually decrease as ρ increases below.

Table 2 Summary of the project firm's optimal decision and expected profit for $\rho \in \{-0.8, 0, 0.8\}$.

		OPT Region			Q_m^*			T_m^*			Expected Profit		
w	w_1	-0.8	0	0.8	-0.8	0	0.8	-0.8	0	0.8	-0.8	0	0.8
1.0	3.0	A	A	A	135.18	140.18	141.85	105.90	105.54	105.07	581.03	575.24	573.62
1.5	3.0	A	A	A	132.11	136.72	138.44	106.35	105.79	105.07	514.25	506.06	503.60
2.0	3.0	A	A	A	129.79	134.13	135.87	106.80	106.05	105.08	448.80	438.36	435.04
2.5	3.0	A	A	A	127.89	132.01	133.79	107.26	106.32	105.09	384.39	371.79	367.65
3.0	3.0	A	A	A	126.27	130.21	132.01	107.72	106.59	105.11	320.86	306.23	301.21
1.0	3.5	A	A	A	135.18	140.18	141.85	105.90	105.54	105.07	581.03	575.24	573.62
1.5	3.5	A	A	A	132.11	136.72	138.44	106.35	105.79	105.07	514.25	506.06	503.60
2.0	3.5	A	A	A	129.79	134.13	135.87	106.80	106.05	105.08	448.80	438.36	435.04
2.5	3.5	A	A	A	127.89	132.01	133.79	107.26	106.32	105.09	384.39	371.79	367.65
3.0	3.5	A	A	A	126.27	130.21	132.01	107.72	106.59	105.11	320.86	306.23	301.21
3.5	3.5	B	A	A	0.00	128.61	130.46	116.36	106.88	105.13	278.95	241.57	235.59
1.0	4.0	A	A	A	135.18	140.18	141.85	105.90	105.54	105.07	581.03	575.24	573.62
1.5	4.0	A	A	A	132.11	136.72	138.44	106.35	105.79	105.07	514.25	506.06	503.60
2.0	4.0	A	A	A	129.79	134.13	135.87	106.80	106.05	105.08	448.80	438.36	435.04
2.5	4.0	A	A	A	127.89	132.01	133.79	107.26	106.32	105.09	384.39	371.79	367.65
3.0	4.0	B	A	A	86.51	130.21	132.01	114.48	106.59	105.11	324.24	306.23	301.21
3.5	4.0	B	A	A	77.00	128.61	130.46	114.48	106.88	105.13	283.20	241.57	235.59
4.0	4.0	B	B	B	0.00	0.00	0.00	114.48	112.84	106.91	249.65	204.98	173.39
1.0	4.5	A	A	A	135.18	140.18	141.85	105.90	105.54	105.07	581.03	575.24	573.62
1.5	4.5	A	A	A	132.11	136.72	138.44	106.35	105.79	105.07	514.25	506.06	503.60
2.0	4.5	A	A	A	129.79	134.13	135.87	106.80	106.05	105.08	448.80	438.36	435.04
2.5	4.5	A	A	A	127.89	132.01	133.79	107.26	106.32	105.09	384.39	371.79	367.65
3.0	4.5	B	A	A	91.38	130.21	132.01	113.12	106.59	105.11	331.78	306.23	301.21
3.5	4.5	B	B	A	84.70	84.71	130.46	113.12	111.67	105.13	287.70	246.36	235.59
4.0	4.5	B	B	B	75.60	75.59	75.60	113.12	111.67	106.41	247.45	206.11	178.55
4.5	4.5	B	B	B	0.00	0.00	0.00	113.12	111.67	106.41	214.48	173.16	145.58
1.0	5.0	A	A	A	135.18	140.18	141.85	105.90	105.54	105.07	581.03	575.24	573.62
1.5	5.0	A	A	A	132.11	136.72	138.44	106.35	105.79	105.07	514.25	506.06	503.60
2.0	5.0	A	A	A	129.79	134.13	135.87	106.80	106.05	105.08	448.80	438.36	435.04
2.5	5.0	B	A	A	100.00	132.01	133.79	112.08	106.32	105.09	385.64	371.79	367.65
3.0	5.0	B	A	A	94.93	130.21	132.01	112.08	106.59	105.11	336.91	306.23	301.21
3.5	5.0	B	B	A	89.51	89.51	130.46	112.08	110.80	105.13	290.78	252.28	235.59
4.0	5.0	B	B	B	83.17	83.17	83.17	112.08	110.80	106.08	247.55	209.05	184.60
4.5	5.0	B	B	B	74.38	74.37	74.38	112.08	110.80	106.08	207.99	169.50	145.04
5.0	5.0	B	B	B	0.00	0.00	0.00	112.08	110.80	106.08	175.52	137.04	112.57

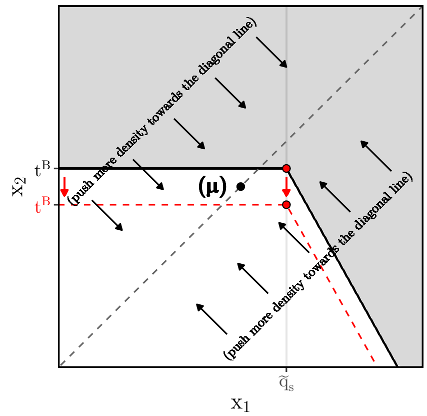
Other parameters: $c = 1$, $w_2 = 5$, $b_1 = 4$, $b_2 = 10$, $a_2 = 5$. $(\mathbf{x}_1, \mathbf{x}_2)$ follows a Gaussian distribution with $\mu = (100, 100)$, $\sigma = (20, 20)$, and $\rho \in \{-0.8, 0, 0.8\}$.

Note: $Q_m^* = 0$ indicates the project firm strictly pulls materials from the supplier (for some wholesale contract where $w = w_1$).

First, we note that in the final example above, Q_m^* lies in region B. As a result, $T_m^* = t^B$. For Gaussian distributions, it is fully anticipated that Q_m^* (and also Q_s^*) does not change with ρ [6]. For T_m^* (or t^B), it is not obvious to infer how it changes with ρ. In Figure 5, we provide a graphical solution for t^B, which has to satisfy Eqn (7). Using the graphical solution we will explain why t^B may decrease when ρ is increased. By Eqn (7), the solution of t^B is a point on the vertical axis such that the probability of $(\mathbf{x}_1, \mathbf{x}_2)$ falling in the gray area equals $\frac{b_1}{b_2}$ in Figure 5. Now suppose ρ increases from 0 to 0.8. What this means is that the increased correlation between $\mathbf{x}_1$ and $\mathbf{x}_2$ is going to "push" more probability density mass toward the diagonal line that represents perfect correlation. This implies that it is quite likely that the probability of the white area is going to increase, and hence the probability of the gray area is going to decrease. Since our choice of t^B

[6] In region B, we have $Q_m^* = q^B, Q_s^* = \tilde{q}_s$. Both of them can be solved from the marginal distribution of a bivariate Gaussian. The marginal distribution of a bivariate Gaussian does not depend on ρ.

Figure 5 Explanation for decreasing T_m^* as the uncertainty correlation (ρ) increases.



is such that the probability of the gray area equals $\frac{b_1}{b_2}$, we must expand the gray area. Note that $\tilde{q}_s$ does not change with ρ, and $\tilde{q}_s$ dictates the intersection of the boundary lines dividing the gray and white areas. The only way to expand the gray area then is to move the boundary lines down, which is illustrated by the red dashed line in Figure 5. This implies t^B should be decreased and explains why we may observe a decreased project due date when the project uncertainty correlation ρ changes from 0 to 0.8.

Intuitively, the project supply chain uses two buffers (time and inventory) to mitigate project completion time and material consumption uncertainties. The inventory buffer assumes a dual role in mitigating these uncertainties. As the correlation ρ between the project activity time and material consumption uncertainties increases, the inventory buffer may cover more risk from project activity time variation. Therefore, the same amount of inventory buffer can carry a bigger probability in meeting the project due date, which can lead the project firm to choose a decreased project due date T_m^*. Similarly, When ρ is negative and decreases, the inventory buffer carries a smaller probability in meeting project due date, and the project firm has to increase the amount of time buffers in order to meet its project due date with a target probability.

On the profit front, we observe that the project firm's optimal profit decreases as ρ increases. In other words, the project firm is going to benefit from more negatively correlated (or less positively correlated) uncertainties between project time and material demand. This observation is consistent with the risk-pooling results (e.g., market, location) that have been derived in the supply chain management literature.

5.2 The Impact of the Responsiveness of the Backup Supplier (a_2).

We next investigate how the responsiveness of the backup supplier (a_2) impacts the project firm's optimal decision and profit. We consider three levels of $a_2 \in \{1,5,10\}$, and fix other parameters at $c = 1$, $w_2 = 5$, $b_1 = 4$, $b_2 = 10$, $\mu = (100,100)$, $\sigma = (20,20)$, $\rho = 0$, and $V = 1200$. Our numerical results show that as a_2 increases: (1) the project firm should push the supply chain more often over a wider range of contracts; and (2) the project firm should always try to drive the supply chain inventories up (e.g., he is pushing the supply chain and carries all inventory in the channel), but may increase or decrease its project due date. A summary of the numerical results is provided in Table 3.

When a_2 increases, the project firm faces a longer project delay (and penalty) when he has to resort to the backup supplier for materials. This puts the project firm at a disadvantageous position, and to deal with such consequences, the project firm should either carry more inventory or set a longer project due date that absorbs some of the financial impact of a prolonged delay. Our results show that the project firm should always carry more inventory if he can, that is, when the firm is pushing the supply chain. The firm may increase or decrease his project due date, depending on whether he is able to drive channel inventories up. For example, when the supply chain operates in a push mode, the project firm can drive the channel inventories up so that it reduces his chance of resorting to the backup supplier. As a result, he may reduce his project due date. When the supply chain operates in a push-pull mode, the channel inventory is determined by $\tilde{q}_s$ and the project firm's own ordering decision will not impact it, and as a result, the firm cannot reduce his chance of resorting to the backup supplier and hence must increase his project due date as a way to mitigate the financial impact. In the latter case, the project firm may also consider taking control of the supply chain by switching it to a push mode. We do observe that the project supply chain is switched from the push-pull mode to the push mode at many contracts, as a_2 is increased. As a final remark, the project firm's profit is always decreased as a_2 is increased.

5.3 The Impact of Project Overhead Cost (b_1) and Project Delay Penalty (b_2).

We next investigate how the project overhead cost (b_1) and project delay penalty (b_2) impact the project firm's optimal decision and the corresponding profit.

For project overhead cost (b_1), we consider three levels of $b_1 \in \{1,4,8\}$ and fix other parameters at $c = 1$, $w_2 = 5$, $b_2 = 10$, $a_2 = 5$, $\mu = (100,100)$, $\sigma = (20,20)$, $\rho = 0$, and $V = 1200$. Our numerical results show that as the project overhead cost b_1 increases, (1) the project firm should push the supply chain more often over a wider range of contracts, and (2) the project firm should always reduce its project due date (T_m^*) and increase the advance material order quantity (Q_m^*). The latter observation was shown analytically for region A and B separately in Proposition 3 and 5. Intuitively, a higher project overhead cost incentivizes the project firm to reduce its time buffers and in return increase its material buffers. The optimal expected profit of the project firm decreases as b_1 increases. We relegate the detailed numerical results for b_1 to Appendix EC.4.

For project delay penalty (b_2), we consider three levels of $b_2 \in \{5,10,20\}$ and fix other parameters at $c = 1$, $w_2 = 5$, $b_1 = 4$, $a_2 = 5$, $\mu = (100,100)$, $\sigma = (20,20)$, $\rho = 0$, and $V = 1200$. Our numerical results show that as b_2 increases, (1) the project firm should push the supply chain more often over a wider range of contracts, and (2) the project firm should increase both his project due date and material order decisions. The latter observation was shown analytically for region A and B separately in Proposition 3 and 5. Intuitively, a harsh delay penalty makes time buffers and material buffers more valuable to the project firm. As a result,

Table 3 Summary of the project firm's optimal decision and expected profit for $a_2 \in \{1,5,10\}$.

		OPT Region			Q_m^*			T_m^*			Expected Profit		
w	w_1	1	5	10	1	5	10	1	5	10	1	5	10
1.0	3.0	A	A	A	126.28	140.18	145.95	105.93	105.54	105.34	587.20	575.24	570.03
1.5	3.0	A	A	A	121.51	136.72	142.83	106.47	105.79	105.48	525.31	506.06	497.87
2.0	3.0	B	A	A	91.39	134.13	140.51	109.36	106.05	105.62	474.57	438.36	427.06
2.5	3.0	B	A	A	80.65	132.01	138.64	109.36	106.32	105.76	438.67	371.79	357.28
3.0	3.0	B	A	A	0.00	130.21	137.05	109.36	106.59	105.91	413.24	306.23	288.38
1.0	3.5	A	A	A	126.28	140.18	145.95	105.93	105.54	105.34	587.20	575.24	570.03
1.5	3.5	A	A	A	121.51	136.72	142.83	106.47	105.79	105.48	525.31	506.06	497.87
2.0	3.5	B	A	A	96.40	134.13	140.51	108.54	106.05	105.62	474.73	438.36	427.06
2.5	3.5	B	A	A	88.68	132.01	138.64	108.54	106.32	105.76	432.92	371.79	357.28
3.0	3.5	B	A	A	78.65	130.21	137.05	108.54	106.59	105.91	396.16	306.23	288.38
3.5	3.5	B	A	A	0.00	128.61	135.68	108.54	106.88	106.05	367.64	241.57	220.22
1.0	4.0	A	A	A	126.28	140.18	145.95	105.93	105.54	105.34	587.20	575.24	570.03
1.5	4.0	A	A	A	121.51	136.72	142.83	106.47	105.79	105.48	525.31	506.06	497.87
2.0	4.0	B	A	A	100.00	134.13	140.51	107.97	106.05	105.62	474.67	438.36	427.06
2.5	4.0	B	A	A	93.63	132.01	138.64	107.97	106.32	105.76	429.42	371.79	357.28
3.0	4.0	B	A	A	86.51	130.21	137.05	107.97	106.59	105.91	387.89	306.23	288.38
3.5	4.0	B	A	A	76.99	128.61	135.68	107.97	106.88	106.05	350.57	241.57	220.22
4.0	4.0	B	B	A	0.00	0.00	134.45	107.97	112.84	106.20	320.58	204.98	152.70
1.0	4.5	A	A	A	126.28	140.18	145.95	105.93	105.54	105.34	587.20	575.24	570.03
1.5	4.5	B	A	A	108.61	136.72	142.83	107.55	105.79	105.48	525.31	506.06	497.87
2.0	4.5	B	A	A	102.79	134.13	140.51	107.55	106.05	105.62	474.36	438.36	427.06
2.5	4.5	B	A	A	97.21	132.01	138.64	107.55	106.32	105.76	426.76	371.79	357.28
3.0	4.5	B	A	A	91.39	130.21	137.05	107.55	106.59	105.91	382.24	306.23	288.38
3.5	4.5	B	B	A	84.71	84.71	135.68	107.55	111.67	106.05	340.89	246.36	220.22
4.0	4.5	B	B	A	75.59	75.59	134.45	107.55	111.67	106.20	303.30	206.11	152.70
4.5	4.5	B	B	A	0.00	0.00	133.35	107.55	111.67	106.36	272.66	173.16	85.75
1.0	5.0	A	A	A	126.28	140.18	145.95	105.93	105.54	105.34	587.20	575.24	570.03
1.5	5.0	B	A	A	110.49	136.72	142.83	107.24	105.79	105.48	526.22	506.06	497.87
2.0	5.0	B	A	A	105.07	134.13	140.51	107.24	106.05	105.62	473.88	438.36	427.06
2.5	5.0	B	A	A	100.00	132.01	138.64	107.24	106.32	105.76	424.52	371.79	357.28
3.0	5.0	B	A	A	94.93	130.21	137.05	107.24	106.59	105.91	377.86	306.23	288.38
3.5	5.0	B	B	A	89.51	89.51	135.68	107.24	110.80	106.05	333.84	252.28	220.22
4.0	5.0	B	B	A	83.17	83.17	134.45	107.24	110.80	106.20	292.69	209.05	152.70
4.5	5.0	B	B	A	74.37	74.37	133.35	107.24	110.80	106.36	255.07	169.50	85.75
5.0	5.0	B	B	B	0.00	0.00	0.00	107.24	110.80	111.94	224.18	137.04	25.72

Other parameters: $c = 1$, $w_2 = 5$, $b_1 = 4$, $b_2 = 10$, $V = 1200$. $(\mathbf{x}_1, \mathbf{x}_2)$ follows a Gaussian distribution with $\mu = (100, 100)$, $\sigma = (20, 20)$, $\rho = 0$.

the firm should increase both type of buffers. The project firm should also consider switching to take control of the supply chain by pushing the chain as b_2 increases. Not surprisingly, the optimal expected profit of the project firm decreases as b_2 increases. We relegate the detailed numerical results for b_2 to Appendix EC.4.

6 Extension: Additive Delay Model

In this section, we consider the case of additive material delay when using the contracted supplier and the backup supplier in meeting project material shortages. As is discussed in Section 3, the delay from using the contracted supplier is $a_1 \cdot \min\{(Q_s - Q_m), (\mathbf{x}_1 - Q_m)^+\}$ and the delay from using the backup supplier is $a_2 \cdot (\mathbf{x}_1 - Q_s^*)^+$. When the two delays are additive, the effective total supply delay is $a_1 \cdot \min\{(Q_s^* - Q_m), (\mathbf{x}_1 - Q_m)^+\} + a_2 \cdot (\mathbf{x}_1 - Q_s^*)^+ - T_m]^+$. This additive delay model may better capture a project environment where the material consumption uncertainty is resolved gradually in distinct phases. Assuming $a_1 < a_2$, the project firm's expected time cost $C_t(Q_m, T_m)$ is altered as

$$\hat{C}_t(Q_m, T_m) = \mathbb{E}_{(\mathbf{x}_1, \mathbf{x}_2)} \left\{ b_1 T_m + b_2 [\mathbf{x}_2 + a_1 \cdot \min\{(Q_s^* - Q_m), (\mathbf{x}_1 - Q_m)^+\} + a_2 \cdot (\mathbf{x}_1 - Q_s^*)^+ - T_m]^+ \right\},$$

and the expected material cost $C_q(Q_m)$ stays the same as defined in Eqn (1b). For the project firm's profit function and optimal solutions in this additive delay model, we apply a hat accent ("ˆ") to differentiate them from the previous model. Hence the project firm's expected profit is denoted by $\hat{\pi}_m(Q_m, T_m) = V - C_q(Q_m) - \hat{C}_t(Q_m, T_m)$. We note that the supplier's problem remains the same as before, and therefore we keep using the same notation of $\pi_s(Q_m)$ and Q_s^* to denote the supplier's expected profit and production decision in this additive model.

To find the project firm's optimal decisions in this model, we let region A and B be the same as is defined in Section 4. When the project firm's decision is constrained in region A, the contracted supplier will set its production quantity to be equal to the project firm's advance order, that is, $Q_s^* = Q_m$. As a result, we have $\hat{C}_t(Q_m, T_m) = C_t(Q_m, T_m)$ and $\hat{\pi}_m^A(Q_m, T_m) = \pi_m^A(Q_m, T_m)$. Therefore $(\hat{Q}_m^A, \hat{T}_m^A) = (Q_m^A, T_m^A)$. In other words, when the project firm pushes the supply chain (taking a decision from region A), the delay from the contracted supplier is immaterial (no safety stock is held by the supplier). Therefore the additive delay model and the maximum delay model are identical. Below we give an update of the optimal decision of the project firm for region B (Proposition 8) and region $A \cup B$ (Theorem 3).

PROPOSITION 8. *$\hat{\pi}_m^B(Q_m, T_m)$ is concave in (Q_m, T_m). Let $w_b = c + a_1 b_2 H_2(\tilde{q}_s, \tilde{t}_s)$. If $w \leq w_b$, the optimal decision of the project firm in region B is $(\hat{Q}_m^B, \hat{T}_m^B) = (\tilde{q}_s, \tilde{t}_s)$; otherwise, $(\hat{Q}_m^B, \hat{T}_m^B) = (\hat{q}^B, \hat{t}^B)$ where $(\hat{q}^B, \hat{t}^B)$ is the unique solution of (q, t) to the following system of equations:*

$$\begin{cases} w_1 H_1(q) + a_1 b_2 H_4(q, t) + a_1 b_2 H_5(q, t) - w = 0 \\ b_2 H_3(q, t) + b_2 H_4(q, t) + b_2 H_5(q, t) - b_1 = 0 \end{cases} \tag{10}$$

Moreover, $(\hat{q}^B, \hat{t}^B) > (q^B, t^B)$.

Proposition 8 indicates that the two decisions of the project firm, Q_m and T_m, can no longer be decoupled in region B, and they have to be jointly solved from Eqn (10). The proposition also indicates that under the additive delay model, the project firm would bear more inventory risk in the chain ($\hat{q}^B > q^B$) and set a longer project due date ($\hat{t}^B > t^B$) when the supply chain is restricted to operate under the risk-sharing mode. Next we provide the unified optimal solution of the project firm for region $A \cup B$ in Theorem 3.

THEOREM 3. *The optimal solution $(\hat{Q}_m^*, \hat{T}_m^*)$ of the project firm can be characterized as follows: for any w_1, there exists some unique $\hat{w} \in (w_b, w_a)$ satisfying $\hat{w} > \widetilde{w}$ such that*

(1) for contracts in $\{(w, w_1) : w < \hat{w}\}$, $(\hat{Q}_m^*, \hat{T}_m^*) = (q^A, t^A)$;

(2) for contracts in $\{(w, w_1) : w = \hat{w}\}$, $(\hat{Q}_m^*, \hat{T}_m^*) = (q^A, t^A)$ or $(\hat{Q}_m^*, \hat{T}_m^*) = (\hat{q}^B, \hat{t}^B)$;

(3) for contracts in $\{(w, w_1) : w > \hat{w}\}$, $(\hat{Q}_m^*, \hat{T}_m^*) = (\hat{q}^B, \hat{t}^B)$.

7 Conclusion

Project firms, whether producing complex ETO products, such as Nooter/Eriksen (N/E), or producing MTO products, such as Emerson (flow control division) and Belden (industrial automation), heavily depend on outsourced critical materials for on-time completion of their projects. Projects are not only subject to activity time variations, but also to other forms of disruptions, such as engineering design changes, quality problems, and low-yield occurrences. These disruptions often impact the material consumption level and may induce additional activity time for a project. If additional materials are caused by these disruptions and they are not planned for, material shortages will occur that can further extend the project duration. As a result, successful project execution needs to encompass an integrated risk management framework that mitigates both activity time and material consumption uncertainties that are prominent in various project settings.

Risk management of project supply chains has been understudied in the supply chain and project management literature. In our paper, we employ a highly stylized model to examine the management of project supply chains facing project activity time and material consumption uncertainties. We allow an arbitrary level of correlation between these uncertainties, which is able to capture all types of disruptions and risks in a project supply chain environment. We provide valuable insights on how a portfolio of risk management strategies, such as time buffers, firm inventory, supplier safety stock, and expedited capacity, can be deployed collectively to effectively mitigate project uncertainties and risks in disruptive environments.

Our model analyzes a two-stage project supply chain consisting of a project firm delivering a custom project that requires a key material supplied by a main supplier under a flexible wholesale price contract. In anticipation of the project activity time and material consumption uncertainties, the project firm decides its project due date and advance material order for the project. Once receiving the advance order, the supplier decides on its production and any safety stock that allows for risk-sharing with the project firm and protects the firm from severe material shortages and project delays.

We start our analysis with the supplier's optimal production decision given an arbitrary advance order quantity from the project firm. We find that the supplier's optimal production is governed by a newsvendor solution with a production constraint. Incorporating the supplier's optimal production decision, we then derive the optimal decisions of the project firm. To do this, we take advantage of the fact that the optimization problem naturally decomposes into two optimization problems in two regions defined as A and B, with region A representing a push supply chain and region B a push-pull chain. Each region represents a unique risk management strategy in dealing with challenging project environments. The strategies in the two regions are differentiated mainly by the use (or not use) of risk-sharing with the supplier (e.g., the supplier produces in excess of the advance order and carries safety stock for the project firm), and the requirement (or no requirement) of interdependent optimization of the project due date and the material order decisions. By comparing the two local optimal decisions in the A and B regions, we then derive the unified optimal solution for the project firm across regions.

We further take the liberty to interpret our model results as managerial insights for two major project supply chain environments. For MTO projects, which are often characterized by low product/process complexity using standardized, mature technology and cheap-to-source materials, our results suggest that the project firm relies on its own levers to manage project risks arising from project activity time and material consumption uncertainties. The project firm uses his own inventory obtained via an advance order from the contract supplier and adjusts his project due date based on project costs and the remaining material shortage risks beyond what his inventory buffer can cover. For ETO projects, which are characterized by complex engineer-to-order products using niche technology and expensive and exotic materials, our results suggest that the project firm should fully leverage his relationship with the contract supplier and exploit the flexibility in the supply contract to induce appropriate risk-sharing within the supply chain. The project firm should set the project due date by taking into account project overhead cost, project delay cost, and the safety stock of the expensive material carried by himself and the contract supplier.

In addition to the above analysis and results, we also discuss how our project supply chain model encompasses the newsvendor supply chain as a special case. Our comparison of the models suggests that project supply chains place a greater emphasis on material availability, which results in the chain (and also the project firm) carrying more inventories when compared with the newsvendor chain.

Our extensive numerical study offers interesting and counter-intuitive managerial insights for project supply chains. First, we find that the correlation between the project activity time and material consumption uncertainties has a great impact on the project firm's decision and profit. A higher correlation between the two project uncertainties makes the risk-sharing strategy less appealing to the project firm. While a higher correlation always leads to a higher advance order quantity and less overall profit for the project firm, we find that the increased correlation may increase or decrease project due date, which is quite counter-intuitive as one would expect the due date decision to be positively correlated with the material order decision. In addition, we find that responsiveness of the backup supplier has interesting implications for the management of project supply chains, too. As the backup supplier becomes less responsive in supplying project material to the project firm, the project firm should always hold more inventory of his own, but he may increase or decrease the project due date. The increased material inventory carried by the project firm can help himself reduce the chance of encountering a material shortage and hence resorting to the backup supplier, to such an extent that the firm may be able to shorten the project due date.

Finally, we provide an extension of the project supply chain model by considering additive total supply delay when both the contract and backup suppliers are effectively used to cover material shortages in a project. We find that in this model, the project firm is willing to bear even more material risk within the supply chain, and there is more dependency between the advance material order and the project due date decisions of the project firm, especially when a push-pull strategy is preferred by the firm.

References

Assaf, Sadi A., Sadiq Al-Hejji. 2006. Causes of delay in large construction projects. *International Journal of Project Management*, 24 (4), 349-357.

Baker, Kenneth R., Gary D. Scudder. 1990. Sequencing with earliness and tardiness penalties: A review. *Operations Research*, 38 (1), 22-36.

Baker, Kenneth R., Dan Trietsch. 2018. *Principles of Sequencing and Scheduling*. 2nd ed. Wiley.

Cachon, Gérard P. 2004. The allocation of inventory risk in a supply chain: Push, pull, and advance-purchase discount contracts. *Management Science*, 50 (2), 222-238. doi:10.1287/mnsc.1030.0190.

Calvo, Eduard, Ruomeng Cui, Juan Camilo Serpa. 2019. Oversight and efficiency in public projects: A regression discontinuity analysis. *Management Science*, 65 (12), 5651-5675.

Cespedes, Frank V, Amy Klopfenstein. 2023. Belden and digital transformation: From product sales to solutions sale, HBS N9-823-002.

Chen, Shi, Hau Lee. 2017. Incentive alignment and coordination of project supply chains. *Management Science*, 63 (4), 1011-1025. doi:10.1287/mnsc.2015.2373.

Chen, Shi, Kamran Moinzadeh. 2018. Inventory control and delivery time quotation for assembly supply chains. *Operations Research*, 66 (4), 1004-1022. doi:10.1287/opre.2017.1704.

Davis, Andrew M, Elena Katok, Natalia Santamaría. 2014. Push, pull, or both? a behavioral study of how the allocation of inventory risk affects channel efficiency. *Management Science*, 60 (11), 2666-2683.

Dell, Michael. 2021. *Play Nice But Win: A CEO's Journey from Founder to Leader*. Findaway World.

Dong, Lingxiu, Kaijie Zhu. 2007. Two-wholesale-price contracts: Push, pull, and advance-purchase discount contracts. *Manufacturing & Service Operations Management*, 9 (3), 291-311. doi:10.1287/msom.1070.0151.

Gou, Qinglong, Suresh Sethi, Jinfeng Yue, Juan Zhang. 2016. Push and pull contracts in a local supply chain with an outside market. *Decision Sciences*, 47 (6), 1150-1177.

Guan, Xu, Shihua Ma, Zhe Yin. 2015. The impact of hybrid push–pull contract in a decentralized assembly system. *Omega*, 50 70-81.

Hiller, Jennifer. 2021. Wind manufacturers blown off course. *The Wall Street Journal*, .

Hou, Chengfan, Mengshi Lu. 2022. Push vs. pull contracts in retail supply chains: Risk aversion, information asymmetry, and outside opportunity. *Working paper*, .

Hughes, Michael William. 1986. Why projects fail: The effects of ignoring the obvious. *Industrial Engineering*, 18 (4), 14.

Katz, Benjamin. 2021. Airbus says it can't meet current demand for single-aisle jets. *The Wall Street Journal*, .

Kazaz, Aynur, Serdar Ulubeyli, Nihan Avvioglu Tuncbilekli. 2012. Causes of delays in construction projects in turkey. *Journal of Civil Engineering and Management*, 18 (3), 426-435. doi:10.3846/13923730.2012.698913.

Keskinocak, Pinar, Sridhar Tayur. 2004. *Due Date Management Policies*. Springer US, Boston, MA, 485-554. doi: 10.1007/978-1-4020-7953-5_12.

Kwon, H., S. Lippman, K. McCardle, C. Tang. 2010. Project management contracts with delayed payments. *Manufacturing & Service Operations Management*, 12 (4), 692–707.

Lariviere, Martin A., Evan L. Porteus. 2001. Selling to the newsvendor: An analysis of price-only contracts. *Manufacturing & Service Operations Management*, 3 (4), 293-305.

Lauren, Nicole. 2023. Supply chain issues cause major delays in miami bridge construction. CBS News Miami. URL https://www.cbsnews.com/miami/news/supply-chain-issues-cause-major-delays-in-miami-bridge-construction/.

Lee, Hau L., Kut C. So, Christopher S. Tang. 2000. The value of information sharing in a two-level supply chain. *Management Science*, 46 (5), 626-643. doi:10.1287/mnsc.46.5.626.12047. URL https://doi.org/10.1287/mnsc.46.5.626.12047.

Majid, M. Z. Abd., Ronald McCaffer. 1998. Factors of non-excusable delays that influence contractors' performance. *Journal of Management in Engineering*, 14 (3), 42-49. doi:10.1061/(ASCE)0742-597X(1998)14:3(42).

Rahman, M M, Y H Yap, N R Ramli, M A Dullah, M S W Shamsuddin. 2017. Causes of shortage and delay in material supply: a preliminary study. *IOP Conference Series: Materials Science and Engineering*, 271 (1), 012037.

Sanders, Peter, Doug Cameron. 2011. Boeing sees long-delayed 787 deliveries starting in third quarter. *Wall Street Journal*, .

Savaşaneril, Seçil, Paul M. Griffin, Pınar Keskinocak. 2010. Dynamic lead-time quotation for an m/m/1 base-stock inventory queue. *Operations Research*, 58 (2), 383-395. doi:10.1287/opre.1090.0717.

Seifert, Ralf W., Richard Markoff. 2017. *Project supply chains: A different world*. IMD working paper. URL https://www.imd.org/research-knowledge/articles/project-supply-chains-a-different-world/.

Simpson, Maddy. 2023. Held hostage by shortages: St. louis construction firms said supply chain issues have improved-just not enough. *St. Louis Business Journal*, April 7-13.

Spurr, Ben, Edward Keenan, Marco Chown Oved, Jayme Poisson, Marina Jimenez, David Rider. 2017. Inside bombardier's delayed streetcar deliveries: Not in service. *Toronto Star*, URL https://projects.thestar.com/bombardier-ttc/.

Tang, Wenjie, Karan Girotra. 2017. Using advance purchase discount contracts under uncertain information acquisition cost. *Production and Operations Management*, 26 (8), 1553-1567.

The 9th Global Project Management Survey. 2017. Success rate rise: Transform the high cost of low performance. Project Management Institute. URL https://www.pmi.org/-/media/pmi/documents/public/pdf/learning/thought-leadership/pulse/pulse-of-the-profession-2017.pdf.

Wang, Yulan, Baozhuang Niu, Pengfei Guo. 2014. The comparison of two vertical outsourcing structures under push and pull contracts. *Production and Operations Management*, 23 (4), 610-625.

Wei, Zhimin, Yun Huang. 2022. Supply chain coordination under carbon emission tax regulation considering greening technology investment. *International Journal of Environmental Research and Public Health*, 19 (15), 9232.

Xia, Yu, Bintong Chen, Jinfeng Yue. 2008. Job sequencing and due date assignment in a single machine shop with uncertain processing times. *European Journal of Operational Research*, 184 (1), 63-75.

Xie, Jinxing, Deming Zhou, Jerry C. Wei, Xiande Zhao. 2010. Price discount based on early order commitment in a single manufacturer–multiple retailer supply chain. *European Journal of Operational Research*, 200 (2), 368-376.

Zhu, Guidong, Jonathan F. Bard, Gang Yu. 2007. A two-stage stochastic programming approach for project planning with uncertain activity durations. *Journal of Scheduling*, 10 (3), 167-180.

E-Companion

The following is an E-Companion for the article titled "Managing material shortages in project supply chains: inventories, time buffers and supplier flexibility".

EC.1 The Heaviside function and its properties.

The Heaviside function:

$$u(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

The derivative of $u(x)$ is the Dirac delta function $\delta(x)$, which satisfies the following two properties:

(1) $\delta(x)$ is almost 0 everywhere except at $x = 0$;

$$\delta(x) = \begin{cases} +\infty & \text{if } x = 0 \\ 0 & \text{if } x \neq 0 \end{cases}$$

(2) for any $\varepsilon > 0$, $\int_{x_0-\varepsilon}^{x_0+\varepsilon} \delta(x-x_0)g(x)dx = g(x_0)$. In particular, if $g(x) \equiv 1$, we have $\int_{-\infty}^{+\infty} \delta(x-x_0)dx = 1$.

EC.2 An illustration of the H_i functions.

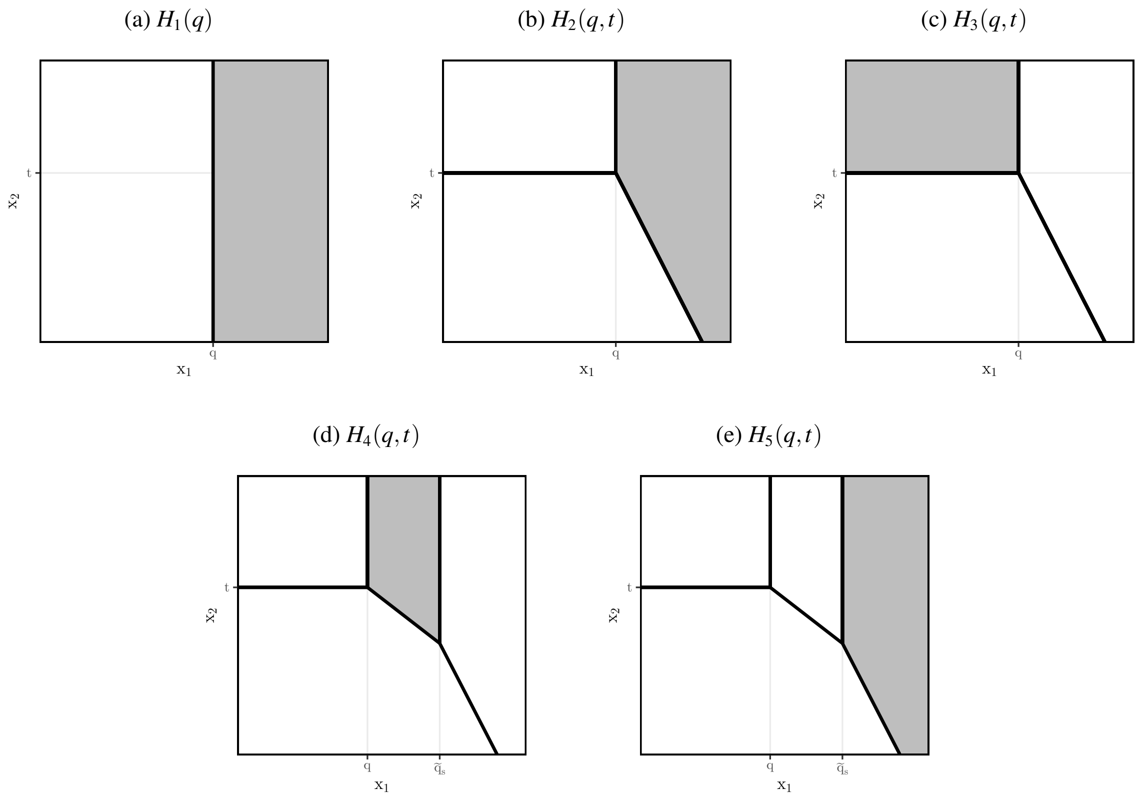
We provide an illustration of the $H_i(\cdot)$ functions in Figure EC.1. The value of a $H_i(\cdot)$ function equals the probability that $(\mathbf{x}_1, \mathbf{x}_2)$ falls into the shaded area in the corresponding figure for the $H_i(\cdot)$ function.

EC.3 Proofs of results.

Proof of Proposition 1. We restrict Q_s to $Q_s \geq Q_m$ throughout this proof. When $Q_s \geq Q_m$, we have $\min(Q_s - Q_m, (x_1 - Q_m)^+) = (x_1 - Q_m)^+ - (x_1 - Q_s)^+$. Hence, the contracted supplier's profit function $\pi_s(Q_s)$ can be simplied as

$$\begin{aligned}
\pi_s(Q_s) &= \mathbb{E}_{\mathbf{x}_1}\{wQ_m + w_1(\mathbf{x}_1 - Q_m)^+ - w_1(\mathbf{x}_1 - Q_s)^+ - cQ_s\} \\
&= \mathbb{E}_{\mathbf{x}_1}\{wQ_m + w_1(\mathbf{x}_1 - Q_m)^+ - w_1 u(\mathbf{x}_1 - Q_s)(\mathbf{x}_1 - Q_s) - cQ_s\} \\
\frac{d\pi_s(Q_s)}{dQ_s} &= \mathbb{E}_{\mathbf{x}_1}\{-w_1\delta(\mathbf{x}_1 - Q_s)(-1)(\mathbf{x}_1 - Q_s) - w_1 u(\mathbf{x}_1 - Q_s)(-1) - c\} \\
&= \mathbb{E}_{\mathbf{x}_1}\{w_1 u(\mathbf{x}_1 - Q_s) - c\} \\
&= w_1 \iint u(x_1 - Q_s) f(x_1, x_2) dx_1 dx_2 - c \\
\frac{d^2\pi_s(Q_s)}{dQ_s^2} &= w_1 \iint [\delta(x_1 - Q_s)(-1)] f(x_1, x_2) dx_1 dx_2 \\
&= -w_1 \int f(Q_s, x_2) dx_2 \\
&\leq 0
\end{aligned}$$

Figure EC.1 An illustration of the H_i functions



Hence $\pi_s(Q_s)$ is concave. The unconstrained optimal solution that maximizes $\pi_s(Q_s)$ is denoted by $\tilde{q}_s$, which can be solved by the first order condition $\frac{d\pi_s(Q_s)}{dQ_s} = 0$. As a result, the constrained optimal solution where $Q_s^* \geq Q_m$ is $Q_s^* = \max\{Q_m, \tilde{q}_s\}$.

■

Proof of Proposition 2. We write out $\pi_m^A(Q_m, T_m)$ explicitly as follows

$$\begin{aligned}
\pi_m^A(Q_m,T_m) &= \mathbb{E}_{\mathbf{x}_1,\mathbf{x}_2}\ \{V - wQ_m - w_2(\mathbf{x}_1 - Q_m)^+ - b_1T_m - b_2[\mathbf{x}_2 + a_2(\mathbf{x}_1 - Q_m)^+ - T_m]^+\} \\
&= \mathbb{E}_{\mathbf{x}_1,\mathbf{x}_2}\ \{V - wQ_m - w_2u(\mathbf{x}_1 - Q_m)[\mathbf{x}_1 - Q_m] - b_1T_m \\
&\qquad -b_2u(\mathbf{x}_1 - Q_m)[\mathbf{x}_2 + a_2(\mathbf{x}_1 - Q_m) - T_m]^+ - b_2[1 - u(\mathbf{x}_1 - Q_m)][\mathbf{x}_2 - T_m]^+\} \\
&= \mathbb{E}_{\mathbf{x}_1,\mathbf{x}_2}\ \{V - wQ_m - w_2u(\mathbf{x}_1 - Q_m)[\mathbf{x}_1 - Q_m] - b_1T_m \\
&\qquad -b_2u(\mathbf{x}_1 - Q_m)u(\mathbf{x}_2 + a_2(\mathbf{x}_1 - Q_m) - T_m)[\mathbf{x}_2 + a_2(\mathbf{x}_1 - Q_m) - T_m] \\
&\qquad -b_2[1 - u(\mathbf{x}_1 - Q_m)]u(\mathbf{x}_2 - T_m)[\mathbf{x}_2 - T_m]\}
\end{aligned} \tag{EC.1}$$

Concavity of $\pi_m^A(Q_m,T_m)$. We show the concavity of $\pi_m^A(Q_m,T_m)$ by showing its Hessian matrix is negative semi-definite. We first derive the first order derivative of $\pi_m^A(Q_m,T_m)$ below.

$$\frac{\partial \pi_m^A(Q_m,T_m)}{\partial Q_m} = \mathbb{E}_{\mathbf{x}_1,\mathbf{x}_2}\{-w - w_2\delta(\mathbf{x}_1 - Q_m)(-1)\mathbf{x}_1 - Q_m] - w_2u(\mathbf{x}_1 - Q_m)[-1]$$

$$
\begin{aligned}
&-b_2\delta(\mathbf{x}_1-Q_m)(-1)u(\mathbf{x}_2+a_2(\mathbf{x}_1-Q_m)-T_m)[\mathbf{x}_2+a_2(\mathbf{x}_1-Q_m)-T_m] \\
&-b_2u(\mathbf{x}_1-Q_m)\delta(\mathbf{x}_2+a_2(\mathbf{x}_1-Q_m)-T_m)(-a_2)[\mathbf{x}_2+a_2(\mathbf{x}_1-Q_m)-T_m] \\
&-b_2u(\mathbf{x}_1-Q_m)u(\mathbf{x}_2+a_2(\mathbf{x}_1-Q_m)-T_m)[-a_2] \\
&-b_2[-\delta(\mathbf{x}_1-Q_m)(-1)]u(\mathbf{x}_2-T_m)[\mathbf{x}_2-T_m]\} \\
=\ & \mathbb{E}_{\mathbf{x}_1,\mathbf{x}_2}\{-w+w_2u(\mathbf{x}_1-Q_m)+a_2b_2u(\mathbf{x}_1-Q_m)u(\mathbf{x}_2+a_2(\mathbf{x}_1-Q_m)-T_m)\} \\
=\ & -w+w_2\iint u(x_1-Q_m)f(x_1,x_2)dx_1dx_2 \\
&+a_2b_2\iint u(x_1-Q_m)u(x_2+a_2(x_1-Q_m)-T_m)f(x_1,x_2)dx_1dx_2
\end{aligned}
$$

where the first equality holds by interchanging derivative and expectation, the second equality holds due to

(1) $\mathbb{E}_{\mathbf{x}_1}\{\delta(\mathbf{x}_1-Q_m)[\mathbf{x}_1-Q_m]\}=0$. This is because

$$
\begin{aligned}
\mathbb{E}_{\mathbf{x}_1}\{\delta(\mathbf{x}_1-Q_m)[\mathbf{x}_1-Q_m]\} &= \iint \delta(x_1-Q_m)(x_1-Q_m)f(x_1,x_2)dx_1dx_2 \\
&= \int (Q_m-Q_m)f(Q_m,x_2)dx_2 \\
&= 0
\end{aligned}
$$

(2) Similarly, $\mathbb{E}_{\mathbf{x}_1,\mathbf{x}_2}\{a_2b_2u(\mathbf{x}_1-Q_m)\delta(\mathbf{x}_2+a_2(\mathbf{x}_1-Q_m)-T_m)[\mathbf{x}_2+a_2(\mathbf{x}_1-Q_m)-T_m]\}=0$.

(3) In addition, we have

$$
\begin{aligned}
&\mathbb{E}_{\mathbf{x}_1,\mathbf{x}_2}\{b_2\delta(\mathbf{x}_1-Q_m)u(\mathbf{x}_2+a_2(\mathbf{x}_1-Q_m)-T_m)[\mathbf{x}_2+a_2(\mathbf{x}_1-Q_m)-T_m] \\
&\qquad -b_2\delta(\mathbf{x}_1-Q_m)u(\mathbf{x}_2-T_m)[\mathbf{x}_2-T_m]\} \\
=&\iint b_2\delta(x_1-Q_m)u(x_2+a_2(x_1-Q_m)-T_m)[x_2+a_2(x_1-Q_m)-T_m]f(x_1,x_2)dx_1dx_2 \\
&-\iint b_2\delta(x_1-Q_m)u(x_2-T_m)[x_2-T_m]f(x_1,x_2)dx_1dx_2 \\
=&\int b_2u(x_2+a_2\cdot 0-T_m)[x_2+a_2\cdot 0-T_m]f(Q_m,x_2)dx_2-\int b_2u(x_2-T_m)[x_2-T_m]f(Q_m,x_2)dx_2 \\
=&0
\end{aligned}
$$

and the last equality holds by writing the expectation as integration. By applying the same technique, for the first order derivative with respect to T_m, we have

$$
\begin{aligned}
\frac{\partial\pi_m^A(Q_m,T_m)}{\partial T_m} =\ & \mathbb{E}_{\mathbf{x}_1,\mathbf{x}_2}\{-b_1-b_2u(\mathbf{x}_1-Q_m)\delta(\mathbf{x}_2+a_2(\mathbf{x}_1-Q_m)-T_m)(-1)[\mathbf{x}_2+a_2(\mathbf{x}_1-Q_m)-T_m] \\
&-b_2u(\mathbf{x}_1-Q_m)u(\mathbf{x}_2+a_2(\mathbf{x}_1-Q_m)-T_m)[-1] \\
&-b_2[1-u(\mathbf{x}_1-Q_m)]\delta(\mathbf{x}_2-T_m)(-1)[\mathbf{x}_2-T_m] \\
&-b_2[1-u(\mathbf{x}_1-Q_m)]u(\mathbf{x}_2-T_m)[-1]\} \\
=\ & \mathbb{E}_{\mathbf{x}_1,\mathbf{x}_2}\{-b_1+b_2u(\mathbf{x}_1-Q_m)u(\mathbf{x}_2+a_2(\mathbf{x}_1-Q_m)-T_m)+b_2[1-u(\mathbf{x}_1-Q_m)]u(\mathbf{x}_2-T_m)\} \\
=\ & -b_1+b_2\iint u(x_1-Q_m)u(x_2+a_2(x_1-Q_m)-T_m)f(x_1,x_2)dx_1dx_2 \\
&+b_2\iint[1-u(x_1-Q_m)]u(x_2-T_m)f(x_1,x_2)dx_1dx_2
\end{aligned}
$$

Next we show that $\pi_m^A(Q_m, T_m)$ is concave by showing its Hessian matrix $\mathbf{H}_A$ is negative semi-definite. The Hessian matrix of $\pi_m^A(Q_m, T_m)$ is defined as

$$\mathbf{H}_A = \begin{bmatrix} \frac{\partial^2 \pi_m^A(Q_m, T_m)}{\partial Q_m^2} & \frac{\partial^2 \pi_m^A(Q_m, T_m)}{\partial Q_m \partial T_m} \\ \frac{\partial^2 \pi_m^A(Q_m, T_m)}{\partial T_m \partial Q_m} & \frac{\partial^2 \pi_m^A(Q_m, T_m)}{\partial T_m^2} \end{bmatrix}$$

The elements of H_A are derived below:

$$\begin{aligned}
\frac{\partial^2 \pi_m^A(Q_m, T_m)}{\partial Q_m^2} &= \mathbb{E}_{\mathbf{x}_1,\mathbf{x}_2}\{w_2\delta(\mathbf{x}_1 - Q_m)(-1) + a_2b_2\delta(\mathbf{x}_1 - Q_m)(-1)u(\mathbf{x}_2 + a_2(\mathbf{x}_1 - Q_m) - T_m) \\
&\qquad + a_2b_2u(\mathbf{x}_1 - Q_m)\delta(\mathbf{x}_2 + a_2(\mathbf{x}_1 - Q_m) - T_m)(-a_2)\} \\
&= -w_2\int f(Q_m, x_2)dx_2 - a_2b_2\int u(x_2 - T_m)f(Q_m, x_2)dx_2 \\
&\qquad - a_2^2b_2\int u(x_1 - Q_m)f(x_1, T_m - a_2(x_1 - Q_m))dx_1 \\
\frac{\partial^2 \pi_m^A(Q_m, T_m)}{\partial T_m^2} &= \mathbb{E}_{\mathbf{x}_1,\mathbf{x}_2}\{b_2u(\mathbf{x}_1 - Q_m)\delta(\mathbf{x}_2 + a_2(\mathbf{x}_1 - Q_m) - T_m)(-1) + b_2[1 - u(\mathbf{x}_1 - Q_m)]\delta(\mathbf{x}_2 - T_m)(-1)\} \\
&= -b_2\int u(x_1 - Q_m)f(x_1, T_m - a_2(x_1 - Q_m))dx_1 - b_2\int [1 - u(x_1 - Q_m)]f(x_1, T_m)dx_1 \\
\frac{\partial^2 \pi_m^A(Q_m, T_m)}{\partial Q_m \partial T_m} &= \mathbb{E}_{\mathbf{x}_1,\mathbf{x}_2}\{-a_2b_2u(\mathbf{x}_1 - Q_m)\delta(\mathbf{x}_2 + a_2(\mathbf{x}_1 - Q_m) - T_m)\} \\
&= \iint -a_2b_2u(x_1 - Q_m)\delta(x_2 + a_2(x_1 - Q_m) - T_m)f(x_1, x_2)dx_1dx_2 \\
&= -a_2b_2\int u(x_1 - Q_m)f(x_1, T_m - a_2(x_1 - Q_m))dx_1 \\
\frac{\partial^2 \pi_m^A(Q_m, T_m)}{\partial T_m \partial Q_m} &= -a_2b_2\int u(x_1 - Q_m)f(x_1, T_m - a_2(x_1 - Q_m))dx_1
\end{aligned}$$

To show $\mathbf{H}_A$ is negative semi-definite, it is sufficient to show $\mathbf{z}^T\mathbf{H}_A\mathbf{z} \leq 0$ for any $\mathbf{z} = (z_1, z_2) \in \mathbb{R}^2$. This is completed below.

$$\begin{aligned}
\mathbf{z}^T\mathbf{H}_A\mathbf{z} &= \left(\frac{\partial^2 \pi_m^A(Q_m, T_m)}{\partial Q_m^2}\right)z_1^2 + 2\left(\frac{\partial^2 \pi_m^A(Q_m, T_m)}{\partial Q_m \partial T_m}\right)z_1z_2 + \left(\frac{\partial^2 \pi_m^A(Q_m, T_m)}{\partial T_m^2}\right)z_2^2 \\
&= -\left[w_2\int f(Q_m, x_2)dx_2 + a_2b_2\int u(x_2 - T_m)f(Q_m, x_2)dx_2\right]z_1^2 \\
&\quad - \left[b_2\int [1 - u(x_1 - Q_m)]f(x_1, T_m)dx_1\right]z_2^2 \\
&\quad - \left[b_2\int u(x_1 - Q_m)f(x_1, T_m - a_2(x_1 - Q_m))dx_1\right](a_2z_1 + z_2)^2 \\
&\leq 0
\end{aligned}$$

Convex Optimization. Now that we have shown $\pi_m^A(Q_m, T_m)$ is convave, since the constraint $(Q_m, T_m) \in A$ is a convex set, we can solve the constrained maximization problem by applying the lagrangian approach.

Specifically, let $L(Q_m, T_m, \lambda_Q, \lambda_T) = \pi_m^A(Q_m, T_m) + \lambda_Q(Q_m - \tilde{q}_s) + \lambda_T T_m$, where λ_Q, λ_T are the lagrangian multipliers. The optimal solution (Q_m^A, T_m^A) can be found by solving the following equations:

$$\frac{\partial \pi_m^A(Q_m, T_m)}{\partial Q_m} + \lambda_Q = 0 \tag{EC.2a}$$

$$\frac{\partial \pi_m^A(Q_m, T_m)}{\partial T_m} + \lambda_T = 0 \tag{EC.2b}$$

$$\lambda_Q(Q_m - \tilde{q}_s) = 0 \tag{EC.2c}$$

$$\lambda_T T_m = 0 \tag{EC.2d}$$

$$\lambda_Q, \lambda_T, (Q_m - \tilde{q}_s), T_m \geq 0 \tag{EC.2e}$$

Let $(Q_m^A, T_m^A, \lambda_Q^A, \lambda_T^A)$ be a solution satisfying eqn (EC.2a) - (EC.2e). The solution must satisfy the following conditions:

(a) $\lambda_T^A = 0$. We prove this by contradiction. Suppose $\lambda_T^A > 0$, since $\lambda_T^A T_m^A = 0$, we must have $T_m^A = 0$. Therefore,

$$\begin{aligned}
&\frac{\partial \pi_m^A(Q_m, T_m)}{\partial T_m} + \lambda_T^A \\
=& -b_1 + b_2 \iint u(x_1 - Q_m^A) u(x_2 + a_2(x - Q_m^A) - T_m^A) f(x_1, x_2) dx_1 dx_2 \\
&+ b_2 \iint [1 - u(x_1 - Q_m^A)] u(x_2 - T_m^A) f(x_1, x_2) dx_1 dx_2 + \lambda_T^A \\
=& -b_1 + b_2 + \lambda_T^A \\
>& \ 0
\end{aligned}$$

which contradicts with Eqn (EC.2b).

(b) $\lambda_Q^A > 0$ if and only if $w > \frac{cw_2}{w_1} + a_2 b_2 \iint u(x_1 - \tilde{q}_s) u(x_2 + a_2(x_1 - \tilde{q}_s) - \tilde{t}_s) f(x_1, x_2) dx_1 dx_2$, where $\tilde{t}_s$ is the unique solution of t to the following equation:

$$\begin{aligned}
&- b_1 + b_2 \iint u(x_1 - \tilde{q}_s) u(x_2 + a_2(x_1 - \tilde{q}_s) - t) f(x_1, x_2) dx_1 dx_2 \\
&\quad + b_2 \iint [1 - u(x_1 - \tilde{q}_s)] u(x_2 - t) f(x_1, x_2) dx_1 dx_2 = 0
\end{aligned}$$

First we show $\lambda_Q^A > 0 \Rightarrow w > \frac{cw_2}{w_1} + a_2 b_2 \iint u(x_1 - \tilde{q}_s) u(x_2 + a_2(x_1 - \tilde{q}_s) - \tilde{t}_s) f(x_1, x_2) dx_1 dx_2$. From Eqn (EC.2c), we get $Q_m^A = \tilde{q}_s$. Then from Eqn (EC.2a), we get the w inequality.

Second we show $\lambda_Q^A > 0 \Leftarrow w > \frac{cw_2}{w_1} + a_2 b_2 \iint u(x_1 - \tilde{q}_s) u(x_2 + a_2(x_1 - \tilde{q}_s) - \tilde{t}_s) f(x_1, x_2) dx_1 dx_2$. We prove by contradiction. Suppose $\lambda_Q^A > 0$ does not hold, e.g., $\lambda_Q^A = 0$. If this is the case, then (Q_m^A, T_m^A) satisfy Eqn (EC.3). Therefore,

$$\begin{aligned}
w = w_2 &\iint u(x_1 - Q_m^A) f(x_1, x_2) dx_1 dx_2 + \\
&\quad a_2 b_2 \iint u(x_1 - Q_m^A) u(x_2 + a_2(x_1 - Q_m^A) - T_m^A) f(x_1, x_2) dx_1 dx_2
\end{aligned}$$

$$\leq w_2 \iint u(x_1 - \tilde{q}_s) f(x_1, x_2) dx_1 dx_2 + a_2 b_2 \iint u(x_1 - \tilde{q}_s) u(x_2 + a_2(x_1 - \tilde{q}_s) - \tilde{t}_s) f(x_1, x_2) dx_1 dx_2$$
$$= \frac{cw_2}{w_1} + a_2 b_2 \iint u(x_1 - \tilde{q}_s) u(x_2 + a_2(x_1 - \tilde{q}_s) - \tilde{t}_s) f(x_1, x_2) dx_1 dx_2$$

where the inequality holds by exploiting the relationship between (Q_m^A, T_m^A) and $(\tilde{q}_s, \tilde{t}_s)$ (It can be shown by a graphical approach and is omitted). This contradicts with the w condition.

Combining the above (a) - (b) results, we have that: if $w > \frac{cw_2}{w_1} + a_2 b_2 \iint u(x_1 - \tilde{q}_s) u(x_2 + a_2(x_1 - \tilde{q}_s) - \tilde{t}_s) f(x_1, x_2) dx_1 dx_2$, we have $Q_m^A = \tilde{q}_s$, and $T_m^A = \tilde{t}_s$; otherwise, (Q_m^A, T_m^A) can be obtained by solving a system of equations: $\frac{\partial \pi_m^A(Q_m, T_m)}{\partial Q_m} = 0$, $\frac{\partial \pi_m^A(Q_m, T_m)}{\partial T_m} = 0$, which is the unique (q, t) to the following system of equations:

$$\begin{cases} w_2 \iint u(x_1 - q) f(x_1, x_2) dx_1 dx_2 \\ \quad + a_2 b_2 \iint u(x_1 - q) u(x_2 + a_2(x_1 - q) - t) f(x_1, x_2) dx_1 dx_2 - w & = 0 \\ b_2 \iint u(x_1 - q) u(x_2 + a_2(x_1 - q) - t) f(x_1, x_2) dx_1 dx_2 \\ \quad + b_2 \iint [1 - u(x_1 - q)] u(x_2 - t) f(x_1, x_2) dx_1 dx_2 - b_1 & = 0 \end{cases} \tag{EC.3}$$

By the definition of H_1, H_2 and H_3, it completes the proof of the proposition.

■

Proof of Proposition 3. We first note that $Q_m^A = \max\{q^A, \tilde{q}_s\}$, $T_m^A = \min\{t^A, \tilde{t}_s\}$. Before we prove the proposition, we first prove the following claim that we will make use of.

CLAIM EC.1. *For the (q^A, t^A) satisfying Eqn. (6), we have*

(1) $\frac{\partial H_1(q^A)}{\partial q^A} < 0$, $\frac{\partial H_1(q^A)}{\partial t^A} = 0$;

(2) $\frac{\partial H_2(q^A, t^A)}{\partial q^A} < 0$, $\frac{\partial H_2(q^A, t^A)}{\partial t^A} < 0$;

(3) $\frac{\partial H_3(q^A, t^A)}{\partial q^A} > 0$, $\frac{\partial H_3(q^A, t^A)}{\partial t^A} < 0$.

Proof of Claim EC.1. We prove case (3) only. The other cases can be proved by the same approach.

$$\begin{aligned} \frac{\partial H_3(q^A, t^A)}{\partial q^A} &= \iint [-\delta(x_1 - q^A)(-1)] u(x_2 - t^A) f(x_1, x_2) dx_1 dx_2 \\ &= \int u(x_2 - t^A) f(q^A, x_2) dx_2 > 0 \\ \frac{\partial H_3(q^A, t^A)}{\partial t^A} &= \iint [1 - u(x_1 - q^A)] \delta(x_2 - t^A)(-1) f(x_1, x_2) dx_1 dx_2 \\ &= -\int [1 - u(x_1 - q^A)] f(x_1, t^A) dx_1 < 0 \end{aligned}$$

Next we prove the proposition by considering the following cases:

(1) $\frac{dq^A}{dw}<0$, $\frac{dt^A}{dw}>0$, $\frac{d\tilde{q}_s}{dw}=0$, $\frac{d\tilde{t}_s}{dw}=0$. As a result, $\frac{dQ_m^A}{dw}\leq 0$, $\frac{dT_m^A}{dw}\geq 0$. To show this, we take the derivative with respect to w in Eqn (6) and get

$$\begin{cases} \frac{\partial}{\partial q^A}[w_2H_1(q^A)+a_2b_2H_2(q^A,t^A)]\cdot\frac{dq^A}{dw}+\frac{\partial}{\partial t^A}[w_2H_1(q^A)+a_2b_2H_2(q^A,t^A)]\cdot\frac{dt^A}{dw}-1=0 \\ \frac{\partial}{\partial q^A}[b_2H_2(q^A,t^A)+b_2H_3(q^A,t^A)]\cdot\frac{dq^A}{dw}+\frac{\partial}{\partial t^A}[b_2H_2(q^A,t^A)+b_2H_3(q^A,t^A)]\cdot\frac{dt^A}{dw}=0 \end{cases} \tag{EC.4}$$

For ease of presentation, we suppress the q^A,t^A in H_i. From the second equation in (EC.4), we have

$$\frac{dt^A}{dw}=-\frac{\frac{\partial H_2}{\partial q^A}+\frac{\partial H_3}{\partial q^A}}{\frac{\partial H_2}{\partial t^A}+\frac{\partial H_3}{\partial t^A}}\cdot\frac{dq^A}{dw} \tag{EC.5}$$

Plug it in the first equation of (EC.4), we have

$$\left[w_1\frac{\partial H_1}{\partial q^A}+a_2b_2\frac{\partial H_2}{\partial q^A}-(a_2b_2\frac{\partial H_2}{\partial t^A})\frac{\frac{\partial H_2}{\partial q^A}+\frac{\partial H_3}{\partial q^A}}{\frac{\partial H_2}{\partial t^A}+\frac{\partial H_3}{\partial t^A}}\right]\cdot\frac{dq^A}{dw}-1=0$$

which is equivalent to

$$\left[w_1\frac{\partial H_1}{\partial q^A}+a_2b_2\frac{\partial H_2}{\partial q^A}\frac{\frac{\partial H_3}{\partial t^A}}{\frac{\partial H_2}{\partial t^A}+\frac{\partial H_3}{\partial t^A}}-(a_2b_2\frac{\partial H_2}{\partial t^A})\frac{\frac{\partial H_3}{\partial q^A}}{\frac{\partial H_2}{\partial t^A}+\frac{\partial H_3}{\partial t^A}}\right]\cdot\frac{dq^A}{dw}-1=0$$

By Claim EC.1, we have the term $[\cdot]<0$. Therefore $\frac{dq^A}{dw}<0$. To prove $\frac{dt^A}{dw}>0$, we also show

$$\begin{aligned}\frac{\partial H_2}{\partial q^A}+\frac{\partial H_3}{\partial q^A} &= \iint[\delta(x_1-q^A)(-1)]u(x_2+a_2(x_1-q^A)-t^A)f(x_1,x_2)dx_1dx_2 \\ &\quad+\iint u(x_1-q^A)[\delta(x_2+a_2(x_1-q^A)-t^A)(-a_2)]f(x_1,x_2)dx_1dx_2 \\ &\quad+\iint[-\delta(x_1-q^A)(-1)]u(x_2-t^A)f(x_1,x_2)dx_1dx_2 \\ &= -\int u(x_2-t^A)f(q^A,x_2)dx_2 \\ &\quad -a_2\int u(x_1-q^A)f(x_1,t^A-a_2(x_1-q^A))dx_1 \\ &\quad +\int u(x_2-t^A)f(q^A,x_2)dx_2 \\ &< 0\end{aligned}$$

Therefore by (EC.5), we have $\frac{dt^A}{dw}>0$. Finally $\frac{d\tilde{q}_s}{dw}=0$, $\frac{d\tilde{t}_s}{dw}=0$ are trivial and thus omitted.

(2) $\frac{dq^A}{dw_1}=0$, $\frac{dt^A}{dw_1}=0$, $\frac{d\tilde{q}_s}{dw_1}>0$, $\frac{d\tilde{t}_s}{dw_1}<0$. As a result, $\frac{dQ_m^A}{dw_1}\geq 0$, $\frac{dT_m^A}{dw_1}\leq 0$. The former two cases are trivial as w_1 does not appear in Eqn (6). To show the latter two, we first take the derivative with respect to w_1 for the equation $w_1H_1(\tilde{q}_s)=c$ and get

$$H_1(\tilde{q}_s)+w_1\frac{dH_1(\tilde{q}_s)}{d\tilde{q}_s}\frac{d\tilde{q}_s}{dw_1}=0$$

where $\frac{dH_1(\tilde{q}_s)}{d\tilde{q}_s}=-\int f(\tilde{q}_s,x_2)dx_2=-f_1(\tilde{q}_s)$. Hence

$$\frac{d\tilde{q}_s}{dw_1}=\frac{H_1(\tilde{q}_s)}{w_1f_1(\tilde{q}_s)}>0 \tag{EC.6}$$

To show $\frac{d\tilde{t}_s}{dw_1} < 0$, we take derivative with respect to w_1 for Eqn (5) and get

$$\frac{\partial}{\partial \tilde{q}_s}[b_2H_2(\tilde{q}_s,\tilde{t}_s)+b_2H_3(\tilde{q}_s,\tilde{t}_s)-b_1]\cdot\frac{d\tilde{q}_s}{dw_1}+\frac{\partial}{\partial \tilde{t}_s}[b_2H_2(\tilde{q}_s,\tilde{t}_s)+b_2H_3(\tilde{q}_s,\tilde{t}_s)-b_1]\cdot\frac{d\tilde{t}_s}{dw_1}=0$$

It is easy to verify that $\frac{\partial}{\partial \tilde{q}_s}[b_2H_2(\tilde{q}_s,\tilde{t}_s)+b_2H_3(\tilde{q}_s,\tilde{t}_s)-b_1] < 0$ and $\frac{\partial}{\partial \tilde{t}_s}[b_2H_2(\tilde{q}_s,\tilde{t}_s)+b_2H_3(\tilde{q}_s,\tilde{t}_s)-b_1] < 0$. As a result, $\frac{d\tilde{t}_s}{dw_1} < 0$.

(3) $\frac{dq^A}{db_1} > 0$, $\frac{dt^A}{db_1} < 0$, $\frac{d\tilde{q}_s}{db_1} = 0$, $\frac{d\tilde{t}_s}{db_1} < 0$. As a result, $\frac{dQ_m^A}{db_1} \geq 0$, $\frac{dT_m^A}{db_1} \leq 0$. To show this, we take the derivative with respect to b_1 in Eqn (6) and get

$$\begin{cases} \frac{\partial}{\partial q^A}[w_2H_1+a_2b_2H_2]\cdot\frac{dq^A}{db_1}+\frac{\partial}{\partial t^A}[w_2H_1+a_2b_2H_2]\cdot\frac{dt^A}{db_1}=0 \\ \frac{\partial}{\partial q^A}[b_2H_2+b_2H_3]\cdot\frac{dq^A}{db_1}+\frac{\partial}{\partial t^A}[b_2H_2+b_2H_3]\cdot\frac{dt^A}{db_1}-1=0 \end{cases} \tag{EC.7}$$

From the first equation, we get

$$\frac{dq^A}{db_1} = -\frac{a_2b_2\frac{\partial H_2}{\partial t^A}}{w_2\frac{\partial H_1}{\partial q^A}+a_2b_2\frac{\partial H_2}{\partial q^A}}\cdot\frac{dt^A}{db_1} \tag{EC.8}$$

Plug it in to the second equation, we have

$$b_2\cdot\left[-(\frac{\partial H_2}{\partial q^A}+\frac{\partial H_3}{\partial q^A})\cdot\frac{a_2b_2\frac{\partial H_2}{\partial t^A}}{w_2\frac{\partial H_1}{\partial q^A}+a_2b_2\frac{\partial H_2}{\partial q^A}}+\frac{\partial H_2}{\partial t^A}+\frac{\partial H_3}{\partial t^A}\right]\cdot\frac{dt^A}{db_1}-1=0$$

which is equivalent to

$$b_2\cdot\left[\frac{\partial H_2}{\partial t^A}\cdot\frac{w_2\frac{\partial H_1}{\partial q^A}}{w_2\frac{\partial H_1}{\partial q^A}+a_2b_2\frac{\partial H_2}{\partial q^A}}-\frac{\partial H_3}{\partial q^A}\cdot\frac{a_2b_2\frac{\partial H_2}{\partial t^A}}{w_2\frac{\partial H_1}{\partial q^A}+a_2b_2\frac{\partial H_2}{\partial q^A}}+\frac{\partial H_3}{\partial t^A}\right]\cdot\frac{dt^A}{db_1}-1=0$$

By Claim EC.1, we have the term $[\cdot] < 0$. Therefore $\frac{dq^A}{db_1} < 0$. By (EC.10), $\frac{dt^A}{db_1} > 0$.

$\frac{d\tilde{q}_s}{db_1} = 0$ is trivial as $\tilde{q}_s$ does not depend on b_1. To show $\frac{d\tilde{t}_s}{db_1} < 0$, we take derivative with respect to w_1 for Eqn (5) and get

$$\frac{\partial}{\partial \tilde{q}_s}[b_2H_2(\tilde{q}_s,\tilde{t}_s)+b_2H_3(\tilde{q}_s,\tilde{t}_s)-b_1]\cdot\frac{d\tilde{q}_s}{db_1}+\frac{\partial}{\partial \tilde{t}_s}[b_2H_2(\tilde{q}_s,\tilde{t}_s)+b_2H_3(\tilde{q}_s,\tilde{t}_s)-b_1]\cdot\frac{d\tilde{t}_s}{db_1}-1=0$$

which is equivalent to

$$\left[b_2\frac{\partial H_2}{\partial \tilde{t}_s}+b_2\frac{\partial H_3}{\partial \tilde{t}_s}\right]\cdot\frac{d\tilde{t}_s}{db_1}-1=0$$

By Claim EC.1, we have the term $[\cdot] < 0$. Therefore $\frac{d\tilde{t}_s}{db_1} < 0$.

(4) $\frac{dq^A}{db_2} > 0$, $\frac{dt^A}{db_2} > 0$, $\frac{d\tilde{q}_s}{db_2} = 0$, $\frac{d\tilde{t}_s}{db_2} > 0$. As a result, $\frac{dQ_m^A}{db_2} \geq 0$, $\frac{dT_m^A}{db_2} \geq 0$. To show this, we take the derivative with respect to b_2 in Eqn (6) and get

$$\begin{cases} \frac{\partial}{\partial q^A}[w_2H_1+a_2b_2H_2]\cdot\frac{dq^A}{db_2}+\frac{\partial}{\partial t^A}[w_2H_1+a_2b_2H_2]\cdot\frac{dt^A}{db_2}+a_2H_2=0 \\ \frac{\partial}{\partial q^A}[b_2H_2+b_2H_3]\cdot\frac{dq^A}{db_2}+\frac{\partial}{\partial t^A}[b_2H_2+b_2H_3]\cdot\frac{dt^A}{db_2}+H_2+H_3=0 \end{cases} \tag{EC.9}$$

From the first equation, we get

$$\frac{dq^A}{db_2} = -\frac{a_2 H_2}{w_2 \frac{\partial H_1}{\partial q^A} + a_2 b_2 \frac{\partial H_2}{\partial q^A}} - \frac{a_2 b_2 \frac{\partial H_2}{\partial t^A}}{w_2 \frac{\partial H_1}{\partial q^A} + a_2 b_2 \frac{\partial H_2}{\partial q^A}} \cdot \frac{dt^A}{db_2} \tag{EC.10}$$

Plug it in to the second equation, we have

$$b_2 \cdot \left[\frac{\partial H_2}{\partial q^A} + \frac{\partial H_3}{\partial q^A}\right] \cdot \left[-\frac{a_2 H_2}{w_2 \frac{\partial H_1}{\partial q^A} + a_2 b_2 \frac{\partial H_2}{\partial q^A}} - \frac{a_2 b_2 \frac{\partial H_2}{\partial t^A}}{w_2 \frac{\partial H_1}{\partial q^A} + a_2 b_2 \frac{\partial H_2}{\partial q^A}} \cdot \frac{dt^A}{db_2}\right] + b_2 \cdot \left[\frac{\partial H_2}{\partial t^A} + \frac{\partial H_3}{\partial t^A}\right] \cdot \frac{dt^A}{db_2}$$
$$+H_2 + H_3 = 0$$

which is equivalent to

$$\left[H_2 \cdot \frac{w_2 \frac{\partial H_1}{\partial q^A}}{w_2 \frac{\partial H_1}{\partial q^A} + a_2 b_2 \frac{\partial H_2}{\partial q^A}} - \frac{\partial H_3}{\partial q^A} \cdot \frac{a_2 b_2 H_2}{w_2 \frac{\partial H_1}{\partial q^A} + a_2 b_2 \frac{\partial H_2}{\partial q^A}} + H_3\right]$$
$$+b_2 \left[-\frac{\partial H_3}{\partial q^A} \cdot \frac{a_2 b_2 \frac{\partial H_2}{\partial t^A}}{w_2 \frac{\partial H_1}{\partial q^A} + a_2 b_2 \frac{\partial H_2}{\partial q^A}} + \frac{\partial H_2}{\partial t^A} \cdot \frac{w_2 \frac{\partial H_1}{\partial q^A}}{w_2 \frac{\partial H_1}{\partial q^A} + a_2 b_2 \frac{\partial H_2}{\partial q^A}} + \frac{\partial H_3}{\partial t^A}\right] \cdot \frac{dt^A}{db_2} = 0$$

By Claim EC.1, we have the first term $[\cdot] > 0$, the second term $[\cdot] < 0$. Therefore $\frac{dt^A}{db_2} > 0$. Similarly we can show $\frac{dq^A}{db_2} > 0$. Moreover, $\frac{d\tilde{q}_s}{db_2} = 0$, $\frac{d\tilde{t}_s}{db_2} > 0$. We omit these proofs.

■

Proof of Proposition 4. Let $g(T_m, x_1, x_2) = x_2 + a_2(x_1 - \tilde{q}_s) - T_m$. For simplicity, we write g instead of $g(T_m, x_1, x_2)$ in the following proof. We can write the project firm's expected profit $\pi_m^B(Q_m, T_m)$ as

$$\begin{aligned}
\pi_m^B(Q_m, T_m) &= \mathbb{E}_{\mathbf{x}_1, \mathbf{x}_2}\{V - wQ_m - w_1(\mathbf{x}_1 - Q_m)^+ - (w_2 - w_1)(\mathbf{x}_1 - \tilde{q}_s)^+ \\
&\qquad - b_1 T_m - b_2[\mathbf{x}_2 + a_2(\mathbf{x}_1 - \tilde{q}_s)^+ - T_m]^+\} \\
&= \mathbb{E}_{\mathbf{x}_1, \mathbf{x}_2}\{V - wQ_m - w_1 u(\mathbf{x}_1 - Q_m)[\mathbf{x}_1 - Q_m] - (w_2 - w_1)u(\mathbf{x}_1 - \tilde{q}_s)[\mathbf{x}_1 - \tilde{q}_s] \\
&\qquad - b_1 T_m - b_2(1 - u(\mathbf{x}_1 - \tilde{q}_s))u(\mathbf{x}_2 - T_m)[\mathbf{x}_2 - T_m] - b_2 u(\mathbf{x}_1 - \tilde{q}_s)u(g)[g]\}
\end{aligned} \tag{EC.11}$$

Concavity of $\pi_m^B(Q_m, T_m)$. We show the concavity of $\pi_m^B(Q_m, T_m)$ by showing its Hessian matrix is negative semi-definite. We first derive the first order derivative of $\pi_m^B(Q_m, T_m)$ below.

$$\begin{aligned}
\frac{\partial \pi_m^B(Q_m, T_m)}{\partial Q_m} &= \mathbb{E}_{\mathbf{x}_1, \mathbf{x}_2}\{-w - w_1 \delta(\mathbf{x}_1 - Q_m)(-1)[\mathbf{x}_1 - Q_m] - w_1 u(\mathbf{x}_1 - Q_m)(-1)\} \\
&= -w + w_1 \iint u(x_1 - Q_m) f(x_1, x_2) dx_1 dx_2 \\
\frac{\partial \pi_m^B(Q_m, T_m)}{\partial T_m} &= \mathbb{E}_{\mathbf{x}_1, \mathbf{x}_2}\{-b_1 - b_2(1 - u(\mathbf{x}_1 - \tilde{q}_s))\delta(\mathbf{x}_2 - T_m)(-1)[\mathbf{x}_2 - T_m] - b_2(1 - u(\mathbf{x}_1 - \tilde{q}_s))u(\mathbf{x}_2 - T_m)[-1] \\
&\qquad - b_2 u(\mathbf{x}_1 - \tilde{q}_s)\delta(g)(-1)[g] - b_2 u(\mathbf{x}_1 - \tilde{q}_s)u(g)[-1]\} \\
&= -b_1 + b_2 \iint (1 - u(x_1 - \tilde{q}_s))u(x_2 - T_m) f(x_1, x_2) dx_1 dx_2 \\
&\qquad + b_2 \iint u(x_1 - \tilde{q}_s)u(g(T_m, x_1, x_2)) f(x_1, x_2) dx_1 dx_2
\end{aligned}$$

Second Order Condition. Next we show that $\pi_m^B(Q_m, T_m)$ is concave by showing its Hessian matrix, denoted by $\mathbf{H}_B$, is negative semi-definite. The Hessian $\mathbf{H}_B$ is defined by

$$\mathbf{H}_B = \begin{bmatrix} \frac{\partial^2 \pi_m^B(Q_m,T_m)}{\partial Q_m^2} & \frac{\partial^2 \pi_m^B(Q_m,T_m)}{\partial Q_m \partial T_m} \\ \frac{\partial^2 \pi_m^B(Q_m,T_m)}{\partial T_m \partial Q_m} & \frac{\partial^2 \pi_m^B(Q_m,T_m)}{\partial T_m^2} \end{bmatrix}$$

where

$$\begin{aligned}
\frac{\partial^2 \pi_m^B(Q_m,T_m)}{\partial Q_m^2} &= w_1 \iint \delta(x_1 - Q_m)(-1) f(x_1,x_2) dx_1 dx_2 \\
&= -w_1 \int f(Q_m, x_2) dx_2 \\
\frac{\partial^2 \pi_m^B(Q_m,T_m)}{\partial T_m^2} &= b_2 \iint (1 - u(x_1 - \tilde{q}_s)) \delta(x_2 - T_m)(-1) f(x_1,x_2) dx_1 dx_2 \\
&\quad + b_2 \iint u(x_1 - \tilde{q}_s) \delta(g(Q_m, T_m, x_1, x_2))(-1) f(x_1,x_2) dx_1 dx_2 \\
&= -b_2 \int (1 - u(x_1 - \tilde{q}_s)) f(x_1, T_m) dx_1 \\
&\quad - b_2 \int u(x_1 - \tilde{q}_s) f(x_1, T_m - a_2(x_1 - \tilde{q}_s)) dx_1 \\
\frac{\partial^2 \pi_m^B(Q_m,T_m)}{\partial Q_m \partial T} &= 0
\end{aligned}$$

To show $\mathbf{H}_B$ is negative semi-definite, it is sufficient to show $\mathbf{z}^T \mathbf{H}_B \mathbf{z} \leq 0$ for any $\mathbf{z} = (z_1, z_2) \in \mathbb{R}^2$.

$$\begin{aligned}
\mathbf{z}^T \mathbf{H}_B \mathbf{z} &= \left(\frac{\partial^2 \pi_m^B(Q_m,T_m)}{\partial Q_m^2}\right) z_1^2 + 2\left(\frac{\partial^2 \pi_m^B(Q_m,T_m)}{\partial Q_m \partial T_m}\right) z_1 z_2 + \left(\frac{\partial^2 \pi_m^B(Q_m,T_m)}{\partial T_m^2}\right) z_2^2 \\
&= -\left[w_1 \int f(Q_m, x_2) dx_2 \right] z_1^2 \\
&\quad - \left[b_2 \int (1 - u(x_1 - \tilde{q}_s)) f(x_1, T_m) dx_1 + b_2 \int u(x_1 - \tilde{q}_s) f(x_1, T_m - a_2(x_1 - \tilde{q}_s)) dx_1 \right] z_2^2 \\
&\leq 0
\end{aligned}$$

Convex Optimization. Let

$$L(Q_m, T_m, \lambda_{Q,1}, \lambda_{Q,2}, \lambda_T) = \pi_m^B(Q_m, T_m) + \lambda_{Q,1} Q_m + \lambda_{Q,2}(\tilde{q}_s - Q_m) + \lambda_T T_m$$

The optimal solution in Region B has to satisfy the following equations:

$$\frac{\partial \pi_m^B(Q_m,T_m)}{\partial Q_m} + \lambda_{Q,1} - \lambda_{Q,2} = 0 \tag{EC.12a}$$

$$\frac{\partial \pi_m^B(Q_m,T_m)}{\partial T_m} + \lambda_T = 0 \tag{EC.12b}$$

$$\lambda_{Q,1} Q_m = 0 \tag{EC.12c}$$

$$\lambda_{Q,2}(\tilde{q}_s - Q_m) = 0 \tag{EC.12d}$$

$$\lambda_T T_m = 0 \tag{EC.12e}$$

$$\lambda_{Q,1}, \lambda_{Q,2}, \lambda_T, Q_m, (\tilde{q}_s - Q_m), T_m \geq 0 \tag{EC.12f}$$

Let $(Q_m^B, T_m^B, \lambda_{Q,1}^B, \lambda_{Q,2}^B, \lambda_T^B)$ be any solution satisfying Eqn (EC.16a) - (EC.16f). The solution must satisfy the following conditions:

(a) $\lambda_T^B = 0$. We prove this by contradiciton. Suppose $\lambda_T^B > 0$, then by Eqn (EC.16e), it must hold that $T_m^B = 0$. Then

$$\begin{aligned}
&\frac{\partial \pi_m^B(Q_m, T_m)}{\partial T_m} + \lambda_T \\
&= -b_1 + b_2 \iint (1 - u(x_1 - \tilde{q}_s))u(x_2) f(x_1, x_2) dx_1 dx_2 \\
&\quad + b_2 \iint u(x_1 - \tilde{q}_s) u(x_2 + a_2(x_1 - \tilde{q}_s)) f(x_1, x_2) dx_1 dx_2 + \lambda_T^B \\
&= -b_1 + b_2 \iint (1 - u(x_1 - \tilde{q}_s)) f(x_1, x_2) dx_1 dx_2 + b_2 \iint u(x_1 - \tilde{q}_s) f(x_1, x_2) dx_1 dx_2 + \lambda_T^B \\
&= -b_1 + b_2 + \lambda_T^B \\
&> 0
\end{aligned}$$

which contracdicts with Eqn (EC.16b).

(b) $\lambda_{Q,1}^B = 0$. We prove this by contradiciton. Suppose $\lambda_{Q,1}^B > 0$, then it must hold that $Q_m^B = 0$ (by Eqn (EC.16c)) and $\lambda_{Q,2}^B = 0$ (by Eqn (EC.16d)) . Then

$$\begin{aligned}
&\frac{\partial \pi_m^B(Q_m, T_m)}{\partial Q_m} + \lambda_{Q,1} - \lambda_{Q,2} \\
&= -w + w_1 \iint u(x_1 - 0) f(x_1, x_2) dx_1 dx_2 + \lambda_{Q,1}^B - 0 \\
&= -w + w_1 + \lambda_{Q,1}^B \\
&> 0
\end{aligned}$$

which contracdicts with Eqn (EC.16a).

(c) $\lambda_{Q,2}^B > 0 \iff w < c$. First we prove $\lambda_{Q,2}^B > 0 \Rightarrow w < c$. Since $\lambda_{Q,2}^B > 0$, we have $Q_m^B = \tilde{q}_s$ and $T_m^B = \tilde{t}_s$. Then from Eqn (EC.16a) we have

$$-w + w_1 \iint u(x_1 - \tilde{q}_s) f(x_1, x_2) dx_1 dx_2 - \lambda_{Q,2}^B = 0$$

Note that by Proposition 1, we have $w_1 \iint u(x_1 - \tilde{q}_s) f(x_1, x_2) dx_1 dx_2 = c$. Hence we have $w = c - \lambda_{Q,2}^B < c$. Second, we prove $\lambda_{Q,2}^B > 0 \Leftarrow w < c$. We prove by contradiction. Suppose $\lambda_{Q,2}^B > 0$ does not hold, then it must be $\lambda_{Q,2}^B = 0$. Note from Eqn (EC.16a), we have

$$-w + w_1 \iint u(x_1 - \tilde{q}_s) f(x_1, x_2) dx_1 dx_2 - \lambda_{Q,2}^B = 0$$

It gives $w = w_1 \iint u(x_1 - \tilde{q}_s) f(x_1, x_2) dx_1 dx_2 = c$, which contradicts with the induction assumption.

Combining the above (a) - (c) cases, we have that: for all $w \geq c$, (Q_m^B, T_m^B) can be obtained by solving a system of equations: $\frac{\partial \pi_m^B(Q_m, T_m)}{\partial Q_m} = 0$, $\frac{\partial \pi_m^B(Q_m, T_m)}{\partial T_m} = 0$, which gives $T_m^B = \tilde{t}_s$ and Q_m^B is the solution to $w_1 H_1(q) - w = 0$.

∎

Proof of Proposition 5. The proof can be done by taking derivatives with respect to the parameters in Eqn (7), similar to the proof provided for Proposition 3. It is omitted.

■

Proof of Theorem 1. We treat $\Delta = \pi_m^A(Q_m^A, T_m^A) - \pi_m^B(Q_m^B, T_m^B)$ as a function of (w, w_1) and express it by $\Delta(w, w_1)$. Next, we analyze the sign of Δ by fixing w_1. Since $w_a > w_b$, by applying Proposition 2 and 4, we have

$$\Delta(w,w_1) = \begin{cases} \pi_m^A(q^A,t^A) - \pi_m^B(\tilde{q}_s,\tilde{t}_s) > 0 & \text{for } w \le w_b \\ \pi_m^A(q^A,t^A) - \pi_m^B(q^B,t^B) & \text{for } w_b < w < w_a \\ \pi_m^A(\tilde{q}_s,\tilde{t}_s) - \pi_m^B(q^B,t^B) < 0 & \text{for } w \ge w_a \end{cases}$$

For $w_b < w < w_a$, plugging in (q^A,t^A) and (q^B,t^B) to π_m^A, π_m^B, where π_m^A and π_m^B are expressed in (EC.1) and (EC.15), we have

$$\begin{aligned} \frac{\partial \Delta(w,w_1)}{\partial w} &= \left[\frac{\partial \pi_m^A(q^A,t^A)}{\partial w} + \frac{\partial \pi_m^A(q^A,t^A)}{\partial q^A}\frac{\partial q^A}{\partial w} + \frac{\partial \pi_m^A(q^A,t^A)}{\partial t^A}\frac{\partial t^A}{\partial w} + \frac{\partial \pi_m^A(q^A,t^A)}{\partial \tilde{q}_s}\frac{\partial \tilde{q}_s}{\partial w}\right] \\ &\quad - \left[\frac{\partial \pi_m^B(q^B,t^B)}{\partial w} + \frac{\partial \pi_m^B(q^B,t^B)}{\partial q^B}\frac{\partial q^B}{\partial w} + \frac{\partial \pi_m^B(q^B,t^B)}{\partial t^B}\frac{\partial t^B}{\partial w} + \frac{\partial \pi_m^B(q^B,t^B)}{\partial \tilde{q}_s}\frac{\partial \tilde{q}_s}{\partial w}\right] \\ &= q^B - q^A \\ &< 0 \end{aligned} \tag{EC.13}$$

Therefore $\Delta(w,w_1)$ decreases in w. Since $\Delta(w,w_1)$ is also continuous in w, and note $\Delta(w_b,w_1) > 0$ and $\Delta(w_a,w_1) < 0$, then there must exists a $\widetilde{w} \in (w_b,w_a)$ such that $\Delta(\widetilde{w},w_1) = 0$. As a result, we have $\Delta(w,w_1) > 0$ for $w < \widetilde{w}$, and $\Delta(w,w_1) < 0$ for $w > \widetilde{w}$. The $\widetilde{w}$ is the w that satisfies the following:

$$\pi_m^A(q^A,t^A) - \pi_m^B(q^B,t^B) = 0$$

which is the equivalent to the w satisfying the following equation:

$$\begin{aligned} & wq^A + b_1 t^A + w_2 \iint u(x_1 - q^A)[x_1 - q^A] f(x_1,x_2) dx_1 dx_2 \\ & \quad + b_2 \iint u(x_1 - q^A) u(x_2 + a_2(x_1 - q^A) - t^A)[x_2 + a_2(x_1 - q^A) - t^A] f(x_1,x_2) dx_1 dx_2 \\ & \quad + b_2 \iint [1 - u(x_1 - q^A)] u(x_2 - t^A)[x_2 - t^A] f(x_1,x_2) dx_1 dx_2 \\ = \; & wq^B + b_1 t^B + w_1 \iint u(x_1 - q^B)[x_1 - q^B] f(x_1,x_2) dx_1 dx_2 \\ & \quad + (w_2 - w_1) \iint u(x_1 - \tilde{q}_s)[x_1 - \tilde{q}_s] f(x_1,x_2) dx_1 dx_2 \\ & \quad + b_2 \iint (1 - u(x_1 - \tilde{q}_s)) u(x_2 - t^B)[x_2 - t^B] f(x_1,x_2) dx_1 dx_2 \\ & \quad + b_2 \iint u(x_1 - \tilde{q}_s) u(x_2 + a_2(x_1 - \tilde{q}_s) - t^B)[x_2 + a_2(x_1 - \tilde{q}_s) - t^B] f(x_1,x_2) dx_1 dx_2 \end{aligned} \tag{EC.14}$$

Note that (q^A,t^A) and (q^B,t^B) have to simultaneously satisfy Eqn. (6) and (7) that also involve w. That said, we can jointly solve (w,q^A,t^A,q^B,t^B) by combining the equations and the solution is unique.

■

Proof of Theorem 2. Let us use a superscript β to denote project firm 2's profit functions and decisions in his project supply chain. For example, let $(q^{A,\beta}, t^{A,\beta})$, $(q^{B,\beta}, t^{B,\beta})$, $(\tilde{q}_s^{\beta}, \tilde{t}_s^{\beta})$ be the counterparts of (q^A, t^A), (q^B, t^B), $(\tilde{q}_s, \tilde{t}_s)$, and $\pi_m^{A,\beta}(q^{A,\beta}, t^{A,\beta})$, $\pi_m^{B,\beta}(q^{B,\beta}, t^{B,\beta})$ be the counterparts of $\pi_m^A(q^A, t^A)$, $\pi_m^B(q^B, t^B)$, where the material costs of project firm 2 are all adjusted by a factor of β. Furthermore, we define

$$\Lambda(\beta) = \Delta(\beta w, \beta w_1 | \beta c, \beta w_2) = \pi_m^{A,\beta}(q^{A,\beta}, t^{A,\beta}) - \pi_m^{B,\beta}(q^{B,\beta}, t^{B,\beta})$$

$\Lambda(\beta)$ is continuous in β because both $\pi_m^{A,\beta}$ and $\pi_m^{B,\beta}$ are continuous in β.

Define $\bar{\beta} = \sup\{\beta : \Delta(\beta w, \beta w_1 | \beta c, \beta w_2) \geq 0\}$. We next prove the following lemma, which basically implies that $\Lambda(\beta) < 0$ (or $=, >$) if and only if $\beta > \bar{\beta}$ (or $=, <$).

LEMMA EC.1. *For any β^0 satisfying $\Lambda(\beta^0) = 0$, we must have $\frac{d\Lambda(\beta)}{d\beta}|_{\beta=\beta^0} < 0$.*

Proof of Lemma EC.1. First, we derive explicitly $\Lambda(\beta)$ and $\frac{d\Lambda(\beta)}{d\beta}$ as follows:

$$\begin{aligned}
\Lambda(\beta) &= \pi_m^{A,\beta}(q^{A,\beta}, t^{A,\beta}) - \pi_m^{B,\beta}(q^{B,\beta}, t^{B,\beta}) \\
&= \left[V - \mathbb{E}_{\mathbf{x}_1}\left\{\beta w \cdot q^{A,\beta} + \beta w_2 \cdot (\mathbf{x}_1 - q^{A,\beta})^+\right\} - \mathbb{E}_{\mathbf{x}_1,\mathbf{x}_2}\left\{b_1 t^{A,\beta} + b_2[\mathbf{x}_2 + a_2 \cdot (\mathbf{x}_1 - q^{A,\beta})^+ - t^{A,\beta}]^+\right\}\right] \\
&\quad - \left[V - \mathbb{E}_{\mathbf{x}_1}\left\{\beta w \cdot q^{B,\beta} + \beta w_1 \cdot \min\{\tilde{q}_s^{\beta} - q^{B,\beta}, (\mathbf{x}_1 - q^{B,\beta})^+\} + \beta w_2 \cdot (\mathbf{x}_1 - \tilde{q}_s^{\beta})^+\right\}\right. \\
&\qquad \left. - \mathbb{E}_{\mathbf{x}_1,\mathbf{x}_2}\left\{b_1 t^{B,\beta} + b_2[\mathbf{x}_2 + a_2 \cdot (\mathbf{x}_1 - \tilde{q}_s^{\beta})^+ - t^{B,\beta}]^+\right\}\right] \\
&= \mathbb{E}_{\mathbf{x}_1}\left\{\beta w \cdot q^{B,\beta} + \beta w_1 \cdot (\mathbf{x}_1 - q^{B,\beta})^+ + \beta(w_2 - w_1) \cdot (\mathbf{x}_1 - \tilde{q}_s^{\beta})^+\right\} \\
&\qquad + \mathbb{E}_{\mathbf{x}_1,\mathbf{x}_2}\left\{b_1 t^{B,\beta} + b_2[\mathbf{x}_2 + a_2 \cdot (\mathbf{x}_1 - \tilde{q}_s^{\beta})^+ - t^{B,\beta}]^+\right\} \\
&\quad - \mathbb{E}_{\mathbf{x}_1}\left\{\beta w \cdot q^{A,\beta} + \beta w_2 \cdot (\mathbf{x}_1 - q^{A,\beta})^+\right\} \\
&\qquad - \mathbb{E}_{\mathbf{x}_1,\mathbf{x}_2}\left\{b_1 t^{A,\beta} + b_2[\mathbf{x}_2 + a_2 \cdot (\mathbf{x}_1 - q^{A,\beta})^+ - t^{A,\beta}]^+\right\}
\end{aligned}$$

and

$$\begin{aligned}
\frac{d\Lambda(\beta)}{d\beta} &= \frac{\partial\Lambda(\beta)}{\partial\beta} + \frac{\partial\Lambda(\beta)}{\partial q^{A,\beta}}\frac{\partial q^{A,\beta}}{\partial\beta} + \frac{\partial\Lambda(\beta)}{\partial t^{A,\beta}}\frac{\partial t^{A,\beta}}{\partial\beta} + \frac{\partial\Lambda(\beta)}{\partial q^{B,\beta}}\frac{\partial q^{B,\beta}}{\partial\beta} + \frac{\partial\Lambda(\beta)}{\partial t^{B,\beta}}\frac{\partial t^{B,\beta}}{\partial\beta} + \frac{\partial\Lambda(\beta)}{\partial\tilde{q}_s^{\beta}}\frac{\partial\tilde{q}_s^{\beta}}{\partial\beta} \\
&= \mathbb{E}_{\mathbf{x}_1}\left\{w \cdot q^{B,\beta} + w_1 \cdot (\mathbf{x}_1 - q^{B,\beta})^+ + (w_2 - w_1) \cdot (\mathbf{x}_1 - \tilde{q}_s^{\beta})^+\right\} \\
&\quad - \mathbb{E}_{\mathbf{x}_1}\left\{w \cdot q^{A,\beta} + w_2 \cdot (\mathbf{x}_1 - q^{A,\beta})^+\right\}
\end{aligned}$$

Next, we show that for any $\Lambda(\beta^0) = 0$, $\frac{d\Lambda(\beta)}{d\beta}|_{\beta=\beta^0} < 0$. We consider two cases.

(1) $q^{A,\beta^0} > \tilde{q}_s^{\beta^0}$. In this case, we have

$$\begin{aligned}
&\mathbb{E}_{\mathbf{x}_1,\mathbf{x}_2}\left\{b_1 t^{A,\beta^0} + b_2[\mathbf{x}_2 + a_2 \cdot (\mathbf{x}_1 - q^{A,\beta^0})^+ - t^{A,\beta^0}]^+\right\} \\
\leq\ &\mathbb{E}_{\mathbf{x}_1,\mathbf{x}_2}\left\{b_1 t^{B,\beta^0} + b_2[\mathbf{x}_2 + a_2 \cdot (\mathbf{x}_1 - q^{A,\beta^0})^+ - t^{B,\beta^0}]^+\right\} \\
<\ &\mathbb{E}_{\mathbf{x}_1,\mathbf{x}_2}\left\{b_1 t^{B,\beta^0} + b_2[\mathbf{x}_2 + a_2 \cdot (\mathbf{x}_1 - \tilde{q}_s^{\beta^0})^+ - t^{B,\beta^0}]^+\right\}
\end{aligned}$$

where this first inequality holds by the optimality of t^{A,β^0} (corresponding to q^{A,β^0}), and the second inequality holds by $q^{A,\beta^0} > \tilde{q}_s^{\beta^0}$. Combining the above inequality with $\Lambda(\beta^0) = 0$, we get

$$\mathbb{E}_{\mathbf{x}_1}\left\{\beta^0 w \cdot q^{B,\beta^0} + \beta^0 w_1 \cdot (\mathbf{x}_1 - q^{B,\beta^0})^+ + \beta^0(w_2 - w_1)\cdot(\mathbf{x}_1 - \tilde{q}_s^{\beta^0})^+\right\} \\ -\mathbb{E}_{\mathbf{x}_1}\left\{\beta^0 w \cdot q^{A,\beta^0} + \beta^0 w_2 \cdot (\mathbf{x}_1 - q^{A,\beta^0})^+\right\} < 0$$

which leads to $\frac{d\Lambda(\beta)}{d\beta}|_{\beta=\beta^0} < 0$.

(2) $q^{A,\beta^0} = \tilde{q}_s^{\beta^0}$. In this case, we have

$$\mathbb{E}_{\mathbf{x}_1,\mathbf{x}_2}\left\{b_1 t^{A,\beta^0} + b_2[\mathbf{x}_2 + a_2\cdot(\mathbf{x}_1 - q^{A,\beta^0})^+ - t^{A,\beta^0}]^+\right\} \\ = \mathbb{E}_{\mathbf{x}_1,\mathbf{x}_2}\left\{b_1 t^{B,\beta^0} + b_2[\mathbf{x}_2 + a_2\cdot(\mathbf{x}_1 - \tilde{q}_s^{\beta^0})^+ - t^{B,\beta^0}]^+\right\}$$

and therefore

$$\mathbb{E}_{\mathbf{x}_1}\left\{\beta^0 w \cdot q^{B,\beta^0} + \beta^0 w_1 \cdot (\mathbf{x}_1 - q^{B,\beta^0})^+ + \beta^0(w_2 - w_1)\cdot(\mathbf{x}_1 - \tilde{q}_s^{\beta^0})^+\right\} \\ -\mathbb{E}_{\mathbf{x}_1}\left\{\beta^0 w \cdot q^{A,\beta^0} + \beta^0 w_2 \cdot (\mathbf{x}_1 - q^{A,\beta^0})^+\right\} = 0$$

Recall that q^{B,β^0} is the minimizer in region B, and the above equality implies $q^{B,\beta^0} = \tilde{q}_s^{\beta^0} = q^{A,\beta^0}$. We note that $q^{B,\beta^0} = \tilde{q}_s^{\beta^0}$ if and only if $w = c$, and when $w = c$, we can show $\tilde{q}_s^{\beta^0} \neq q^{A,\beta^0}$. This case does not exist.

The above two cases conclude the proof for Lemma EC.1. The lemma implies that $\Lambda(\beta)$ crosses 0 from above **at most** once.

By the definition of $\bar{\beta}$ and the continuity of $\Lambda(\beta)$, we proved that $\Lambda(\beta) < 0$ (or $=, >$) if and only if $\beta > \bar{\beta}$ (or $=, <$). This completes the proof for the three cases stated in Theorem 2.

For the remainder of Theorem 2, note that if project firm 1 prefers to push-pull, it implies $\Lambda(1) \le 0$. By the single crossing property of $\Lambda(\beta)$ and the definition of $\bar{\beta}$, we have $\bar{\beta} \le 1$. Similarly we can show that if project firm 1 prefers to push, we have $\bar{\beta} \ge 1$.

■

Proof of Corollary 1. The corollary follows directly from Theorem 1 by setting $b_1 = b_2 = 0$, and the proof is omitted.

■

Proof of Proposition 6. Let $\widetilde{w}^{(b_1,b_2)}$ denote the w such that $\Delta(w, w_1, b_1, b_2) = 0$.

$$\begin{aligned}\frac{d\Delta(w,w_1,b_1,b_2)}{db_1} &= \left[\frac{\partial\pi_m^A(q^A,t^A)}{\partial b_1} + \frac{\partial\pi_m^A(q^A,t^A)}{\partial q^A}\frac{\partial q^A}{\partial b_1} + \frac{\partial\pi_m^A(q^A,t^A)}{\partial t^A}\frac{\partial t^A}{\partial b_1} + \frac{\partial\pi_m^A(q^A,t^A)}{\partial \tilde{q}_s}\frac{\partial \tilde{q}_s}{\partial b_1}\right] \\ &\quad - \left[\frac{\partial\pi_m^B(q^B,t^B)}{\partial b_1} + \frac{\partial\pi_m^B(q^B,t^B)}{\partial q^B}\frac{\partial q^B}{\partial b_1} + \frac{\partial\pi_m^B(q^B,t^B)}{\partial t^B}\frac{\partial t^B}{\partial b_1} + \frac{\partial\pi_m^B(q^B,t^B)}{\partial \tilde{q}_s}\frac{\partial \tilde{q}_s}{\partial b_1}\right] \\ &= \frac{\partial\pi_m^A(q^A,t^A)}{\partial b_1} - \frac{\pi_m^B(q^B,t^B)}{\partial b_1} \\ &= t^B - t^A \\ &> 0\end{aligned}$$

Hence, $\Delta(w, w_1, b_1, b_2) > \Delta(w, w_1, 0, b_2)$. Let $\widetilde{w}^{(b_1,b_2)}$, $\widetilde{w}^{(0,b_2)}$ satisfy $\Delta(\widetilde{w}^{(b_1,b_2)}, w_1, b_1, b_2) = 0$ and $\Delta(\widetilde{w}^{(0,b_2)}, w_1, b_1, b_2) = 0$. First we have $\Delta(\widetilde{w}^{(0,b_2)}, w_1, b_1, b_2) > \Delta(\widetilde{w}^{(0,b_2)}, w_1, 0, b_2) = 0$ due to the above result. Hence $\Delta(\widetilde{w}^{(0,b_2)}, w_1, b_1, b_2) > \Delta(\widetilde{w}^{(b_1,b_2)}, w_1, b_1, b_2)$. Then since Δ is decreasing in w, we must have $\widetilde{w}^{(0,b_2)} < \widetilde{w}^{(b_1,b_2)}$.

Next we show $\widetilde{w}^{(0,b_2)} = \widetilde{w}^{(0,0)}$. We first show $\Delta(w, w_1, 0, b_2) = \Delta(w, w_1, 0, 0)$. As a result of this, if $\widetilde{w}^{(0,b_2)}$ satisfies $\Delta(\widetilde{w}^{(0,b_2)}, w_1, 0, b_2) = 0$, it must also satisfy $\Delta(\widetilde{w}^{(0,b_2)}, w_1, 0, 0) = 0$. Therefore $\widetilde{w}^{(0,b_2)} = \widetilde{w}^{(0,0)}$. Let $q^A(b_1, b_2), t^A(b_1, b_2)$ be the optimal decision in region A that is contingent on (b_1, b_2), and similarly for $q^B(b_1, b_2), t^B(b_1, b_2)$. By Eqn (6) we have $t^A(0, b_2) = +\infty$. Similarly, $t^B(0, b_2) = +\infty$. Moreover, by Eqn (6) we can have $q^A(0, b_2) = q^A(0, 0)$, since $H_2(q^A, +\infty) = 0$. One can then verify that $\pi_m^A(q^A(0, b_2), t^A(0, b_2)) = \pi_m^A(q^A(0, 0), t^A(0, 0))$ and $\pi_m^B(q^B(0, b_2), t^B(0, b_2)) = \pi_m^B(q^B(0, 0), t^B(0, 0))$. As a result, $\Delta(w, w_1, 0, b_2) = \pi_m^A(q^A(0, b_2), t^A(0, b_2)) - \pi_m^B(q^B(0, b_2), t^B(0, b_2)) = \pi_m^A(q^A(0, 0), t^A(0, 0)) - \pi_m^B(q^B(0, 0), t^B(0, 0)) = \Delta(w, w_1, 0, 0)$.

Combining the above, we have $\widetilde{w}^{(0,0)} = \widetilde{w}^{(0,b_2)} < \widetilde{w}^{(b_1,b_2)}$, which completes the proof (note that there is a slight difference of notation used in the proposition: $\widetilde{w}^{(b_1,b_2)} = \widetilde{w}$ and $\widetilde{w}^{(0,0)} = \widetilde{w}^{\phi}$).

■

Proof of Proposition 7. We consider the following contract cases:

(1) Contracts in $\{(w, w_1) : w < \widetilde{w}\}$. By Proposition 6, we further consider two sub-cases:

- $w < \widetilde{w}^{\phi}$. In this case, by Corollary 1, we have $Q_m^{\phi} = q^{\phi,A}$; and by Theorem 1, $Q_m^* = q^A$. Therefore, we have

$$
\begin{aligned}
w_2 H_1(q^A) &= w - a_2 b_2 H_2(q^A, t^A) \\
w_2 H_1(q^{\phi,A}) &= w
\end{aligned}
$$

  where the first equation is from Eqn (6), and the second is from Eqn (8). Since $H_2(q^A, t^A) > 0$, we have $H_1(q^A) < H_1(q^{\phi,A})$. Hence, $q^A > q^{\phi,A}$. That is, $Q_m^* > Q_m^{\phi}$. Note that this case is the push region, and therefore $Q_m^* = Q_s^*$ $Q_m^{\phi} = Q_s^{\phi}$. Hence $Q_s^* > Q_s^{\phi}$.

- $\widetilde{w}^{\phi} < w < \widetilde{w}$. In this case, by Corollary 1, we have $Q_m^{\phi} = q^{\phi,B}$; and by Theorem 1, $Q_m^* = q^A$. Therefore, we have

$$
\begin{aligned}
w_2 H_1(q^A) &= w - a_2 b_2 H_2(q^A, t^A) \\
w_1 H_1(q^{\phi,B}) &= w
\end{aligned}
$$

  where the first equation is from Eqn (6), and the second is from Eqn (9). Since $w_1 \le w_2$, we have $H_1(q^A) < H_1(q^{\phi,B})$. Hence, $q^A > q^{\phi,B}$. That is, $Q_m^* > Q_m^{\phi}$. Note that in this case we also have $Q_s^* = Q_m^* > \tilde{q}_s = Q_s^{\phi}$. Hence $Q_s^* > Q_s^{\phi}$.

- $w = \widetilde{w}^{\phi}$. In this case, $Q_m^{\phi} = q^{\phi,A}$ or $Q_m^{\phi} = q^{\phi,B}$; and $Q_m^* = q^A$. For either case of Q_m^{ϕ}, we have $Q_m^{\phi} < Q_m^*$ as is shown above. The result for Q_s^{ϕ} and Q_s^* holds as well.

$Q_m^* > Q_m^\phi$ and $Q_s^* > Q_s^\phi$;

(2) Contracts in $\{(w, w_1) : w > \widetilde{w}\}$. In this case, by Corollary 1, we have $Q_m^\phi = q^{\phi,B}$; and by Theorem 1, $Q_m^* = q^B$. $q^B = q^{\phi,B}$ by Eqn (7) and (9). Moreover, $Q_s^* = Q_s^\phi = \tilde{q}_s$.

(3) Contracts in $\{(w, w_1) : w = \widetilde{w}\}$. The stated result follows immediately by combining the analysis for the above two cases.

■

Proof of Proposition 8. Let $g_1(Q_m, T_m, x_1, x_2) = x_2 + a_1(x_1 - Q_m) - T_m$, and $g_2(Q_m, T_m, x_1, x_2) = x_2 + a_1(x_1 - Q_m) + (a_2 - a_1)(x_1 - \tilde{q}_s) - T_m$. For simplicity, we write g_1, g_2 and drop their dependencies on the parameters when the context is clear. Based on these, we can write the project firm's expected profit $\hat{\pi}_m^B(Q_m, T_m)$ as

$$\begin{aligned}
\hat{\pi}_m^B(Q_m, T_m) &= \mathbb{E}_{\mathbf{x}_1,\mathbf{x}_2}\{V - wQ_m - w_1(\mathbf{x}_1 - Q_m)^+ - (w_2 - w_1)(\mathbf{x}_1 - \tilde{q}_s)^+ \\
&\qquad -b_1T_m - b_2[\mathbf{x}_2 + a_1(\mathbf{x}_1 - Q_m)^+ + (a_2 - a_1)(\mathbf{x}_1 - \tilde{q}_s)^+ - T_m]^+\} \\
&= \mathbb{E}_{\mathbf{x}_1,\mathbf{x}_2}\{V - wQ_m - w_1u(\mathbf{x}_1 - Q_m)[\mathbf{x}_1 - Q_m] - (w_2 - w_1)u(\mathbf{x}_1 - \tilde{q}_s)[\mathbf{x}_1 - \tilde{q}_s] \\
&\qquad -b_1T_m - b_2(1 - u(\mathbf{x}_1 - Q_m))u(\mathbf{x}_2 - T_m)[\mathbf{x}_2 - T_m] \\
&\qquad -b_2u(\mathbf{x}_1 - Q_m)(1 - u(\mathbf{x}_1 - \tilde{q}_s))u(g_1)[g_1] \\
&\qquad -b_2u(\mathbf{x}_1 - \tilde{q}_s)u(g_2)[g_2]\}
\end{aligned} \tag{EC.15}$$

Concavity of $\hat{\pi}_m^B(Q_m, T_m)$. We show the concavity of $\hat{\pi}_m^B(Q_m, T_m)$ by showing its Hessian matrix is negative semi-definite. We first derive the first order derivative of $\hat{\pi}_m^B(Q_m, T_m)$ below.

$$\begin{aligned}
\frac{\partial \hat{\pi}_m^B(Q_m, T_m)}{\partial Q_m} &= \mathbb{E}_{\mathbf{x}_1,\mathbf{x}_2}\{-w - w_1\delta(\mathbf{x}_1 - Q_m)(-1)[\mathbf{x}_1 - Q_m] - w_1u(\mathbf{x}_1 - Q_m)(-1) \\
&\qquad -b_2(-\delta(\mathbf{x}_1 - Q_m))(-1)u(\mathbf{x}_2 - T_m)[\mathbf{x}_2 - T_m] \\
&\qquad -b_2\delta(\mathbf{x}_1 - Q_m)(-1)(1 - u(\mathbf{x}_1 - \tilde{q}_s))u(g_1)[g_1] \\
&\qquad -b_2u(\mathbf{x}_1 - Q_m)(1 - u(\mathbf{x}_1 - \tilde{q}_s))\delta(g_1)(-a_1)[g_1] \\
&\qquad -b_2u(\mathbf{x}_1 - Q_m)(1 - u(\mathbf{x}_1 - \tilde{q}_s))u(g_1)[-a_1] \\
&\qquad -b_2u(\mathbf{x}_1 - \tilde{q}_s)\delta(g_2)(-a_1)[g_2] \\
&\qquad -b_2u(\mathbf{x}_1 - \tilde{q}_s)u(g_2)[-a_1]\} \\
&= -w + w_1\iint u(x_1 - Q_m)f(x_1, x_2)dx_1dx_2 \\
&\qquad +a_1b_2\iint u(x_1 - Q_m)(1 - u(x_1 - \tilde{q}_s))u(g_1(Q_m, T_m, x_1, x_2))f(x_1, x_2)dx_1dx_2 \\
&\qquad +a_1b_2\iint u(x_1 - \tilde{q}_s)u(g_2(Q_m, T_m, x_1, x_2))f(x_1, x_2)dx_1dx_2
\end{aligned}$$

where the first equality holds due to interchanging derivative and expectation, and the second equality holds due to the property of the Dirac delta (δ) function (see similar property in the proof of Proposition 2). Similarly, for T_m we have

$$\begin{aligned}
\frac{\partial \hat{\pi}_m^B(Q_m,T_m)}{\partial T_m} &= \mathbb{E}_{\mathbf{x}_1,\mathbf{x}_2}\{-b_1 - b_2(1-u(\mathbf{x}_1-Q_m))\delta(\mathbf{x}_2-T_m)(-1)[\mathbf{x}_2-T_m] - b_2(1-u(\mathbf{x}_1-Q_m))u(\mathbf{x}_2-T_m)[-1] \\
&\qquad -b_2u(\mathbf{x}_1-Q_m)(1-u(\mathbf{x}_1-\tilde{q}_s))\delta(g_1)(-1)[g_1] \\
&\qquad -b_2u(\mathbf{x}_1-Q_m)(1-u(\mathbf{x}_1-\tilde{q}_s))u(g_1)[-1] \\
&\qquad -b_2u(\mathbf{x}_1-\tilde{q}_s)\delta(g_2)(-1)[g_2] \\
&\qquad -b_2u(\mathbf{x}_1-\tilde{q}_s)u(g_2)[-1]\} \\
&= -b_1 + b_2\iint(1-u(x_1-Q_m))u(x_2-T_m)f(x_1,x_2)dx_1dx_2 \\
&\qquad +b_2\iint u(x_1-Q_m)(1-u(x_1-\tilde{q}_s))u(g_1(Q_m,T_m,x_1,x_2))f(x_1,x_2)dx_1dx_2 \\
&\qquad +b_2\iint u(x_1-\tilde{q}_s)u(g_2(Q_m,T_m,x_1,x_2))f(x_1,x_2)dx_1dx_2
\end{aligned}$$

Next we show that $\hat{\pi}_m^B(Q_m,T_m)$ is concave by showing its Hessian matrix, denoted by $\mathbf{H}_B$, is negative semi-definite. The Hessian $\mathbf{H}_B$ is defined by

$$\mathbf{H}_B = \begin{bmatrix} \frac{\partial^2\hat{\pi}_m^B(Q_m,T_m)}{\partial Q_m^2} & \frac{\partial^2\hat{\pi}_m^B(Q_m,T_m)}{\partial Q_m\partial T_m} \\ \frac{\partial^2\hat{\pi}_m^B(Q_m,T_m)}{\partial T_m\partial Q_m} & \frac{\partial^2\hat{\pi}_m^B(Q_m,T_m)}{\partial T_m^2} \end{bmatrix}$$

The elements of $\mathbf{H}_B$ are derived below:

$$\begin{aligned}
\frac{\partial^2\hat{\pi}_m^B(Q_m,T_m)}{\partial Q_m^2} &= -w_1\int f(Q_m,x_2)dx_2 - a_1b_2\int u(x_2-T_m)f(Q_m,x_2)dx_2 \\
&\quad -a_1^2b_2\int u(x_1-Q_m)(1-u(x_1-\tilde{q}_s))f(x_1,T_m-a_1(x_1-Q_m))dx_1 \\
&\quad -a_1^2b_2\int u(x_1-\tilde{q}_s)f(x_1,T_m-a_1(x_1-Q_m)-(a_2-a_1)(x_1-\tilde{q}_s))dx_1 \\
\frac{\partial^2\hat{\pi}_m^B(Q_m,T_m)}{\partial T_m^2} &= -b_2\int(1-u(x_1-Q_m))f(x_1,T_m)dx_1 \\
&\quad -b_2\int u(x_1-Q_m)(1-u(x_1-\tilde{q}_s))f(x_1,T_m-a_1(x_1-Q_m))dx_1 \\
&\quad -b_2\int u(x_1-\tilde{q}_s)f(x_1,T_m-a_1(x_1-Q_m)-(a_2-a_1)(x_1-\tilde{q}_s))dx_1 \\
\frac{\partial^2\hat{\pi}_m^B(Q_m,T_m)}{\partial Q_m\partial T} &= -a_1b_2\int u(x_1-Q_m)(1-u(x_1-\tilde{q}_s))f(x_1,T_m-a_1(x_1-Q_m))dx_1 \\
&\quad -a_1b_2\int u(x_1-\tilde{q}_s)f(x_1,T_m-a_1(x_1-Q_m)-(a_2-a_1)(x_1-\tilde{q}_s))dx_1
\end{aligned}$$

To show $\mathbf{H}_B$ is negative semi-definite, it is sufficient to show $\mathbf{z}^T\mathbf{H}_B\mathbf{z} \leq 0$ for any $\mathbf{z} = (z_1,z_2) \in \mathbb{R}^2$.

$$\mathbf{z}^T\mathbf{H}_B\mathbf{z} = \left(\frac{\partial^2\hat{\pi}_m^B(Q_m,T_m)}{\partial Q_m^2}\right)z_1^2 + 2\left(\frac{\partial^2\hat{\pi}_m^B(Q_m,T_m)}{\partial Q_m\partial T_m}\right)z_1z_2 + \left(\frac{\partial^2\hat{\pi}_m^B(Q_m,T_m)}{\partial T_m^2}\right)z_2^2$$

$$
\begin{aligned}
&= -\left[w_1 \int f(Q_m, x_2)dx_2 + a_1 b_2 \int u(x_2 - T_m) f(Q_m, x_2)dx_2\right] z_1^2 \\
&\quad - \left[b_2 \int [1 - u(x_1 - Q_m)] f(x_1, T_m)dx_1\right] z_2^2 \\
&\quad - \left[b_2 \int u(x_1 - Q_m)(1 - u(x_1 - \tilde{q}_s)) f(x_1, T_m - a_1(x_1 - Q_m))dx_1\right] (a_1 z_1 + z_2)^2 \\
&\quad - \left[b_2 \int u(x_1 - \tilde{q}_s) f(x_1, T_m - a_1(x_1 - Q_m) - (a_2 - a_1)(x_1 - \tilde{q}_s))dx_1\right] (a_1 z_1 + z_2)^2 \\
&\leq 0
\end{aligned}
$$

Convex Optimization. Since $\hat{\pi}_m^B(Q_m, T_m)$ is concave and $B = \{(Q_m, T_m) : Q_m \leq \tilde{q}_s, Q_m \geq 0, T_m \geq 0\}$ is a convex set, we can apply Lagrangian approach to derive the solution to the optimization problem. Let

$$L(Q_m, T_m, \lambda_{Q,1}, \lambda_{Q,2}, \lambda_T) = \hat{\pi}_m^B(Q_m, T_m) + \lambda_{Q,1} Q_m + \lambda_{Q,2}(\tilde{q}_s - Q_m) + \lambda_T T_m$$

where $, \lambda_{Q,2}, \lambda_T$ are lagrangian multipliers. The optimal solution (Q_m^B, T_m^B) can be found by solving the following equations:

$$\frac{\partial \hat{\pi}_m^B(Q_m, T_m)}{\partial Q_m} + \lambda_{Q,1} - \lambda_{Q,2} = 0 \tag{EC.16a}$$

$$\frac{\partial \hat{\pi}_m^B(Q_m, T_m)}{\partial T_m} + \lambda_T = 0 \tag{EC.16b}$$

$$\lambda_{Q,1} Q_m = 0 \tag{EC.16c}$$

$$\lambda_{Q,2}(\tilde{q}_s - Q_m) = 0 \tag{EC.16d}$$

$$\lambda_T T_m = 0 \tag{EC.16e}$$

$$\lambda_{Q,1}, \lambda_{Q,2}, \lambda_T, Q_m, (\tilde{q}_s - Q_m), T_m \geq 0 \tag{EC.16f}$$

Let $(Q_m^B, T_m^B, \lambda_{Q,1}^B, \lambda_{Q,2}^B, \lambda_T^B)$ be a solution satisfying Eqn (EC.16a) - (EC.16f). The solution must satisfy the following conditions:

(a) $\lambda_T^B = 0$. We prove this by contradiciton. Suppose $\lambda_T^B > 0$, then by Eqn (EC.16e), it must hold that $T_m^B = 0$. Then

$$
\begin{aligned}
&\frac{\partial \hat{\pi}_m^B(Q_m, T_m)}{\partial T_m} + \lambda_T \\
&= -b_1 + b_2 \iint (1 - u(x_1 - Q_m^B)) u(x_2 - T_m^B) f(x_1, x_2)dx_1 dx_2 \\
&\quad + b_2 \iint u(x_1 - Q_m^B)(1 - u(x_1 - \tilde{q}_s)) u(g_1(Q_m^B, T_m^B, x_1, x_2)) f(x_1, x_2)dx_1 dx_2 \\
&\quad + b_2 \iint u(x_1 - \tilde{q}_s) u(g_2(Q_m^B, T_m^B, x_1, x_2)) f(x_1, x_2)dx_1 dx_2 + \lambda_T^B \\
&= -b_1 + b_2 + \lambda_T^B \\
&> 0
\end{aligned}
$$

which contracdicts with Eqn (EC.16b).

(b) $\lambda^B_{Q,1} = 0$. We prove this by contradiciton. Suppose $\lambda^B_{Q,1} > 0$, then it must hold that $Q^B_m = 0$ (by Eqn (EC.16c)) and $\lambda^B_{Q,2} = 0$ (by Eqn (EC.16d)) . Then

$$
\begin{aligned}
&\frac{\partial \hat{\pi}^B_m(Q_m, T_m)}{\partial Q_m} + \lambda_{Q,1} - \lambda_{Q,2} \\
=& -w + w_1 \iint u(x_1 - Q^B_m) f(x_1,x_2) dx_1 dx_2 \\
&+ a_1 b_2 \iint u(x_1 - Q^B_m)(1 - u(x_1 - \tilde{q}_s)) u(g_1(Q^B_m, T^B_m, x_1, x_2)) f(x_1,x_2) dx_1 dx_2 \\
&+ a_1 b_2 \iint u(x_1 - \tilde{q}_s) u(g_2(Q^B_m, T^B_m, x_1, x_2)) f(x_1,x_2) dx_1 dx_2 + \lambda^B_{Q,1} - 0 \\
\geq& -w + w_1 \iint u(x_1 - 0) f(x_1,x_2) dx_1 dx_2 + \lambda^B_{Q,1} \\
=& -w + w_1 + \lambda^B_{Q,1} \\
>& \ 0
\end{aligned}
$$

which contracdicts with Eqn (EC.16a).

(c) $\lambda^B_{Q,2} > 0 \iff w < c + a_1 b_2 \iint u(x_1 - \tilde{q}_s) u(x_2 + a_2(x_1 - \tilde{q}_s) - \tilde{t}_s) f(x_1,x_2) dx_1 dx_2$.

First we prove $\lambda^B_{Q,2} > 0 \Rightarrow w < c + a_1 b_2 \iint u(x_1 - \tilde{q}_s) u(x_2 + a_2(x_1 - \tilde{q}_s) - \tilde{t}_s) f(x_1,x_2) dx_1 dx_2$. Since $\lambda^B_{Q,2} > 0$, we have $Q^B_m = \tilde{q}_s$ and $T^B_m = \tilde{t}_s$. Then from Eqn (EC.16a) we have

$$
-w + w_1 \iint u(x_1 - \tilde{q}_s) f(x_1,x_2) dx_1 dx_2 + a_1 b_2 \iint u(x_1 - \tilde{q}_s) u(g_2(\tilde{q}_s, \tilde{t}_s, x_1, x_2)) f(x_1,x_2) dx_1 dx_2 - \lambda^B_{Q,2} = 0
$$

which gives $w < c + a_1 b_2 \iint u(x_1 - \tilde{q}_s) u(x_2 + a_2(x_1 - \tilde{q}_s) - \tilde{t}_s) f(x_1,x_2) dx_1 dx_2$.

Second, we prove $\lambda^B_{Q,2} > 0 \Leftarrow w < c + a_1 b_2 \iint u(x_1 - \tilde{q}_s) u(x_2 + a_2(x_1 - \tilde{q}_s) - \tilde{t}_s) f(x_1,x_2) dx_1 dx_2$. We prove by contradiction. Suppose $\lambda^B_{Q,2} > 0$ does not hold, then it must be $\lambda^B_{Q,2} = 0$. Note from Eqn (EC.16b), we have

$$
\begin{aligned}
&-b_1 + b_2 \iint (1 - u(x_1 - Q^B_m)) u(x_2 - T^B_m) f(x_1,x_2) dx_1 dx_2 \\
&+ b_2 \iint u(x_1 - Q^B_m)(1 - u(x_1 - \tilde{q}_s)) u(x_2 + a_1(x_1 - Q^B_m) - T^B_m) f(x_1,x_2) dx_1 dx_2 \\
&+ b_2 \iint u(x_1 - \tilde{q}_s) u(x_2 + a_1(x_1 - Q^B_m) + (a_2 - a_1)(x_1 - \tilde{q}_s) - T^B_m)) f(x_1,x_2) dx_1 dx_2 = 0
\end{aligned}
$$

and by definition of $\tilde{t}_s$, we have

$$
\begin{aligned}
&-b_1 + b_2 \iint u(x_1 - \tilde{q}_s) u(x_2 + a_2(x_1 - \tilde{q}_s) - \tilde{t}_s) f(x_1,x_2) dx_1 dx_2 \\
&+ b_2 \iint [1 - u(x_1 - \tilde{q}_s)] u(x_2 - \tilde{t}_s) f(x_1,x_2) dx_1 dx_2 = 0
\end{aligned}
$$

Therefore by Eqn (EC.16a), we have

$$
w = w_1 \iint u(x_1 - Q^B_m) f(x_1,x_2) dx_1 dx_2
$$

$$
\begin{aligned}
&+a_1b_2\iint u(x_1-Q_m^B)(1-u(x_1-\tilde{q}_s))u(x_2+a_1(x_1-Q_m^B)-T_m^B)f(x_1,x_2)dx_1dx_2\\
&+a_1b_2\iint u(x_1-\tilde{q}_s)u(x_2+a_1(x_1-Q_m^B)+(a_2-a_1)(x_1-\tilde{q}_s)-T_m^B))f(x_1,x_2)dx_1dx_2\\
=\ & w_1\iint u(x_1-Q_m^B)f(x_1,x_2)dx_1dx_2\\
&+a_1b_2\iint u(x_1-\tilde{q}_s)u(x_2+a_2(x_1-\tilde{q}_s)-\tilde{t}_s)f(x_1,x_2)dx_1dx_2\\
&+a_1b_2\iint [1-u(x_1-\tilde{q}_s)]u(x_2-\tilde{t}_s)f(x_1,x_2)dx_1dx_2\\
&-a_1b_2\iint (1-u(x_1-Q_m^B))u(x_2-T_m^B)f(x_1,x_2)dx_1dx_2\\
\geq\ & w_1\iint u(x_1-\tilde{q}_s)f(x_1,x_2)dx_1dx_2\\
&+a_1b_2\iint u(x_1-\tilde{q}_s)u(x_2+a_2(x_1-\tilde{q}_s)-\tilde{t}_s)f(x_1,x_2)dx_1dx_2\\
=\ & c+a_1b_2\iint u(x_1-\tilde{q}_s)u(x_2+a_2(x_1-\tilde{q}_s)-\tilde{t}_s)f(x_1,x_2)dx_1dx_2
\end{aligned}
$$

which contradicts with the induction assumption.

Combining the above (a) - (c) results, we proved that: if $w < c + a_1b_2\iint u(x_1-\tilde{q}_s)u(x_2+a_2(x_1-\tilde{q}_s)-\tilde{t}_s)f(x_1,x_2)dx_1dx_2$, we have $Q_m^B=\tilde{q}_s$ and $T_m^B=\tilde{t}_s$; otherwise, (Q_m^B,T_m^B) can be obtained by solving a system of equations: $\frac{\partial\hat{\pi}_m^B(Q_m,T_m)}{\partial Q_m}=0$, $\frac{\partial\hat{\pi}_m^B(Q_m,T_m)}{\partial T_m}=0$, which is the unique (q,t) to the following system of equations:

$$
\begin{cases}
w_1\iint u(x_1-q)f(x_1,x_2)dx_1dx_2 & \\
\quad +a_1b_2\iint u(x_1-q)(1-u(x_1-\tilde{q}_s))u(x_2+a_1(x_1-q)-t)f(x_1,x_2)dx_1dx_2 & \\
\quad +a_1b_2\iint u(x_1-\tilde{q}_s)u(x_2+a_1(x_1-q)+(a_2-a_1)(x_1-\tilde{q}_s)-t))f(x_1,x_2)dx_1dx_2-w & =0\\
b_2\iint (1-u(x_1-q))u(x_2-t)f(x_1,x_2)dx_1dx_2 & \\
\quad +b_2\iint u(x_1-q)(1-u(x_1-\tilde{q}_s))u(x_2+a_1(x_1-q)-t)f(x_1,x_2)dx_1dx_2 & \\
\quad +b_2\iint u(x_1-\tilde{q}_s)u(x_2+a_1(x_1-q)+(a_2-a_1)(x_1-\tilde{q}_s)-t)f(x_1,x_2)dx_1dx_2-b_1 & =0
\end{cases}
\tag{EC.17}
$$

By the definition of H_1, H_3, H_4 and H_5, Eqn (EC.17) is exactly Eqn (10) in the proposition. Let the solution to Eqn (EC.17) be denoted by $(\hat{q}^B,\hat{t}^B)$.

Finally, we prove that $(\hat{q}^B,\hat{t}^B) > (q^B,t^B)$. First by Eqn (7) and (10), we have

$$
\begin{aligned}
w_1H_1(q^B)-w&=0\\
w_1H_1(\hat{q}^B)+a_1b_2H_4(\hat{q}^B,\hat{t}^B)+a_1b_2H_5(\hat{q}^B,\hat{t}^B)-w&=0
\end{aligned}
$$

Since $H_4(\hat{q}^B,\hat{t}^B)>0$ and $H_5(\hat{q}^B,\hat{t}^B)>0$, we must have $H_1(\hat{q}^B)<H_1(q^B)$. As a result, $\hat{q}^B>q^B$. To prove $\hat{t}^B>t^B$, we show by contradiction. Suppose $\hat{t}^B\leq t^B$, by Eqn (10), we have

$$
0=b_2H_3(\hat{q}^B,\hat{t}^B)+b_2H_4(\hat{q}^B,\hat{t}^B)+b_2H_5(\hat{q}^B,\hat{t}^B)-b_1
$$

$$
\begin{aligned}
&= b_2 \iint (1-u(x_1-\hat{q}^B))u(x_2-\hat{t}^B)f(x_1,x_2)dx_1dx_2 \\
&\quad + b_2 \iint u(x_1-\hat{q}^B)(1-u(x_1-\tilde{q}_s))u(x_2+a_1(x_1-\hat{q}^B)-\hat{t}^B)f(x_1,x_2)dx_1dx_2 \\
&\quad + b_2 \iint u(x_1-\tilde{q}_s)u(x_2+a_1(x_1-\hat{q}^B)+(a_2-a_1)(x_1-\tilde{q}_s)-\hat{t}^B))f(x_1,x_2)dx_1dx_2 - b_1 \\
&> b_2 \iint (1-u(x_1-\hat{q}^B))u(x_2-\hat{t}^B)f(x_1,x_2)dx_1dx_2 \\
&\quad + b_2 \iint u(x_1-\hat{q}^B)(1-u(x_1-\tilde{q}_s))u(x_2-\hat{t}^B)f(x_1,x_2)dx_1dx_2 \\
&\quad + b_2 \iint u(x_1-\tilde{q}_s)u(x_2+a_2(x_1-\tilde{q}_s)-t^B)f(x_1,x_2)dx_1dx_2 - b_1 \\
&\geq b_2H_3(\tilde{q}_s,t^B)+b_2H_2(\tilde{q}_s,t^B)-b_1 \\
&= 0
\end{aligned}
$$

where the last equality hols by Eqn (7). This is a contradiction that $0>0$.

■

Proof of Theorem 3. The main proof is similar to that of Theorem 1 and is omitted. We provide a sketch of the proof for $\hat{w}>\tilde{w}$ below. Since $a_1>0$, for any (w,w_1), we must have $\hat{\pi}_m^B(\hat{Q}_m^B,\hat{T}_m^B)<\pi_m^B(Q_m^B,T_m^B)$. On the other hand, $\hat{\pi}_m^A(\hat{Q}_m^A,\hat{T}_m^A)=\pi_m^A(Q_m^A,T_m^A)$. Therefore,

$$\hat{\Delta}(w,w_1)\triangleq\hat{\pi}_m^A(\hat{Q}_m^A,\hat{T}_m^A)-\hat{\pi}_m^B(\hat{Q}_m^B,\hat{T}_m^B)>\pi_m^A(Q_m^A,T_m^A)-\pi_m^B(Q_m^B,T_m^B)\triangleq\Delta(w,w_1)$$

This implies that for any $\tilde{w}$ satisfying $\Delta(\tilde{w},w_1)=0$, we have $\hat{\Delta}(\tilde{w},w_1)>0$. Note that $\hat{\Delta}(w,w_1)$ is decreasing in w (the proof is omitted, but it is a property similar to that has been shown for $\Delta(w,w_1)$). Therefore for the $\hat{w}$ satisfying $\hat{\Delta}(\hat{w},w_1)=0$, we must have $\hat{w}>\tilde{w}$.

■

EC.4 Additional numerical results.

We provide detailed numerical results on the project firm's optimal decision (Q_m^*,T_m^*) and expected profit for project overhead cost $b_1\in\{1,4,8\}$ and project delay penalty $b_2\in\{5,10,20\}$ in Table EC.1 and EC.2, respectively. Other parameters used in the numerical test are provided under each table.

Table EC.1 Summary of the project firm's optimal decision and expected profit for $b_1 \in \{1,4,8\}$.

		OPT Region			Q_m^*			T_m^*			Expected Profit		
w	w_1	1	4	8	1	4	8	1	4	8	1	4	8
1.0	3.0	A	A	A	137.49	140.18	141.50	127.36	105.54	83.38	919.91	575.24	195.24
1.5	3.0	A	A	A	133.88	136.72	138.09	128.37	105.79	83.49	852.10	506.06	125.38
2.0	3.0	B	A	A	91.39	134.13	135.52	187.65	106.05	83.60	824.28	438.36	57.03
2.5	3.0	B	A	A	80.65	132.01	133.44	187.65	106.32	83.72	810.56	371.79	-10.21
3.0	3.0	B	A	A	0.00	130.21	131.67	187.65	106.59	83.84	775.70	306.23	-76.48
1.0	3.5	A	A	A	137.49	140.18	141.50	127.36	105.54	83.38	919.91	575.24	195.24
1.5	3.5	A	A	A	133.88	136.72	138.09	128.37	105.79	83.49	852.10	506.06	125.38
2.0	3.5	B	A	A	96.40	134.13	135.52	174.28	106.05	83.60	805.41	438.36	57.03
2.5	3.5	B	A	A	88.68	132.01	133.44	174.28	106.32	83.72	779.83	371.79	-10.21
3.0	3.5	B	A	A	78.65	130.21	131.67	174.28	106.59	83.84	738.59	306.23	-76.48
3.5	3.5	B	A	A	0.00	128.61	130.12	174.28	106.88	83.96	704.38	241.57	-141.92
1.0	4.0	A	A	A	137.49	140.18	141.50	127.36	105.54	83.38	919.91	575.24	195.24
1.5	4.0	A	A	A	133.88	136.72	138.09	128.37	105.79	83.49	852.10	506.06	125.38
2.0	4.0	B	A	A	100.00	134.13	135.52	163.57	106.05	83.60	796.90	438.36	57.03
2.5	4.0	B	A	A	93.63	132.01	133.44	163.57	106.32	83.72	761.90	371.79	-10.21
3.0	4.0	B	A	A	86.51	130.21	131.67	163.57	106.59	83.84	717.76	306.23	-76.48
3.5	4.0	B	A	A	76.99	128.61	130.12	163.57	106.88	83.96	676.71	241.57	-141.92
4.0	4.0	B	B	B	0.00	0.00	0.00	163.57	112.84	86.78	643.18	204.98	-201.46
1.0	4.5	A	A	A	137.49	140.18	141.50	127.36	105.54	83.38	919.91	575.24	195.24
1.5	4.5	A	A	A	133.88	136.72	138.09	128.37	105.79	83.49	852.10	506.06	125.38
2.0	4.5	B	A	A	102.79	134.13	135.52	155.40	106.05	83.60	792.97	438.36	57.03
2.5	4.5	B	A	A	97.21	132.01	133.44	155.40	106.32	83.72	752.00	371.79	-10.21
3.0	4.5	B	A	A	91.39	130.21	131.67	155.40	106.59	83.84	705.70	306.23	-76.48
3.5	4.5	B	B	A	84.71	84.71	130.12	155.40	111.67	83.96	661.63	246.36	-141.92
4.0	4.5	B	B	B	75.59	75.59	75.59	155.40	111.67	86.29	621.39	206.11	-196.38
4.5	4.5	B	B	B	0.00	0.00	0.00	155.40	111.67	86.29	588.43	173.16	-229.34
1.0	5.0	A	A	A	137.49	140.18	141.50	127.36	105.54	83.38	919.91	575.24	195.24
1.5	5.0	A	A	A	133.88	136.72	138.09	128.37	105.79	83.49	852.10	506.06	125.38
2.0	5.0	B	A	A	105.07	134.13	135.52	149.52	106.05	83.60	791.48	438.36	57.03
2.5	5.0	B	A	A	100.00	132.01	133.44	149.52	106.32	83.72	746.84	371.79	-10.21
3.0	5.0	B	A	A	94.93	130.21	131.67	149.52	106.59	83.84	698.88	306.23	-76.48
3.5	5.0	B	B	A	89.51	89.51	130.12	149.52	110.80	83.96	652.77	252.28	-141.92
4.0	5.0	B	B	B	83.17	83.17	83.17	149.52	110.80	85.92	609.54	209.05	-190.50
4.5	5.0	B	B	B	74.37	74.37	74.37	149.52	110.80	85.92	569.99	169.50	-230.05
5.0	5.0	B	B	B	0.00	0.00	0.00	149.52	110.80	85.92	537.53	137.04	-262.51

Other parameters: $c = 1$, $w_2 = 5$, $b_2 = 10$, $a_2 = 5$, $V = 1200$. $(\mathbf{x}_1, \mathbf{x}_2)$ follows a Gaussian distribution with $\mu = (100, 100)$, $\sigma = (20, 20)$, $\rho = 0$.

Table EC.2 Summary of the project firm's optimal decision and expected profit for $b_2 \in \{5, 10, 20\}$.

		OPT Region			Q_m^*			T_m^*			Expected Profit		
w	w_1	5	10	20	5	10	20	5	10	20	5	10	20
1.0	3.0	A	A	A	136.34	140.18	143.80	83.56	105.54	117.30	627.78	575.24	537.37
1.5	3.0	A	A	A	132.56	136.72	140.63	83.78	105.79	117.55	560.61	506.06	466.30
2.0	3.0	A	A	A	129.68	134.13	138.25	84.00	106.05	117.81	495.08	438.36	396.61
2.5	3.0	A	A	A	127.31	132.01	136.33	84.24	106.32	118.07	430.86	371.79	327.99
3.0	3.0	A	A	A	125.26	130.21	134.70	84.47	106.59	118.35	367.71	306.23	260.27
1.0	3.5	A	A	A	136.34	140.18	143.80	83.56	105.54	117.30	627.78	575.24	537.37
1.5	3.5	A	A	A	132.56	136.72	140.63	83.78	105.79	117.55	560.61	506.06	466.30
2.0	3.5	A	A	A	129.68	134.13	138.25	84.00	106.05	117.81	495.08	438.36	396.61
2.5	3.5	A	A	A	127.31	132.01	136.33	84.24	106.32	118.07	430.86	371.79	327.99
3.0	3.5	A	A	A	125.26	130.21	134.70	84.47	106.59	118.35	367.71	306.23	260.27
3.5	3.5	B	A	A	0.00	128.61	133.28	87.48	106.88	118.63	331.48	241.57	193.29
1.0	4.0	A	A	A	136.34	140.18	143.80	83.56	105.54	117.30	627.78	575.24	537.37
1.5	4.0	A	A	A	132.56	136.72	140.63	83.78	105.79	117.55	560.61	506.06	466.30
2.0	4.0	A	A	A	129.68	134.13	138.25	84.00	106.05	117.81	495.08	438.36	396.61
2.5	4.0	A	A	A	127.31	132.01	136.33	84.24	106.32	118.07	430.86	371.79	327.99
3.0	4.0	B	A	A	86.51	130.21	134.70	86.78	106.59	118.35	372.36	306.23	260.27
3.5	4.0	B	A	A	76.99	128.61	133.28	86.78	106.88	118.63	331.31	241.57	193.29
4.0	4.0	B	B	A	0.00	0.00	132.01	86.78	112.84	118.91	297.78	204.98	126.98
1.0	4.5	A	A	A	136.34	140.18	143.80	83.56	105.54	117.30	627.78	575.24	537.37
1.5	4.5	A	A	A	132.56	136.72	140.63	83.78	105.79	117.55	560.61	506.06	466.30
2.0	4.5	A	A	A	129.68	134.13	138.25	84.00	106.05	117.81	495.08	438.36	396.61
2.5	4.5	A	A	A	127.31	132.01	136.33	84.24	106.32	118.07	430.86	371.79	327.99
3.0	4.5	B	A	A	91.39	130.21	134.70	86.29	106.59	118.35	376.97	306.23	260.27
3.5	4.5	B	B	A	84.71	84.71	133.28	86.29	111.67	118.63	332.89	246.36	193.29
4.0	4.5	B	B	B	75.59	75.59	75.59	86.29	111.67	129.82	292.65	206.11	129.90
4.5	4.5	B	B	B	0.00	0.00	0.00	86.29	111.67	129.82	259.69	173.16	96.94
1.0	5.0	A	A	A	136.34	140.18	143.80	83.56	105.54	117.30	627.78	575.24	537.37
1.5	5.0	A	A	A	132.56	136.72	140.63	83.78	105.79	117.55	560.61	506.06	466.30
2.0	5.0	A	A	A	129.68	134.13	138.25	84.00	106.05	117.81	495.08	438.36	396.61
2.5	5.0	A	A	A	127.31	132.01	136.33	84.24	106.32	118.07	430.86	371.79	327.99
3.0	5.0	B	A	A	94.93	130.21	134.70	85.92	106.59	118.35	380.11	306.23	260.27
3.5	5.0	B	B	A	89.51	89.51	133.28	85.92	110.80	118.63	333.98	252.28	193.29
4.0	5.0	B	B	B	83.17	83.17	83.17	85.92	110.80	127.81	290.75	209.05	136.75
4.5	5.0	B	B	B	74.37	74.37	74.37	85.92	110.80	127.81	251.20	169.50	97.19
5.0	5.0	B	B	B	0.00	0.00	0.00	85.92	110.80	127.81	218.74	137.04	64.74

Other parameters: $c = 1$, $w_2 = 5$, $b_1 = 4$, $a_2 = 5$, $V = 1200$. $(\mathbf{x}_1, \mathbf{x}_2)$ follows a Gaussian distribution with $\mu = (100, 100)$, $\sigma = (20, 20)$, $\rho = 0$.