Globalization

and

Jobless Recoveries

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Abstract

Slow rebounds in employment have become a salient feature of recoveries from recessions over the past few decades. During this time, U.S. production has become increasingly globalized. In this paper, I provide evidence that offshoring contributes to slow recoveries in labor markets. Using data from the Current Population Survey, I show that employment in offshorable occupations mimics employment in routine occupations, recovering more slowly than other types of occupations. Additionally, I use data from the Bureau of Economic Analysis on activities of multinationals to show that offshoring contributes directly to this phenomenon. I then provide a theoretical framework that rationalizes these observations in the context of a modified growth model.

1 Introduction

For much of the post-war era, labor markets began to recover one to two quarters after GDP reached its trough. Moreover, this growth tended to be robust. However, after the three most recent recessions (1990, 2001, and 2007), labor recoveries have been both slow and weak relative to their predecessors, earning them the moniker "jobless recoveries." Even when one accounts for the fact that GDP growth was slower in these recoveries than in earlier ones, the sluggishness of labor market recoveries is remarkable. Much attention has been paid to this change and the uncharacteristically high and persistent unemployment rate that followed the Great Recession renewed interest in the jobless recovery phenomenon. In this paper, I argue that jobless recoveries are related to trend growth in emerging markets and the related increase in global production opportunities. Emerging markets offer companies alternative means of expansion, but the costliness of reallocation encourages companies to wait until the potential benefit of reallocation outweighs the costs. Recessions provide an opportunity to reallocate because lower productivity in the advanced country lowers the relative cost of reallocating resources during recessionary periods. I offer evidence that this reallocation occurred over the same time period in which jobless recoveries emerged, and that occupations that are more easily offshored are the ones that recover most slowly from recessions. I then provide a counterfactual exercise which shows that if multinationals had expanded U.S. employment at the same rate that they expanded employment in developing countries, jobless recoveries could have been mitigated.

In order to explore my hypothesis further, I build a modified growth model in which multinational corporations choose to produce in either an advanced, high-productivity country whose productivity is not growing or in an emerging, lower-productivity country with growing productivity. The multinational produces a final consumption good using labor and managerial services[1] which are produced in the advanced economy but can be reallocated and used for production in the emerging economy. There are two forces operating in the model. The first is the relative growth of the emerging country, which leads to a secular shift in production to the emerging market. The second mechanism slows this secular shift, ensuring that it occurs primarily during recessions: a cost of adjusting resources from one country to the next. The particular resource that is shifted in the model is managerial services. Adjustment costs cause the shift in production to occur primarily during recessions, leading to the emergence of jobless recoveries. Essentially, recessions are "cheap" times to reallocate resources.

I show that the theoretical model is consistent, under certain conditions, with the emer-

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[1]Similar to those proposed by Burstein and Monge-Naranjo (2009).

gence of jobless recoveries. I then use a numerical exercise to show that falling production of the consumption good in the advanced economy does not coincide with falling GDP. Thus, without adjustment costs, the model produces increasing GDP and falling labor; moreover, labor productivity rises as factors are reallocated. With adjustment costs, recessions become a time when firms are willing to pay to make adjustments, shifting resources to the more efficient production location. Therefore, the model produces large and sustained drops in labor in the advanced economy following a recession, while GDP recovers as the emerging market grows and production shifts from the advanced to the emerging economy. Thus, the model is able to produce a jobless recovery. Additionally, the model is consistent with increasing income inequality across individuals in the advanced economy. This is due, in part, to a decrease in labor demand for the laborer households in the economy. It is also because labor by managerial households becomes relatively more valuable as productivity in the emerging market grows.

Related Literature

This paper is most closely related to the recent work of Jaimovich and Siu (2015). They hypothesize that jobless recoveries can be tied to the recent reduction of routine jobs in the economy and to the increased concentration of employment in the tails of the occupational skills distribution. They show that the vast majority of the shift in the occupational distribution occurred around and during the three most recent recessions, and that the recoveries that followed were jobless. Following the literature on job polarization,[2] they attribute the drop in employment in jobs that predominately rely on routine skills to technological change which is skill- or routine-biased. I propose a different mechanism for the shift in the labor composition of employment. As has been recognized in several empirical studies,[3] routine jobs are also those that can be easily offshored. I see my work as complementary to that of Jaimovich and Siu, who recognize that there may be a role for offshoring and outsourcing in the job polarization literature, but do not focus on it. In this paper, I show that offshorable occupations follow similar time paths as routine occupations and that multinational employment decisions impact the overall labor market in the U.S., suggesting that international forces could be a contributing factor to both trend and cycle declines in employment.

My paper is related to two additional strands of literature. The first is a growing body of work that explores the jobless recovery phenomenon. Bachmann (2009) offers an increase in labor hoarding as an explanation for jobless recoveries. According to this theory, when firms retain redundant workers during downturns, hiring is weak during the subsequent recovery.

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[2]Autor (2010), Autor et al. (2006), and Goos et al. (2014)

[3]See, for example, Ebenstein et al. (2014), Goos et al. (2014), and Liu and Trefler (2011).

This theory implies, counterfactually, that the recent recoveries should be associated with increasingly pro-cyclical labor productivity. In a related work, Berger (2012) builds a model in which firms use recessions as opportunities to streamline their workforce. He argues that firms have a greater ability to do this in the recent past than was historically the case due to the decline of union power. While Berger is able to generate weak labor recoveries and acyclical labor productivity, his paper suggests, counterfactually, that the pattern of joblessness arises from increased job destruction rates. However, the data show that while job losses certainly increase around recessions, jobless *recoveries* are related to low job *creation* rates. He relies on firms growing "fat" in good times, or booms, and shedding some of the inefficiencies during recessions. The mechanism that I propose is able to generate acyclical labor productivity and weak job creation after recessions. In another related study, Garin, Pries, and Sims (2013) present a theory in which the Great Moderation and jobless recoveries are related. They hypothesize that reallocation shocks have become relatively more important than aggregate shocks. My work is complementary to theirs in that it offers an reason that this may be the case.

This paper also contributes the theoretical literature on job market impacts of globalization, included international competition via both trade and increased multinational activity. I draw on the observations of Autor, Dorn, and Hanson (2013), who show that increased competition from China, in the form of imported goods, can account for a large portion of the decline in U.S. manufacturing employment. They show that rising import exposure increases unemployment and lowers labor force participation. They concentrate on an empirical exploration of the downward trend from 1990 to 2007, while I offer a reason that this trend exists; moreover, I tie the trend to jobless recoveries. In a related paper, Kondo (2012) uses an alternate measure of import competition, finding larger impacts on the broader labor market, not just the market for manufacturing employees. He finds that in addition to reducing unemployment and labor force participation, increased import competition is associated with lower job creation rates and high job destruction rates. There is also a large theoretical literature which ties offshoring to declining domestic employment, including Antras et al (2006) and Grossman and Rossi-Hansberg (2008). To my knowledge, I am the first to consider the way in which an economy recovers from a recession in a world with increased exposure to globalization.

The rest of the paper is organized as follows. Section 2 documents the existence and features of jobless recoveries and then provides a link between this observation and the extent to which occupations are easily offshored. Section 3 lays out a simple theoretical model, while Section 4 provides the main theoretical results from the model. In Section 5, I conduct a quantitative experiment to demonstrate that the model produces jobless recoveries,

and, Section 6 concludes.

2   Empirical Evidence

This section begins by documenting the existence of jobless recoveries. I then show that routine jobs and offshorable jobs follow similar patterns in the data and both fail to recover after recent recessions. Finally, I show evidence that offshored jobs can account for some of the sluggishness in labor market recoveries in the United States.

2.1   Jobless Recoveries

Slow growth in labor markets has become a consistent feature of economic recoveries in the past three decades. In order to illustrate this, in Figure 1, I plot the percent change in total non-farm establishment employment relative to its level at the trough of GDP for the post-war recession, with each line representing a different recovery. The blue lines are all of the pre-1990 recoveries and the red lines are the three recoveries that occurred after 1990. As can be seen, all pre-1990 recessions feature labor market rebounds within a few months of inflection point for GDP. In contrast, the three most recent recoveries feature employment that continues to decline for at least 18 months after output begins to recover.
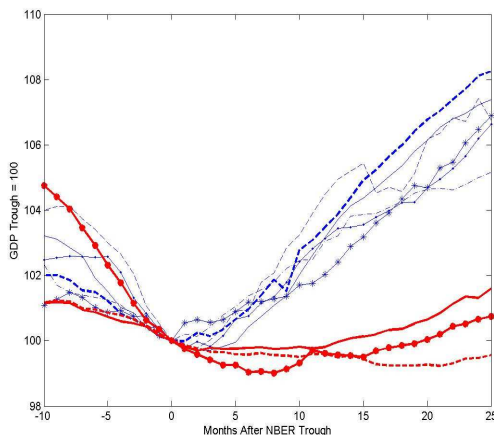

Figure 1: Employment relative to Trough of Recession

This is further highlighted by Table 1, which measures the number of months that elapsed before total non-farm employment per capita recovered to its level at the trough of the business cycle. By construction, GDP begins to recover zero months after each recession. The first row reports the number of months elapsed before non-farm employment per capita

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	Early			Recent		
Months to:	1970	1975	1982	1991	2002	2009
Turn-around	1	2	1	18	23	41
Return to Trough	2	2	4	33	55	78

Table 1: Measures of Recovery

hit its inflection point, moving from negative to positive growth. The second row of the table reports the number of months elapsed before non-farm employment per capita to returned to its level at the trough of the business cycle. Note that the second row does not count the number of months elapsed before employment to reached its cyclical peak, but rather the number of months elapsed before employment to rebounded to its level when GDP began to exhibit positive growth. The first row shows that, in earlier recessions, recovery in output was followed quickly by a turnaround in labor markets, while in more recent recessions, labor markets continued to decline for many months after GDP began to expand. The second row illustrates the decreased speed of labor market recoveries, with pre-1990 employment per capita returning to its trough level within a few months and post-1990 employment per capita taking up to 78 months to do the same. Thus, the jobless recovery phenomenon is characterized by a marked decrease in recovery speed of all measures of labor market health.

2.2 Joblessness and Offshoring

In order to explore the impact of offshoring on labor market recoveries, I follow the recent work of Jaimovich and Siu (2015), who show that routine occupations experience both larger downturns than non-routine occupations in recessions and fail to recover from these large job losses. Ebenstein et al. (2014) show that increased offshoring shifts workers within sectors into less offshorable jobs and puts downward pressure on wages. I combine the approaches of these two works to examine whether offshorable occupations recover more slowly from recessions. I use data from the monthly files from the Current Population Survey (CPS) and construct employment by occupation in order to address this question. I follow Autor and Dorn (2013) in constructing a measure of the "offshorability" and "routineness" of an occupation. These authors create a measure of how offshorable a job is using the Dictionary of Occupational Tasks and the Occupational Information Network (O*NET) in order to quantify the extent to which a given occupation requires face-to-face interaction or onsite work. Those occupations whose responsibilities include many tasks that can easily be completed remotely are more offshorable than those occupations that require on-site interaction. Likewise, occupations that require tasks that are easily codified are more routine than those that do not.
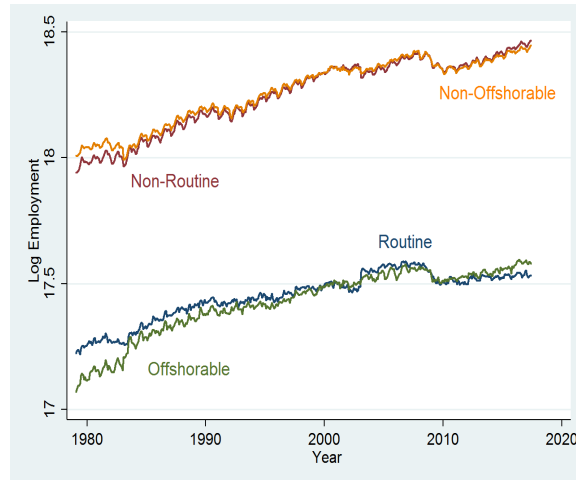
Figure 2: Routine vs. Offshorable Employment

I compare my findings with those of Jaimovich and Siu by plotting routine versus non-routine employment on the same plot as offshorable versus non-offshorable employment in Figure 2. This figure uses Autor and Dorn's definitions of these groups to display the time series in question.[4] The figure shows that employment in offshorable and routine occupations follow very similar paths.[5] Furthermore, it also illustrates the finding from Jaimovich and Siu that employment in routine occupations has been flat or falling, whereas employment in non-routine occupations has been rising. This same pattern holds when occupations are subdivided into offshorable versus non-offshorable; employment those occupations that are most easily offshored has been flat or declining while employment in non-offshorable occupations has been increasing.[6]

Jaimovich and Siu (2015) showed that routine occupations fail to recover from recessions, whereas non-routine occupations recover quickly. I take this finding as given and illustrate that employment in offshorable occupations mimics that in routine occupations following recessions by showing that routine and offshorable occupations follow the same time paths.[7] In Figures 3 and 4, I conduct the same exercise as is conducted in Jaimovich and Siu (2015), but add employment per capita in offshorable occupations. Following their work, the data have been deseasonalized and band pass filtered to remove fluctuations at frequencies higher

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[4]All employment numbers have been logged.

[5]In fact, the correlation between occupations that are offshorable and those that are routine over the entire sample is 0.74.

[6]One might worry that the fact that manufacturing has been declining as a share of total employment implies that fewer and fewer jobs might be offshorable. See Figure 15 in the Appendix to see that this is not the case. Many service sector jobs are offshorable; for example, customer service jobs such as call center workers or tech support agents are both easily offshored.

[7]This should no be surprising, given the high correlation between occupations that are routine and those that are offshorable.

than 18 months. In Figure 3, I plot employment in routine, non-routine, offshorable, and non-offshorable occupations in the 1982 recession as a benchmark, with the solid black line representing employment in routine occupations and the solid red line representing employment in offshorable occupations. The dashed lines represent employment in non-routine (black) and non-offshorable (red) occupations. As the figure shows, employment in all types of occupations recovered quickly, beginning to rise within a few months of the recession's trough.
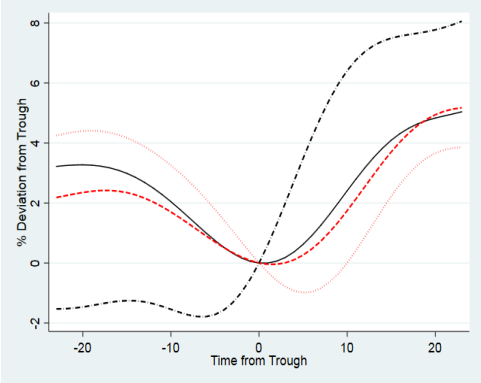

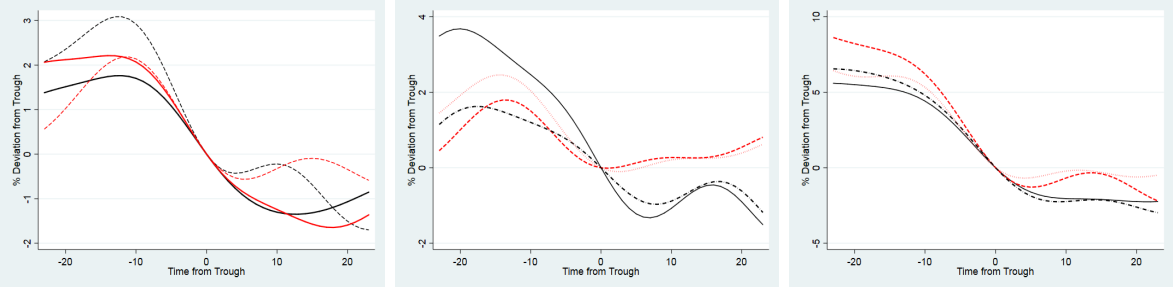Figure 3: Benchmark - 1982 Recession/Recovery

Figure 4: Jobless Recoveries

In contrast, when we look at the panels of Figure 4, we can see that the picture changes. In all three recessions, offshorable and routine occupations recover slowly, while employment in non-routine and non-offshorable occupations recovers in a more robust way. From this, I infer that both the replacement of routine occupations by new technologies, a phenomenon which has been explored in other papers, and the offshoring of eligible occupations may have contributed to the jobless recovery phenomenon.

2.3 Jobs Offshored

Given the findings in the previous section, one might naturally wonder whether jobs have actually been offshored. In order to address this question, I turn to data gathered by the Bureau of Economic Analysis on the activities of multinational enterprises. The panels of Figure 5 display data from the Bureau of Economic Analysis's Multinational Enterprise database on log employment by all affiliates of U.S. parents, separated by location of employment from 1983 to 2013.[8] I have aggregated employment by U.S.-based multinational enterprises (MNEs) in emerging economies[9] and I plot employees of multinational enterprises in both the U.S. and emerging economies. These plots show that emerging market employment has accounted for the vast majority of employment growth for MNEs, with employment in emerging economies almost tripling between 1990 and 2013. In contrast, domestic employment by U.S. multiantionals has grown by about 20% over the same period.

(a) Log Employees   (b) Employees relative to 1990

Figure 5: Multinational Employment

Though multinationals account for less than 1% of all firms in the United States, they employ a large percentage of all employees in the U.S. economy. For example, in 2015, U.S. multinational parent companies accounted for 20% of all non-farm employment[10] in the United States, which only accounts for the employees of those multinational enterprises

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[8]The data is available until 2015; however, a change in sampling creates a jump between 2013 and 2014. In order to avoid that issue, I restrict my sample to observations before 2014. Moreover, the data before 2009 includes only non-bank multinationals and after includes all multinationals. Therefore, I have subtracted employment by depository institutions from aggregate employment by both parents and affiliates for 2009 forward.

[9]I hold the set of emerging economies fixed as defined by the World Bank in 1990.

[10]Current Employment Survey, BLS.

within the United States. Foreign employees of those same firms are equal to about 12% of total non-farm employment in the U.S. Although it's not clear that we should think of foreign employment by multinationals as jobs that would otherwise be done by U.S. workers, these measures confirm that multinationals are major players within the American labor market. Furthermore, one might worry that the importance of multinationals might be falling as the U.S. moves towards a more service-oriented work force; however, the share of total employment accounted for by multinationals has been relatively steady, ranging from roughly 17% to 20% of U.S. employment during the period of interest.

One might also wonder what determines the locations into which multinationals expand. In order to explore this question, I combine the multinational employment data with information on gross domestic product from the World Bank for all of the countries for which U.S. MNE employment data is available. I then calculate the percent changes in employment and in GDP in order to run the following simple regression,

$$\Delta employees_{i,t} = \beta_0 + \beta_1 \Delta GDP_{i,t-1} + \gamma_i + \epsilon_{i,t},$$

where $\Delta employees_{i,t}$ measures the year-to-year change in multinational employees in a given country, $\Delta GDP_{i,t-1}$ is the GDP growth rate in the previous year, and γ_i is a country fixed effect.[11] The goal of estimating this relationship to is establish whether multinationals respond to GDP growth in the host country by expanding employment. The idea here is that multinationals make decisions about where to invest using the country's past performance as an indicator of future growth. Table 2 indicates that GDP growth has a strong positive correlation with increased multinational employment in the following year. As is well known, developing countries have accounted for much of the growth in world GDP that has occurred in the past 30 years. This evidence indicates that the production location decisions of multinationals are correlated with the past performance of GDP or productivity of a given country, meaning that multinationals expand more readily into countries with stronger past productivity growth.

Variable	Coefficient
Constant	−0.530
	(0.585)
Lagged GDP	0.846***
	(0.204)
Country FE?	Yes

Table 2: MNE Employment and GDP Growth

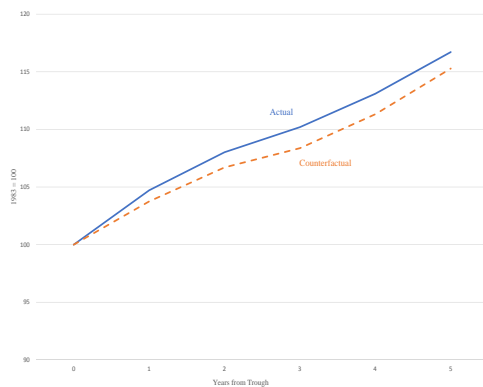[11]The equation is estimated using robust standard errors.
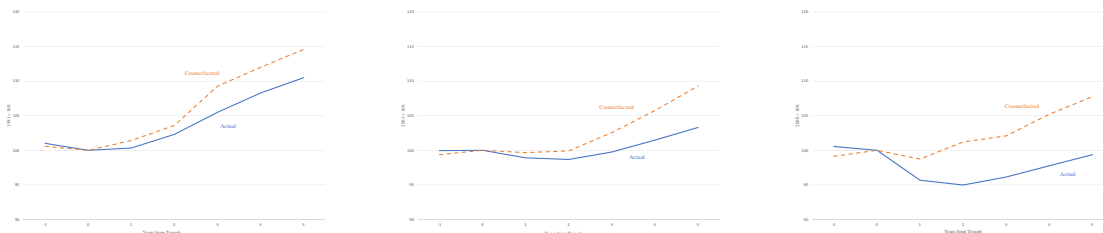
Figure 6: Benchmark - 1982 Recovery

Figure 7: Jobless Recoveries - Actual vs. Counterfactual

I now conduct a counterfactual in order to assess the importance of the shift in multi-national employment from the United States to emerging economies on overall employment in the United States. It's not immediately obvious what kind of counterfactual would best illustrate this impact, given that the economy experienced many changes over the course of the decades of interest. I will use a counterfactual which supposes what might have occurred if multinationals did not have the option or desire to expand into emerging markets, but continued to grow their world-wide employment by the same amount as is observed in the data. The hypothesis I am exploring in this paper is that multinationals choose to expand employment in emerging markets as a substitute for expanding employment domestically. Therefore, the thought experiment that I am conducting explores the situation wherein multinationals were not willing or able to expand their employment abroad and instead had to satisfy the increase in world demand for their goods via an increase in domestic employment.

In order to do this, I combine data on non-farm payroll employees with multinational employment in the United States and in emerging economies. In Figures 6 and 7, I first plot the actual total non-farm payroll employment relative to the trough of the recession as

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the solid blue line for the 1982, 1991, 2001, and 2008 recessions. I then include how total non-farm payroll would have behaved if the growth rate of U.S. multinational employment had been the same as the growth rate of U.S. multinational employment in emerging markets in the given year.[12] In order to construct this measure, I build a counterfactual time series for U.S. multinational employment starting at the beginning of the observation period. I hold fixed the number of domestic employees of multinationals in 1983 and then replace each subsequent year's employment with what domestic employment would have been had it grown at the same rate as MNE employment in emerging economies. I then subtract actual multinational domestic employment from total non-farm payroll employment and replace it with this constructed series. Figure 8 displays the counterfactual series for total non-farm payroll employment over the entire sample period.
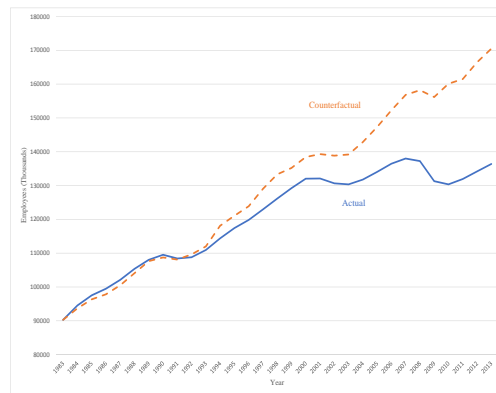

Figure 8: Counterfactual Domestic MNE Employment

As can be seen, had domestic multinational employment grown at the rate that foreign multinational employment grew, labor market recoveries would have been much more robust. Furthermore, had multinationals expanded in the United States in a way similar to their expansion in emerging economies, overall growth in employment in the United States would have been much stronger, mirroring the employment growth observed in the 1980s.

I interpret these observations as suggestive evidence that offshoring contributes to the emergence of jobless recoveries. Next, I build a theoretical model that is consistent with this mechanism.

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[12]I conduct several different counterfactuals and the picture is very similar. Those results are available upon request.

3 Theory

In this section, I develop a modified growth model which is consistent with the empirical evidence above. In the model, growth in an emerging country can lead to jobless recoveries in the advanced country. I first describe the problem solved by firms (multinationals) and then the problem solved by two types of households.

3.1 The Environment

Time is infinite and discrete. There are two countries: Advanced (A) and Emerging (E). In Country A, there are two types of households, whereas in Country E, there is only one type of household. Each household consumes a single consumption good and saves with a one-period bond. Households of type "M," which only exist in country A, operate a linear backyard technology in order to produce managerial services, which they then rent to firms. These services can be used in either country but are produced only in Country A. L-type households live in both countries and rent labor services to firms' production facilities in the country where they reside. The proportion of M-type to L-type households in Country A is fixed at $\frac{\alpha}{1-\alpha}$. The single consumption good is produced by a mass of perfectly competitive multinational firms, which are headquartered in Country A. These firms can choose production locations and will rent labor from the households that are located in the country of production. The two countries differ in their productivities. Country A is more productive than Country E at $t = 0$ but does not grow over time, whereas Country E's productivity grows deterministically, approaching the productivity of Country A over time.

3.2 Multinationals

The multinationals operating in Country i produce output (y_t^i) at time t by combining labor (l_t^i) and managerial "know-how" or services (m_t^i),

$$y_t^i = z_t^i (m_t^i)^\theta (l_t^i)^\nu, \tag{1}$$

where

$$\theta + \nu \leq 1.$$

Managerial services are rented from M-type households in Country A and may be reallocated by the multinationals across countries. The amount of managerial services that are used in country i are denoted by m_t^i and the total amount of managerial services hired by the firms by $\bar{m}_t$, which is equal to the sum of managerial services used in Country A, m_t^A, and managerial

services used in Country E, m_t^E. Notice that managerial services are mobile across borders, but labor is not. The term z_t^i represents country-specific productivity. The multinationals will face productivity processes whose growth varies across countries. In particular,

$$z_0^A > z_0^E, \tag{2}$$

$$z_t^A = z_0^A, \quad \forall t, \tag{3}$$

$$z_t^E = \rho^t z_0^E + (1 - \rho^t) z_0^A. \tag{4}$$

This means that productivity will be constant in Country A, whereas productivity in Country E will be growing at rate ρ, asymptotically approaching the productivity level of Country A.

Multinationals pay a local wage to laborers $(w_{L,t}^i)$. The rent paid to managerial services will be the same no matter where the services are used, since they are all produced in Country A. The multinational pays a cost for adjusting the level of managerial services in each country. Therefore, taking prices as given, they maximize the present value of the stream of dividends:

$$\max_{m_t, m_t^A, m_t^E, l_t^A, l_t^E} \sum_t p_t D_t \tag{5}$$

subject to

$$D_t = d_t^A + d_t^E, \tag{6}$$

$$d_t^i = y_t^i - w_{L,t}^i l_t^i - w_{M,t}\overline{m_t} - \frac{\varphi}{2}\left(m_t^i - m_{t-1}^i\right)^2, \tag{7}$$

$$y_t^i = z_t^i(m_t^i)^\theta (l_t^i)^\nu. \tag{8}$$

As can be seen in Equation 6, in a given period, total dividends are the sum of dividends earned in Country A and in Country E. Dividends earned in a Country i, expressed in Equation 7, are equal to output produced in Country i, y_t^i, minus the wage bill for laborers, $w_{L,t}^i l_t^i$, minus the rental bill for all managerial services used, $w_{M,t}\bar{m}_t$, minus the cost of adjusting managerial services in Country i, $\frac{\varphi}{2}\left(m_t^i - m_{t-1}^i\right)^2$. The adjustment cost associated with changing the level of managerial services used in a given country can be thought of as the cost associated with managers making contacts in the new country or learning a different language in order to be able to do business in the new location. The adjustment cost will slow the relocation of production to Country E, even as Country E becomes more productive.

3.3 Households

There are two types of households in the advanced economy: L-type households and M-type households. L-type households make up a fraction $1-\alpha$ of the total economy, whereas M-type

households make up a fraction α. The emerging economy has only L-type households.

In each period, t, L-type households in Country i receive labor income, $w_{L,t}^i n_{L,t}^i$; earnings on their asset position, $(1 + r_{b,t})b_{L,t}^i$; and some fraction ϕ_L^i of the total dividends of the firm (D_t). Households choose consumption, $c_{L,t}^i$, labor supply, $n_{L,t}^i$, and asset position, $b_{L,t+1}^i$. The maximization problem for the household is thus

$$\max_{c_{L,t}^i, n_{L,t}^i, b_{L,t+1}^i} \sum_t \beta^t \frac{\left(c_{L,t}^i - \frac{n_{L,t}^i{}^{1+\gamma}}{1+\gamma}\right)^{1-\sigma}}{1-\sigma}, \tag{9}$$

subject to

$$p_t(c_{L,t}^i + b_{L,t+1}^i) = p_t \left(w_{L,t}^i n_{L,t}^i + (1 + r_{b,t})b_{L,t}^i + \phi_L^i D_t\right) \tag{10}$$

$$0 \leq n_{L,t}^i \leq 1. \tag{11}$$

Households take all prices $(p_t, w_{L,t}^i, r_{b,t})$ as given.

M-type households have access to a linear backyard technology which they operate in order to produce an intermediate input, "managerial services" $\bar{m}_t$, which they sell to the firm at price $w_{M,t}$. Income is thus composed of rental income, $w_{m,t}\bar{m}_t$; earnings from bonds, $(1 + r_{b,t})b_{M,t}$; and a fraction ϕ_M of the firm's dividend payments. They choose consumption, $c_{M,t}$, labor supply, $n_{M,t}$, and an asset position, $b_{M,t+1}$. The maximization problem for these households is thus

$$\max_{c_{M,t}, n_{M,t}, \bar{m}_t, b_{M,t+1}} \sum_t \beta^t \frac{\left(c_{M,t}^i - \frac{n_{M,t}^{1+\gamma}}{1+\gamma}\right)^{1-\sigma}}{1-\sigma}, \tag{12}$$

subject to

$$p_t(c_{M,t} + b_{M,t+1}) \quad = \quad p_t \left(w_{M,t}\bar{m}_t + (1 + r_{b,t})b_{M,t} + \phi_M D_t\right) \tag{13}$$

$$\bar{m}_t \quad = \quad n_{M,t} \tag{14}$$

$$0 \leq \quad n_{M,t} \leq \quad 1. \tag{15}$$

The households take all prices $(p_t, w_{M,t}, r_{b,t})$ as given.

There is no difference between the two types of households other than the service that they provide to the economy. L-type households provide labor that can only be used in-country, whereas M-type households provide a service that is transferable across countries which allows the multinational to produce goods both domestically and abroad. It is important to the theoretical results that both households actually work within Country A, although the service produced by the M-type household may then be rented to the production location in

Country E.[13] Note that Type-M households do not participate in the formal labor market, as they are self-employed and operate a backyard technology to produce a service, which will be sold on the market. Therefore, their labor will not be included in the labor market clearing condition and their services will clear a separate market. However, their labor hours will be included in the aggregate measure of employment, to be consistent with the way that aggregate labor is measured in the data.

3.4   Competitive Equilibrium

A competitive equilibrium in this economy is a set of quantities $\{d_t^i, y_t^i, \overline{m_t}, m_t^i, n_{L,t}^i, c_{L,t}^i, c_{M,t}, b_{L,t}^i, b_{M,t}\}_{i \in \{A,E\}}$ and prices $\{p_t, r_{b,t}, w_{L,t}^i, w_{M,t}\}$ that are consistent with

1. the firm's maximization problem,

2. the household maximization problems,

3. managerial hours ,
$$\overline{m}_t = n_{M,t}, \tag{16}$$

4. managerial services market clearing,

$$\overline{m}_t = m_t^A + m_t^E, \tag{17}$$

5. labor market clearing in each country i,

$$l_t^i = n_{L,t}^i, \tag{18}$$

6. bond market clearing,
$$\sum_{i \in \{A,E\}} b_{L,t}^i + b_{M,t} = 0, \tag{19}$$

and

7. the aggregate resource constraint

$$\sum_i c_{L,t}^i + c_{M,t} = \sum_i y_t^i. \tag{20}$$

---

[13]Both types of households are subject to Greenwood–Hercowitz–Huffman preferences (Greenwood et al 1998), which are common in open economy macro studies. I employ these preferences so that I can isolate the impact of globalization on labor supply without the conflating effects of a decline in income on labor supply. Moreover, they allow me to analytically derive the conditions under which jobless recoveries emerge.

It is also useful to define GDP and employment for the advanced economy. GDP can be defined using the income approach as

$$GDP_t = p_t \left[\alpha \left(w_{M,t} \overline{m}_t \right) + (1 - \alpha) \left(w_{L,t}^A n_{L,t}^A \right) + D_t \right], \tag{21}$$

where $w_{M,t} \overline{m}_t$ is the income received by the managerial household in exchange for their services, $w_{L,t}^A n_{L,t}^A$ is the labor income earned by laborer households, and D_t are the dividends that are rebated back to both households. Recall that there are a fraction α managerial households in the economy. The measure of total employment that will be used is

$$Emp_t = \alpha n_{M,t} + (1 - \alpha) n_{L,t}^A, \tag{22}$$

where $n_{M,t}$ are the hours that the managerial household spends producing their services and $n_{L,t}^A$ are the working hours of the laborer household. Note that the managerial household can be thought of as being self-employed, but their labor will be included in the total measure of employment, as it is in the data.

4 Effects of Increasing Productivity in Country E: Secular Decrease in Labor

In this section, I explore the first the theoretical and then the qualitative effects of an increase in the productivity of the emerging market, relative to that of the advanced economy. The goal is to show that, under certain conditions, growth in Country E causes labor in Country A to fall while GDP rises. This exercise illustrates that the model is able to generate the trend of increasing GDP with falling employment in the advance country.

4.1 Abstracting from Adjustment Costs: Secular Changes

For the moment, let us abstract from adjustment costs in order to explore the impact of growth in the developing country in a clear way. In this case, the firm solves the following problem:

$$\max_{m_t, m_t^A, m_t^E, l_t^A, l_t^E} \sum_t p_t D_t$$

subject to

$$D_t = d_t^A + d_t^E \tag{23}$$

$$d_t^i = y_t^i - w_{L,t}^i l_t^i - w_{M,t} m_t \tag{24}$$

$$y_t^i = z_t^i (m_t^i)^\theta (l_t^i)^\nu, \tag{25}$$

so that the firms no have to pay an adjustment cost in order move managerial services from one country to the next (See Equations 7 and 24).

Proposition 1. *If $\gamma > 0$ and there are no adjustment costs ($\varphi = 0$), then production will reallocate toward Country E as Country E's productivity grows closer to the productivity of Country A, i.e., as $\frac{z_t^E}{z_t^A}$ increases.*

Proof. Recall that

$$\nu + \theta \leq 1,$$
$$1 > \nu > 0,$$
$$1 > \theta > 0.$$

Combining the first order conditions from the firm and the household problems yields the condition:

$$\left(\frac{z_t^E}{z_t^A} \right)^{\gamma+1} = \left(\frac{m_t^A}{\overline{m_t} - m_t^A} \right)^{(\theta-1)(\gamma+1)+\nu}. \tag{26}$$

The left-hand side of Equation 26 will be increasing as z^E/z^A increases so long as $\gamma > -1$, so we are searching for conditions under which the right-hand side is inversely related to the left-hand side. Notice that the right-hand side is simply the expression for the relative use of managerial services in Country A versus Country E, where I have exploited the market clearing condition on the bottom of the right-hand side of Equation 26. Therefore, I am trying to find conditions under which

$$(\theta - 1)(\gamma + 1) + \nu < 0,$$

or when the relative use of managerial services in Country A is falling relative to the use of managerial services in Country E. This can be rewritten as

$$(1 - \theta)(\gamma + 1) > \nu$$

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,or

$$(1 - \theta)\gamma + 1 \quad > \quad \nu + \theta.$$

Because $\nu + \theta \leq 1$ and $\theta \in (0, 1)$, this will be true as long as $\gamma > 0$.

Intuition. The key assumption needed for this proposition to hold is that households have preferences such that the substitution effect dominates the income effect, meaning that increasing wages will induce the household to supply more labor. Wages for the L-type household are directly proportional to the productivity of the country in which the household lives and works since there are no distortions to wage-setting in this model. Therefore, all else equal, an increase in productivity in Country E induces an increase in wages in Country E, which results in an increase in L-type labor supplied in that country. Furthermore, the return to managerial services is determined both by the productivity in Country A and the productivity in Country E. If productivity in Country E grows, the wage paid to the managerial household increases which induces said household to supply more managerial services overall. Given that I have abstracted from adjustment costs here, the firm can costlessly reallocate managerial services toward production in Country E, increasing production therein.

Proposition 2. *If $\gamma > \frac{\nu}{1-\theta} - 1$, $\gamma > \nu$, and there are no adjustment costs ($\varphi = 0$), then output in Country A will increase and employment therein will decrease as the relative productivity of Country E rises.*

Proof. Details in the appendix. The key to the proof is that everything in the model can be written as a function of $\overline{m}$, including GDP and employment. Solving for the equilibrium conditions and plugging in, we can re-write Equations 21 and 22 as

$$GDP \;=\; \alpha \overline{m}^{\gamma+1} + (1-\alpha)\left(\nu z_A\right)^{\frac{\gamma+1}{\gamma-\nu+1}} \overline{m}^{\frac{\theta(\gamma+1)}{(\theta-1)(\gamma+1)+\nu}} \left(\frac{z_E}{z_A}\right)^{\frac{\theta(\gamma+1)^2}{(\gamma-\nu+1)((\theta-1)(\gamma+1)+\nu)}} \left(1 + \left(\frac{z_E}{z_A}\right)^{\frac{\gamma+1}{(\theta-1)(\gamma+1)+\nu}}\right)^{-\frac{\theta(\gamma+1)}{\gamma-\nu+1}} \quad (27)$$

$$Emp \;=\; \alpha \overline{m} + (1-\alpha)\left(\nu z_A\right)^{\frac{1}{\gamma-\nu+1}} \overline{m}^{\frac{\theta}{(\theta-1)(\gamma+1)+\nu}} \left(\frac{z_E}{z_A}\right)^{\frac{\theta(\gamma+1)}{(\gamma-\nu+1)((\theta-1)(\gamma+1)+\nu)}} \left(1 + \left(\frac{z_E}{z_A}\right)^{\frac{\gamma+1}{(\theta-1)(\gamma+1)+\nu}}\right)^{-\frac{\theta}{\gamma-\nu+1}} \quad (28)$$

Recall that α is the fraction of M-types households in the economy and that the managerial households are self-employed in producing managerial services, equal to $\overline{m}$. Furthermore, these households are operating a linear technology to produce these services, so their labor hours are also equal to the total services that they produce, $\overline{m}$. It is easily shown that $\overline{m}$, the total employment of the managerial households, is increasing in the ratio of the productivity of Country E relative to Country A. Therefore, because $\overline{m}$ contributes relatively more to GDP than to employment, as can be seen in Equations 27 and 28, GDP increases while

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employment decreases whenever the weight of managerial services is sufficiently high that is able to counterbalance the decrease in income coming from the laborer household in Country A. It is important to note that although managerial services are being used in both countries, all of the employment by managers and all of their services are contributing to employment and GDP of Country A. Note that in the case of a Cobb-Douglas production function, ν will equal $1 - \theta$ and so the condition that $\gamma > \frac{\nu}{1-\theta} - 1$ will be true any time that $\gamma > 0$, which is the same condition as in Proposition 1.

Intuition. The key assumption here is that all managerial services are produced in Country A and treated as exports, so all wages paid to the managerial households are income in Country A. Therefore, GDP will increase as a result of an increase in income for the managerial household. In the case with no adjustment costs, the firm is exactly equalizing the marginal product of managerial services in Country A with the marginal product of managerial services in Country E. Therefore, as the productivity of managerial services in Country E rises, the firm will move production from Country A to Country E, thus decreasing the amount of L-type employment in Country A. The overall return to managerial services in Country A is increasing as a result of their usefulness in Country E, where their marginal product is increasing. Therefore, their wage income increases as long as they are sufficiently willing to supply more labor as their wage increases. The employment in Country A will fall if L-type employment falls enough to counterbalance the increase in M-type employment. However, GDP may rise if the increase in income for the M-types is sufficiently high that it counterbalances the decrease in L-type income. This happens when managerial service's share of income is sufficiently high relative to labor's share of income.

4.2   Quantitative Exercise

Secular Decline in Labor in Country A

I now turn to a quantitative exercise in which the above conditions are satisfied and show that, in the absence of adjustment costs, the model generates a trend decrease in labor, while GDP continues to grow. Table 3 reports the parameter values that were used in the quantitative exercise.

Parameter	Value	Governs
ρ	0.95	Persistence of initial productivity of Country E
β	0.96	Discount rate
γ	2	Labor elasticity
θ	0.7	Service share
ν	0.3	Labor share
α	0.5	Share of managerial households
φ	0	Cost of adjustment

Table 3: Parameter Values

Parameters were chosen to be consistent with the bounds that were derived in Propositions 1 and 2. The specific values chosen for the quantitative exercise are consistent with what has been used in existing studies. First, ρ is set to 0.95 and $1 - \rho$ can be thought of as the rate of convergence between Country A and Country E.[14] This number is in line with estimates for the rate of catch up between the United States and its emerging market counterparts. For example, the average growth rate of labor productivity for China relative to the United States from 1995 to 2015 is 6.5%, whereas the average growth rate of India relative to the United States over the same period is 4%. I choose β so that the annual risk-free interest rate is 4% and this choice doesn't impact the results in any way. The parameters that matter most for being able to qualitatively generate rising GDP with falling employment are γ, θ, and ν. Assuming γ to be greater than 0 is innocuous, as $\frac{1}{\gamma}$ is the labor elasticity in this model, and so assuming it to be greater than 0 is the same as saying that it is finite. Moreover, a labor elasticity of 0.5 is in line with what is typically used in macro models, as well as estimates from the micro data for labor elasticity of the entire population. Notice, from Proposition 2, that as long as $\gamma > \frac{\nu}{1-\theta} - 1$ and $\gamma > \nu$, we arrive at the desired result. When managerial services and labor are combined via a Cobb-Douglas aggregator, these two conditions will hold any time that $\gamma > 1$, since $\nu \in (0, 1)$. Again, this results in a labor elasticity that is less than 1, in line with the micro data. The particular choices of θ and ν are guided by data from the BEA's benchmark surveys of multinational enterprises. In the early benchmark surveys, the BEA provided information on the compensation of production versus non-production workers. Using the data from 1989, which is the last year that this information is available, I calculate that the share for production workers ranges between 27% and 34%. I set ν to be the middle of this range. Since non-production workers account for the rest of employee compensation, I set θ equal to $1 - \nu$. Finally, I set the share of managerial households, α, to 0.5. The value of α does not matter for the existence of the

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[14]More precisely, $\frac{Z_{E_t}}{Z_{A_t}} = \rho^t \frac{Z_{E_0}}{Z_{A_0}} + (1 - \rho^t)$. So the distance between the two productivities shrinks by $1 - \rho$ each period.

secular decline in employment in the United States or the existence of jobless recoveries,[15] but it will impact the size of each effect. I choose α sufficiently large that it's easy to see the two phenomena. The exact interpretation of this parameter isn't clear. In the model, it represents those households that produce something in the U.S. that can then be used as an intermediate for production in U.S. locations of multinationals or in their foreign affiliates. I use the same data from the survey of multinationals from 1989 as I used to set ν, which shows about half of employees of these multinationals are non-production workers. All parameter choices are consistent with the theoretical results and are within reasonable bounds to conduct an illustrative numerical experiment.

Figures 9a through 10 show the model predictions in a frictionless economy for the above parameterization. Notice, in Figure 9a, that GDP in the advanced economy grows, even as total labor in that economy falls. Managerial services become more valuable as more and more labor is used worldwide. In the background, productivity in the emerging country is rising, increasing demand for both m_t^E and n_t^E. Since all managerial services are produced in the advanced country, GDP rises as a result for increasing world demand for managerial services. Figure 10 shows the change in equilibrium outcomes. Notice that $\overline{m}_t$ is rising as n_t^A is falling. Generating falling labor is essentially a horse race between these two forces. Mechanically, it must be the case that labor productivity (GDP per worker) is rising in this economy, since GDP is rising as labor is falling. Figure 9b shows that this is, indeed, the case. Here labor productivity is output divided by total labor, which is the sum of labor provided by laborer households and by managerial households in Country A.

In order for GDP to rise, it must be the case that total income in the economy is rising. It must be true that wages for the L-type household in the advanced economy are falling, since wages simply equal $(n_t^A)^\gamma$ in equilibrium, which is falling. Therefore, the income of the laborer household is falling in the model and income for the managerial household must be rising in order to generate overall growth in income in the economy. This implies that income dispersion increases as the emerging country grows and the world demands more labor inputs from the managerial households and fewer labor inputs from the laborer households in the advanced country. Therefore, as an added feature, the model generates rising income inequality.

It is important to note that, while the focus here is not on the benefits of globalization, the increase in GDP experienced by Country A is being driven by its access to a growing foreign economy, Country E. Country A is able to benefit from increased globalization through higher productivity, which is driven by the ability to use resources abroad. Therefore, even though

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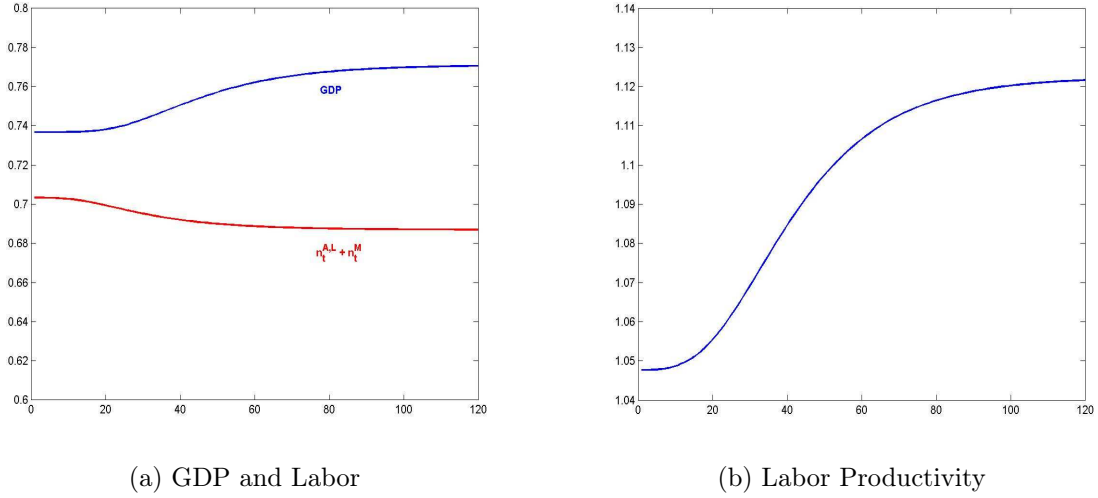[15]Note that in the proofs of the propositions, α plays no role in determining whether or not these phenomena exist.

(a) GDP and Labor

(b) Labor Productivity

Figure 9: Output and Labor Responses

Parameter	Value	Governs
ρ	0.95	Growth in Country E
β	0.96	Discount rate
γ	2	Labor elasticity
θ	0.7	Service share
ν	0.3	Labor share
α	0.5	Share of managerial households
φ	0.05	Cost of adjustment

Table 4: Parameter Values

inequality is increased through this secular change, the managerial households benefit and the overall economy is able to grow, even in the face of constant total factor productivity. Openness, therefore, has positive effect on the economy overall.

Adding Adjustment Costs: Generating Jobless Recoveries

In this section, I allow for adjustment costs and illustrate that the model generates jobless recoveries, or sustained losses in employment accompanied by growth in GDP, via a negative productivity shock to z^A. I return to the model developed in Section 3, which features an adjustment cost that a firm must pay any time it changes m_t^A or m_t^E. Therefore, even though Country E may be growing, the firm may not want to pay the adjustment cost until z^E grows sufficiently or z^A falls sufficiently. It is well known that adjustment costs slow firms' responsiveness to changes in fundamentals.

I parameterize the model as in the previous section and set adjustment costs such that
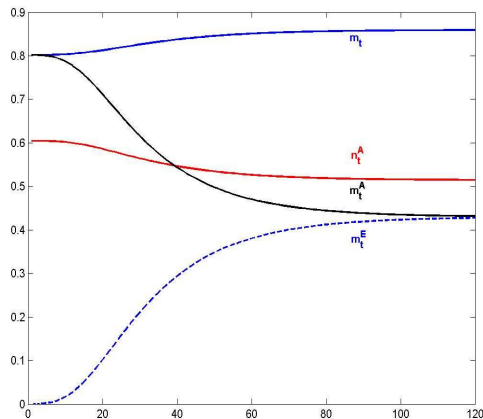
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Figure 10: Change in Allocations

when productivity is sufficiently low in Country E, the firm will choose to maintain resources in Country A rather than pay the cost of adjustment. Table 4 shows the parameterization used in the simulations. I then conduct a simulation in order to illustrate the jobless recoveries created by the model. The experiment is to allow growth in Country E, as governed by the growth parameter ρ, and then shock Country A with a one period negative productivity shock equal to one percent drop in productivity and allow this shock to decay over ten periods.

Figures 11a through 12a show the results of this experiment. As expected, before the negative productivity shock in Country A, which occurs at period 0, the firm chooses not to reallocate workers. Once the negative shock occurs, the firm chooses to reduce the proportion of managers used in Country A, m_t^A. This can best be seen in Figure 11b. Here we can also see that the reduction in labor is still a horse race between increasing overall demand for managerial services (and thus $n_{M,t}$ since managerial services are created using a linear technology with $n_{M,t}$ as its only input) and falling demand for labor in Country A. During the recovery, we see stagnant labor markets, even as GDP is increasing. In this sense, the model is able to qualitatively match the features of a jobless recovery. Moreover, as Figure 12a shows, labor productivity falls initially, but recovers very quickly. In fact, labor productivity grows even as labor inputs are stagnant or even falling. This is a feature of recent recessions which has been puzzling in the context of a standard real business-cycle model. However, this simple growth model with asymmetric growth is able to generate this feature.

Here, again, it should be noted that the economy is recovering more quickly from a recession than it would if it did not have access to the labor in Country E. In a closed economy, the recession would last until TFP, and therefore GDP, recovered to its original

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level. Instead, here, Country A is able to essentially "import" an increase in productivity from Country E. Therefore, the recovery is generated, in part, by the growth in Country E.
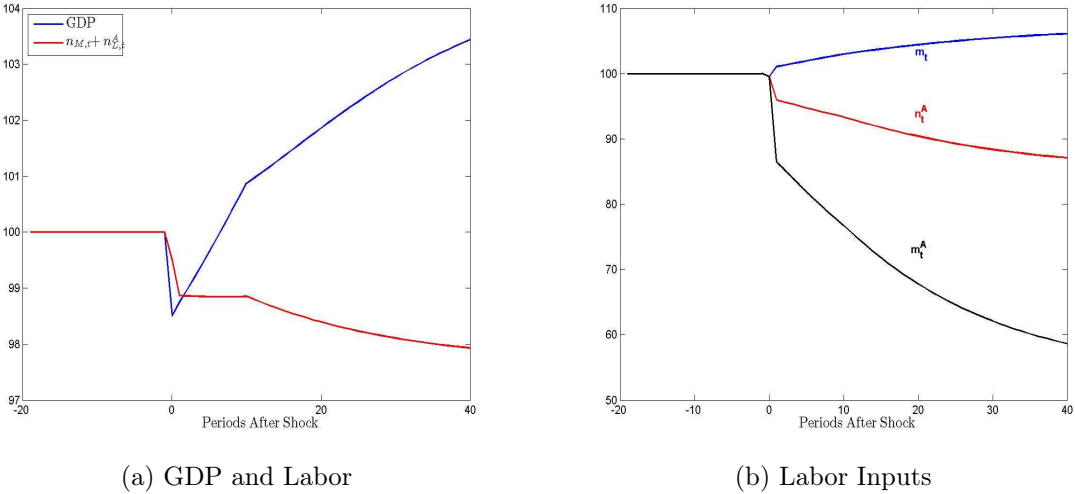

(a) GDP and Labor

(b) Labor Inputs

Figure 11: Allocations and Production

Figure 12b shows the predictions of the model for trade. The model predicts that imports should increase, causing the trade balance to fall, over the course of the recession. In fact, it predicts a large drop in the trade balance just as the negative productivity shock hits Country A. This is because reallocation occurs during this period, causing more consumption goods to be produced in Country E. Perfect risk-sharing implies that households in Country A simply borrow in order to continue to consume these goods when their income falls during the recession.

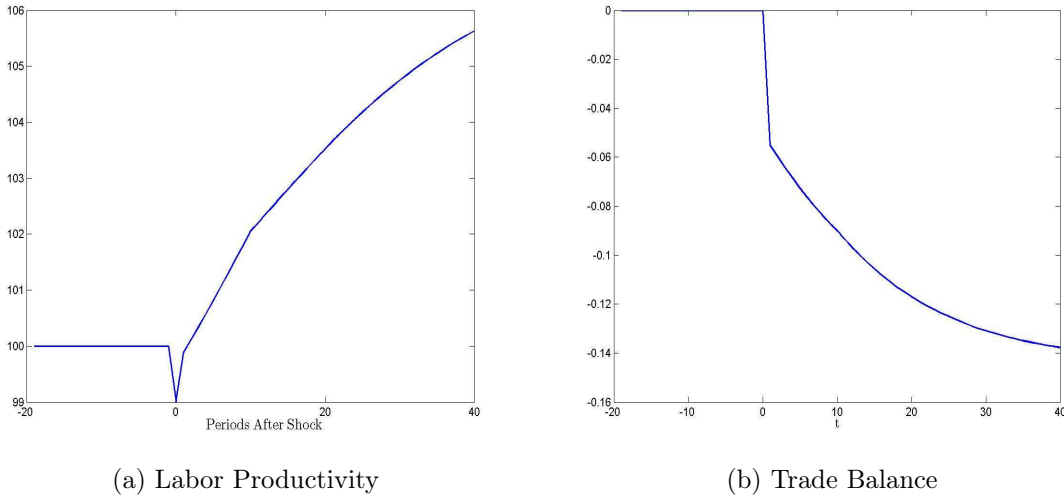(a) Labor Productivity   (b) Trade Balance

Figure 12: Labor Productivity and Trade Balance

It is notable that labor markets in Country A will never fully recover, despite the fact that the productivity shock decays after 10 periods. The permanent fall in employment comes about because, although productivity recovers, it is permanently lower relative to the growing productivity of Country E. The multinational is taking advantage of improved productivity in the foreign country, using the recession as an optimal time to move production to the growing foreign country. It is exactly during a recession that the relative benefit of producing in the foreign location rises sufficiently that the firm is willing to pay the cost associated with this move. In this sense, recessions are part of the impetus towards increasing offshore production.

5   Conclusion

In this paper, I explore the connection between globalization, offshoring, and jobless recoveries. I make a case that the increase in opportunities for firms to harness growth in emerging markets has contributed both to GDP growth in the United States and to the slow labor market recoveries that have been observed after the three most recent U.S. recessions. I show that offshorable occupations respond to recessions similarly to routine occupations, in that they recover to their pre-recession levels after the recession in the early 1980's, but fail to do so in the three most recent recessions. Furthermore, the impact of offshoring seems to be increasing over time, as production has become increasingly globalized. Further work must be done to disentangle the impact of skill-biased technical change versus import competition

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and offshoring on both the trend change and the short-term cyclical changes in employment in occupations that can be easily replaced by machines or foreign workers.

Furthermore, this paper lays the ground work for a more complete exploration of the business cycle effects of increased globalization on labor markets. An interesting and important extension of this model would be to conduct a more complete and realistically calibrated real business cycle model which incorporates the feature of increasing productivity abroad with stagnating productivity in the United States. The model that I have built suggests that increased opportunities for multinational expansion abroad may have implications for the way in which the U.S. economy responds to and recovers from recessionary pressures. A more fully calibrated model would allow researchers to understand more fully the quantitative importance of increased globalization.

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Appendix

Proof of Proposition 2:

Recall that I am trying to show that in the event of a negative productivity shock in the United States, GDP will begin to rise even as employment continues to fall. I will use the expressions from Equations 27 and 28 to show that this is true. I'll start with establishing some preliminaries. First, recall from the household's problem, we have (suppressing the time subscripts):

$$
\begin{aligned}
w_A &= n_A^{\gamma} \\
w_E &= n_E^{\gamma} \\
w_m &= \overline{m}^{\gamma},
\end{aligned}
$$

where $w_A = w_{Lt}^A, w_E = w_{Lt}^E, w_m = w_{Mt}$ and $n_A = n_{Lt}^A, n_E = n_{Lt}^E$. I will use this same notation throughout the appendix.

From the firms' problem, we have:

$$
\begin{aligned}
w_A &= \nu z_A m_A^{\theta} l_A^{\nu-1} \\
w_E &= \nu z_E m_E^{\theta} l_E^{\nu-1} \\
w_m &= \theta z_A m_A^{\theta-1} l_A^{\nu} \\
w_m &= \theta z_A m_A^{\theta-1} l_A^{\nu}.
\end{aligned}
$$

We can use the market clearing conditions

$$
\begin{aligned}
\overline{m} &= m_A + m_E \\
n_A &= l_A \\
n_E &= l_E
\end{aligned}
$$

to substitute in and get:

$$
\begin{aligned}
w_A &= l_A^{\gamma} \\
w_E &= l_E^{\gamma} \\
w_m &= \overline{m}^{\gamma}.
\end{aligned}
$$

Then, we can combine the solutions to the firm and the household problems to solve for labor

allocations in terms of managerial services:

$$l_A = (\nu z_A m_A^\theta)^{1/(\gamma-\nu+1)}$$
$$l_E = (\nu z_A m_E^\theta)^{1/(\gamma-\nu+1)}.$$

Also, we can combine the two expressions for the rental rate on managerial services w_m to solve for m_A in terms of $\overline{m}$:

$$\theta z_A m_A^{\theta-1} l_A^\nu = \theta z_E m_E^{\theta-1} l_E^\nu$$

$$\theta z_A m_A^{\theta-1} (\nu z_A m_A^\theta)^{\nu/(\gamma-\nu+1)} = \theta z_E m_E^{\theta-1} (\nu z_E m_E^\theta)^{\nu/(\gamma-\nu+1)}$$

$$z_A^{\frac{\gamma+1}{\gamma-\nu+1}} m_A^{\theta-1+\theta\nu/(\gamma-\nu+1)} = z_E^{\frac{\gamma+1}{\gamma-\nu+1}} m_E^{\theta-1+\theta\nu/(\gamma-\nu+1)}$$

$$m_A^{\frac{\theta\gamma+\theta-\gamma+\nu-1}{\gamma-\nu+1}} = \frac{z_E^{\frac{\gamma+1}{\gamma-\nu+1}}}{z_A} (\overline{m} - m_A)^{\frac{\theta\gamma+\theta-\gamma+\nu-1}{\gamma-\nu+1}}$$

$$m_A^{\theta\gamma+\theta-\gamma+\nu-1} = \left(\frac{z_E}{z_A}\right)^{\gamma+1} (\overline{m} - m_A)^{\theta\gamma+\theta-\gamma+\nu-1}$$

$$m_A = \frac{z_E^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}}}{z_A} (\overline{m} - m_A)$$

$$m_A \left(1 + \frac{z_E^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}}}{z_A}\right) = \frac{z_E^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}}}{z_A} \overline{m}$$

$$m_A = \overline{m} \frac{z_E^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}}}{z_A} \left(1 + \frac{z_E^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}}}{z_A}\right)^{-1}$$

I can use this expression, combined with the optimality condition for managerial services and the optimality condition from the managerial household's problem to solve for $\overline{m}$ in terms of parameters

and relative productivity:

$$\overline{m}^\gamma = \theta z_A m_A^{\theta-1}(\nu z_A m_A^\theta)^{\nu/(\gamma-\nu+1)}$$

$$\overline{m}^\gamma = \theta(\nu^\nu z_A^{\gamma+1})^{\frac{1}{\gamma-\nu+1}} m_A^{\theta-1+\theta\nu/(\gamma-\nu+1)}$$

$$\overline{m}^\gamma = \theta(\nu^\nu z_A^{\gamma+1})^{\frac{1}{\gamma-\nu+1}} m_A^{\frac{\theta\gamma+\theta-\gamma+\nu-1}{\gamma-\nu+1}}$$

$$\overline{m}^\gamma = \theta(\nu^\nu z_A^{\gamma+1})^{\frac{1}{\gamma-\nu+1}} \left(\overline{m}\frac{z_E}{z_A}^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}}\left(1+\frac{z_E}{z_A}^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}}\right)^{-1}\right)^{\frac{\theta\gamma+\theta-\gamma+\nu-1}{\gamma-\nu+1}}$$

$$\overline{m}^\gamma = \theta(\nu^\nu z_A^{\gamma+1})^{\frac{1}{\gamma-\nu+1}} \left(\overline{m}^{\frac{\theta\gamma+\theta-\gamma+\nu-1}{\gamma-\nu+1}}\frac{z_E}{z_A}^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}\frac{\theta\gamma+\theta-\gamma+\nu-1}{\gamma-\nu+1}}\left(1+\frac{z_E}{z_A}^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}}\right)^{-\frac{\theta\gamma+\theta-\gamma+\nu-1}{\gamma-\nu+1}}\right)$$

$$\overline{m}^{\gamma-\frac{\theta\gamma+\theta-\gamma+\nu-1}{\gamma-\nu+1}} = \theta(\nu^\nu z_A^{\gamma+1})^{\frac{1}{\gamma-\nu+1}} \left(\frac{z_E}{z_A}^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}\frac{\theta\gamma+\theta-\gamma+\nu-1}{\gamma-\nu+1}}\left(1+\frac{z_E}{z_A}^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}}\right)^{-\frac{\theta\gamma+\theta-\gamma+\nu-1}{\gamma-\nu+1}}\right)$$

$$\overline{m}^{\frac{(\gamma+1)(\gamma-\nu+1-\theta)}{\gamma-\nu+1}} = \theta(\nu^\nu z_A^{\gamma+1})^{\frac{1}{\gamma-\nu+1}} \left(\frac{z_E}{z_A}^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}\frac{\theta\gamma+\theta-\gamma+\nu-1}{\gamma-\nu+1}}\left(1+\frac{z_E}{z_A}^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}}\right)^{-\frac{\theta\gamma+\theta-\gamma+\nu-1}{\gamma-\nu+1}}\right)$$

$$\overline{m} = (\theta^{\gamma-\nu+1}\nu^\nu z_A^{\gamma+1})^{\frac{1}{(\gamma+1)(\gamma-\nu+1-\theta)}} \frac{z_E}{z_A}^{\frac{1}{\gamma-\nu+1-\theta}}\left(1+\frac{z_E}{z_A}^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}}\right)^{-\frac{\theta\gamma+\theta-\gamma+\nu-1}{(\gamma+1)(\gamma-\nu+1-\theta)}}$$

I will later write GDP and employment as a function of $\overline{m}$, so I want to see what happens to $\overline{m}$ as $\frac{z_E}{z_A}$ increases. For my purposes, I will assume that z_A is constant and z_E is rising (relative to z_A). Denote by $Z = \frac{z_E}{z_A}$ and by $x = (\theta^{\gamma-\nu+1}\nu^\nu z_A^{\gamma+1})^{\frac{1}{(\gamma+1)(\gamma-\nu+1-\theta)}}$ and take the derivative of $\overline{m}$ with respect to Z:

$$\frac{\partial\overline{m}}{\partial Z} = x\frac{1}{\gamma-\nu+1-\theta}Z^{\frac{1}{\gamma-\nu+1-\theta}-1}\left(1+Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}}\right)^{-\frac{\theta\gamma+\theta-\gamma+\nu-1}{(\gamma+1)(\gamma-\nu+1-\theta)}}$$

$$- x\frac{\theta\gamma+\theta-\gamma+\nu-1}{(\gamma+1)(\gamma-\nu+1-\theta)}\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}Z^{\frac{1}{\gamma-\nu+1-\theta}+\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}-1}$$

$$* \left(1+Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}}\right)^{-\frac{\theta\gamma+\theta-\gamma+\nu-1}{(\gamma+1)(\gamma-\nu+1-\theta)}-1}$$

Simplifying the expression:

$$\frac{\partial\overline{m}}{\partial Z} = x\frac{1}{\gamma-\nu+1-\theta}Z^{\frac{1}{\gamma-\nu+1-\theta}}\left(1+Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}}\right)^{-\frac{\theta\gamma+\theta-\gamma+\nu-1}{(\gamma+1)(\gamma-\nu+1-\theta)}}$$

$$* \left(Z^{-1}-Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}-1}\left(1+Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}}\right)^{-1}\right)$$

Notice that the top line of this expression will always be positive by assumption because all parameters are assumed to be positive and productivities will be positive. the only expression for which this

may not be immediately obvious is $\frac{1}{\gamma-\nu+1-\theta}$. Rearranging this expression to yield $\frac{1}{\gamma+1-(\nu+\theta)}$ and recall that, by assumption $\nu + \theta \leq 1$. Therefore, this expression will be positive for all parameter values. Now, it is left to show that the bottom line will be positive. Rewriting this expression, assuming it's positive (to later be verified):

$$\frac{1}{Z} > \frac{Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}-1}}{1+Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}}}$$

$$1+Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}} > Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}-1+1}$$

$$1+Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}} > Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}}$$

This is always true; therefore the derivative of $\overline{m}$ with respect to Z is positive for all relevant parameter values.

Now, I can write GDP as a function of $\overline{m}$, noting that $\overline{m}$ is a function of Z. Recall that, using the income approach to GDP, GDP can be written:

$$GDP = \alpha w_m \overline{m} + (1-\alpha)w_A l_A$$

We can substitute in for $w_m = \overline{m}^\gamma$ and $w_A = l_A^\gamma$, yielding:

$$GDP = \alpha\overline{m}^{\gamma+1} + (1-\alpha)l_A^{\gamma+1}$$

Substituting in for l_A and then m_A and simplifying yields:

$$
\begin{aligned}
GDP &= \alpha\overline{m(Z)}^{\gamma+1} + (1-\alpha)\left(\nu z_A m_A^\theta\right)^{\frac{\gamma+1}{\gamma-\nu+1}} \\
&= \alpha\overline{m(Z)}^{\gamma+1} + (1-\alpha)(\nu z_A)^{\frac{\gamma+1}{\gamma-\nu+1}}(m_A)^{\frac{\theta(\gamma+1)}{\gamma-\nu+1}} \\
&= \alpha\overline{m(Z)}^{\gamma+1} + (1-\alpha)(\nu z_A)^{\frac{\gamma+1}{\gamma-\nu+1}}\left(\overline{m}(Z)Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}}\left(1+Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}}\right)^{-1}\right)^{\frac{\theta(\gamma+1)}{\gamma-\nu+1}} \\
&= \alpha\overline{m(Z)}^{\gamma+1} + (1-\alpha)(\nu z_A)^{\frac{\gamma+1}{\gamma-\nu+1}}\overline{m}(Z)^{\frac{\theta(\gamma+1)}{\gamma-\nu+1}}Z^{\frac{\theta(\gamma+1)^2}{(\gamma-\nu+1)(\theta\gamma+\theta-\gamma+\nu-1)}}\left(1+Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}}\right)^{-\frac{\theta(\gamma+1)}{\gamma-\nu+1}}
\end{aligned}
$$

Now, in order to see how GDP moves with Z, we can take the derivative of this with respect to Z:

$$\frac{\partial GDP}{\partial Z} = \alpha(\gamma+1)\overline{m(Z)}^{\gamma}\frac{\partial \overline{m}}{\partial Z} + (1-\alpha)\left(\nu z_A\right)^{\frac{\gamma+1}{\gamma-\nu+1}}\frac{\theta(\gamma+1)}{\gamma-\nu+1}\overline{m(Z)}^{\frac{\theta(\gamma+1)}{\gamma-\nu+1}-1}\frac{\partial \overline{m}}{\partial Z}Z^{\frac{\theta(\gamma+1)^2}{(\gamma-\nu+1)(\theta\gamma+\theta-\gamma+\nu-1)}}$$

$$* \quad \left(1+Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}}\right)^{-\frac{\theta(\gamma+1)}{\gamma-\nu+1}} + (1-\alpha)\left(\nu z_A\right)^{\frac{\gamma+1}{\gamma-\nu+1}}\frac{\theta(\gamma+1)^2}{(\gamma-\nu+1)(\theta\gamma+\theta-\gamma+\nu-1)}$$

$$* \quad \overline{m(Z)}^{\frac{\theta(\gamma+1)}{\gamma-\nu+1}}Z^{\frac{\theta(\gamma+1)^2}{(\gamma-\nu+1)(\theta\gamma+\theta-\gamma+\nu-1)}-1}\left(1+Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}}\right)^{-\frac{\theta(\gamma+1)}{\gamma-\nu+1}}$$

$$- \quad (1-\alpha)\left(\nu z_A\right)^{\frac{\gamma+1}{\gamma-\nu+1}}\frac{\theta(\gamma+1)}{\gamma-\nu+1}\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}\overline{m(Z)}^{\frac{\theta(\gamma+1)}{\gamma-\nu+1}}Z^{\frac{\theta(\gamma+1)^2}{(\gamma-\nu+1)(\theta\gamma+\theta-\gamma+\nu-1)}+\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}-1}$$

$$* \quad \left(1+Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}}\right)^{-\frac{\theta(\gamma+1)}{\gamma-\nu+1}-1}$$

First, notice that the top line will be positive because, again, productivities are positive and $\alpha > 0, (1-\alpha) > 0, \nu < 1$. So, I will simplify and combine the second and third lines:

$$\frac{\partial GDP}{\partial Z} \propto (1-\alpha)\frac{\theta(\gamma+1)^2}{(\gamma-\nu+1)(\theta\gamma+\theta-\gamma+\nu-1)}\left(\nu z_A\right)^{\frac{\gamma+1}{\gamma-\nu+1}}\overline{m(Z)}^{\frac{\theta(\gamma+1)}{\gamma-\nu+1}}Z^{\frac{\theta(\gamma+1)^2}{(\gamma-\nu+1)(\theta\gamma+\theta-\gamma+\nu-1)}}\left(1+Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}}\right)$$

$$* \quad \left(Z^{-1}-Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}-1}\left(1+Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}}\right)^{-1}\right)$$

This will be positive whenever

$$\frac{1}{Z} > \frac{Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}-1}}{\left(1+Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}}\right)}$$

which is equivalent to

$$\left(1+Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}}\right) > Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}-1+1}$$

$$1+Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}} > Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}}$$

which is always true. This tells us that when z_A is constant and z_E is increasing, the GDP in Country A is increasing, no matter what the parameter values are. Essentially, it must be true that the productivity in Country E is increasing more quickly than the productivity in Country A following a recession.

I now turn to employment, which is the weighted average of employment of Type-M households and Type-L households:[16]

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[16]Note that employment of Type-M households is self-employment, whereas employment of Type-

$$Emp = \alpha\overline{m} + (1-\alpha)l_A.$$

Again, I can substitute in for l_A, then for m_A to arrive at an expression of employment as a function of $\overline{m}$:

$$
\begin{aligned}
Emp &= \alpha\overline{m} + (1-\alpha)(\nu z_A m_A^\theta)^{\frac{1}{\gamma-+1}} \\
&= \alpha\overline{m} + (1-\alpha)(\nu z_A)^{\frac{1}{\gamma-\nu+1}} m_A^{\frac{\theta}{\gamma-\nu+1}} \\
&= \alpha\overline{m} + (1-\alpha)(\nu z_A)^{\frac{1}{\gamma-\nu+1}} \left(\overline{m} Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}} \left(1 + Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}}\right)^{-1} \right)^{\frac{\theta}{\gamma-\nu+1}} \\
&= \alpha\overline{m} + (1-\alpha)(\nu z_A)^{\frac{1}{\gamma-\nu+1}} Z^{\frac{\theta(\gamma+1)}{\theta\gamma+\theta-\gamma+\nu-1}} \left(1 + Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}}\right)^{-\frac{\theta}{\gamma-\nu+1}} \overline{m}^{\frac{\theta}{\gamma-\nu+1}}
\end{aligned}
$$

Again, recalling that $\overline{m}$ is a function of Z, I can take the derivative of this with respect to Z:

$$
\begin{aligned}
\frac{\partial Emp}{\partial Z} &= \alpha\frac{\partial \overline{m(Z)}}{\partial Z} + (1-\alpha)(\nu z_A)^{\frac{1}{\gamma-+1}} \frac{\theta}{\gamma-\nu+1} \overline{m}^{\frac{\theta}{\gamma-\nu+1}-1} \frac{\partial \overline{m(Z)}}{\partial Z} Z^{\frac{\theta(\gamma+1)}{\theta\gamma+\theta-\gamma+\nu-1}} \left(1 + Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}}\right)^{-\frac{\theta}{\gamma-\nu+1}} \\
&+ (1-\alpha)\frac{\theta(\gamma+1)}{\theta\gamma+\theta-\gamma+\nu-1}(\nu z_A)^{\frac{1}{\gamma-+1}} Z^{\frac{\theta(\gamma+1)}{\theta\gamma+\theta-\gamma+\nu-1}-1} \left(1 + Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}}\right)^{-\frac{\theta}{\gamma-\nu+1}} \overline{m}^{\frac{\theta}{\gamma-\nu+1}} \\
&- (1-\alpha)\frac{\theta}{\gamma-\nu+1}\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}(\nu z_A)^{\frac{1}{\gamma-+1}} Z^{\frac{\theta(\gamma+1)}{\theta\gamma+\theta-\gamma+\nu-1}+\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}-1} \left(1 + Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}}\right)^{-\frac{\theta}{\gamma-\nu+1}-1} \\
&* \overline{m}^{\frac{\theta}{\gamma-\nu+1}}
\end{aligned}
$$

Again, the first line of this expression will be positive because productivities are positive and $\alpha > 0, (1-\alpha) > 0, \nu < 1$. So, I will again simplify and combine the second and third lines:

$$
\begin{aligned}
\frac{\partial Emp}{\partial Z} &\propto (1-\alpha)(\nu z_A)^{\frac{1}{\gamma-+1}} \overline{m}^{\frac{\theta}{\gamma-\nu+1}} Z^{\frac{\theta(\gamma+1)}{\theta\gamma+\theta-\gamma+\nu-1}} \left(1 + Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}}\right)^{-\frac{\theta}{\gamma-\nu+1}} \frac{\theta(\gamma+1)}{\theta\gamma+\theta-\gamma+\nu-1} \\
&* \left(Z^{-1} - \frac{1}{\gamma-\nu+1} Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}-1} \left(1 + Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}}\right)^{-1}\right).
\end{aligned}
$$

L households is market employment.

Simplifying again

$$\frac{\partial Emp}{\partial Z} \propto (1-\alpha)(\nu z_A)^{\frac{1}{\gamma-+1}} \overline{m}^{\frac{\theta}{\gamma-\nu+1}} Z^{\frac{\theta(\gamma+1)}{\theta\gamma+\theta-\gamma+\nu-1}} \left(1+Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}}\right)^{-\frac{\theta}{\gamma-\nu+1}} \frac{\theta(\gamma+1)}{\theta\gamma+\theta-\gamma+\nu-1}$$

$$* \quad Z^{-1}\left(1 - \frac{1}{\gamma-\nu+1}\frac{Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}}}{1+Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}}}\right).$$

The top line of this expression will be negative if

$$\theta\gamma + \theta - \gamma + \nu - 1 \quad < \quad 0.$$

Because the allocation, productivities, and parameters are all positive. Furthermore, $\theta(\gamma+1) > 0$ since both $\gamma > 0$ and $\theta > 0$. Therefore the top line will be positive when

$$\gamma + 1 - \nu \quad > \quad \theta\gamma + \theta$$
$$\gamma \quad > \quad \frac{\nu}{1-\theta} - 1.$$

In the case of Cobb-Douglas production, $\nu = 1 - \theta$, so this becomes

$$\gamma \quad > \quad 0.$$

It is left to show the conditions under which the bottom line is positive, so that the entire derivative is negative. The expression will be positive whenever the following holds:

$$1 \quad > \quad \frac{1}{\gamma-\nu+1}\frac{Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}}}{1+Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}}}$$
$$(\gamma-\nu+1)\left(1+Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}}\right) \quad > \quad Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}}$$
$$(\gamma-\nu+1) + (\gamma-\nu+1)Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}} \quad > \quad Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}}$$
$$(\gamma-\nu+1) + (\gamma-\nu)Z^{\frac{\gamma+1}{\theta\gamma+\theta-\gamma+\nu-1}} \quad > \quad 0$$

As noted before $\gamma - \nu + 1 > 0$ so long as $\nu < 1$ and $Z > 0$, so, this expression is positive as long as $\gamma > \nu$. Therefore, as Country E's productivity grows relative to Country A's productivity, GDP in Country A increases and employment decreases so long as $\gamma > \nu, \gamma > \frac{\nu}{1-\theta} - 1$. Both of these expressions are generically true in the case with Cobb-Douglas production so long as $\gamma > 0$.