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Radiative Corrections for Deuteron Electrodisintegration

G P Gilfoyle[1] and A.Afansev[2]
[1] *Physics Department, University of Richmond, Richmond, VA*
[2] *Jefferson Lab, Newport News, VA*

Abstract

Full exclusive calculations of radiative corrections for the $d(e,e'p)n$ and $d(e,e'n)p$ reactions have been performed using a code originally written and tested for pion electroproduction. We find significant differences between radiative corrections calculated with this method and a Schwinger calculation. The program can be obtained from a website referenced in this CLAS-Note.

1 Introduction

Radiative corrections (RC) are calculations of the effect of unmeasured photons that are emitted during lepton scattering events. These photons change the distribution of the yield as a function of the kinematics of the scattering and are a common phenomenon in almost all lepton scattering reactions including the $d(\vec{e},e'p)n$ and $d(\vec{e},e'n)p$ reaction studied here. In this CLAS-Note we discuss our approach to radiative corrections and include a user's guide for others to obtain and use the program.

We begin with a short summary of the current status of radiative correction calculations. There are several, well-studied methods for calculating RC's, but most resort to the methods originally developed by Schwinger and Mo and Tsai [1, 2]. In that approach, it was assumed that only the scattered electron was detected (inclusive scattering). As a result that method suffers from several shortcomings if it is to be applied to exclusive reactions like those measured at JLab. First, detecting the ejected hadron (exclusive reactions) alters the phase space that is allowed for the final radiated photon. Second, more structure functions can contribute in exclusive reactions. Only the longitudinal and transverse pieces contribute in the Schwinger/Mo and Tsai method while many experiments performed at JLab investigate processes associated with the f_{LT}, f_{TT} and f'_{LT} structure functions. Third, the Schwinger/Mo and Tsai approach relies on an unphysical parameter to split the hard and soft regions of the radiated photon's phase space and cancel the infrared divergence.

These shortcoming have been addressed by A.Afanasev, *et al.* first in exclusive pion electroproduction [3]. They developed analytical formulas for the lowest-order QED radiative correction to the cross section and polarization beam asymmetry using the kinematic variables Q^2 (squared virtual photon momentum), W (invariant mass of the object recoiling against the lepton projectile) and the polar and azimuthal angles of the detected hadron in

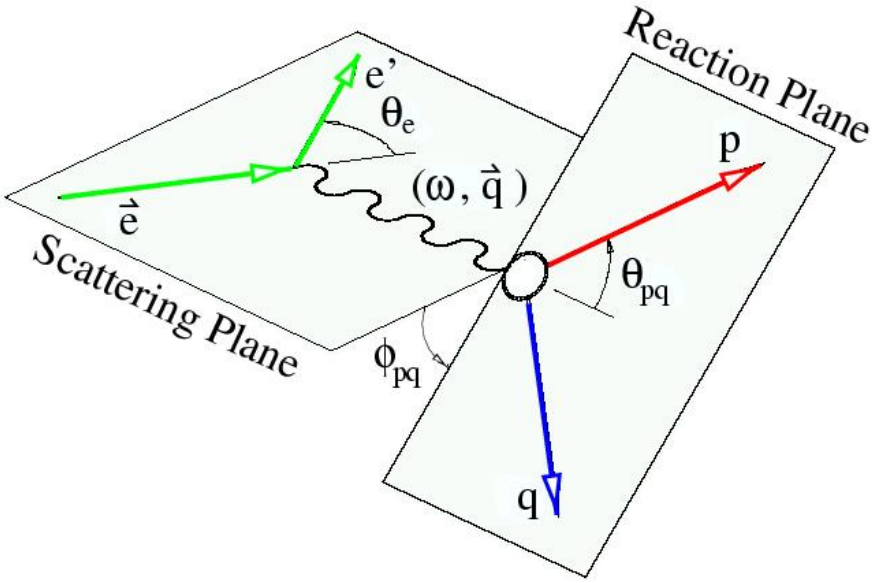

Figure 1: Kinematic definitions for the $d(e,e'p)n$ and $d(e,e'n)p$ reactions.

the center-of-mass of the final hadrons. See Figure 1 for the definitions of these angles. This formulation addresses the first shortcoming of the Schwinger/Mo and Tsai method).

The Feynman diagrams used in their approach are shown in Figure 2. They include the Born term (1.a), QED processes for undetected photons before and after the interaction of the lepton beam with the target (1.b and 1.c), lepton-photon vertex corrections (1.d) and vacuum polarization (1.e). They used the leptonic tensor for a longitudinally polarized lepton beam and a general covariant form of the hadronic tensor that includes all five structure functions appropriate for exclusive reactions with a polarized lepton beam (addressing the second shortcoming of the Schwinger/Mo and Tsai method). Finally, to deal with the infrared divergence in the Schwinger/Mo and Tsai method Afanasev, *et al.* used a covariant procedure of infrared divergence cancellation which does not require the splitting [4]. A FORTRAN code named EXCLURAD was developed for the reactions $p(e,e'\pi^+)n$ and $p(e,e'p)\pi^0$ [3, 5]. We have taken this program EXCLURAD and modified it for the $d(e,e'p)n$ and $d(e,e'n)p$ reactions.

In this CLAS-Note we describe the changes made to the original program EXCLURAD to go from calculations of pion electroproduction to deuteron electrodisintegration. We present results from testing and performing these calculations and include a users manual for acquiring and running the code.

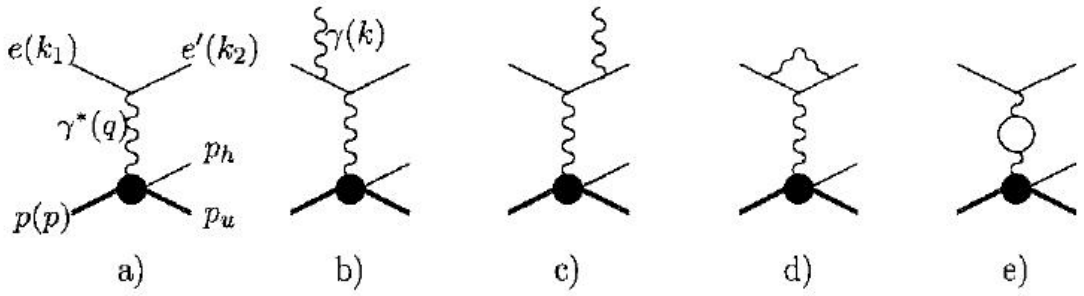

Figure 2: Feynman diagrams contributing to the Born and the next-order electroproduction cross sections. a) Born process, b) and c) bremsstrahlung, d) vertex correction, and e) vacuum polarization. The momentum $p_h(p_u)$ is assigned to the detected (undetected) hadron.

2 Modifying EXCLURAD for Deuteron Electrodisintegration

These calculations are being done with a modified version of the computer program EXCLURAD written originally by Afanasev, *et al.* to calculate RCs for the $p(e,e'\pi^+)X$ reaction [3]. It has been modified here to work for the $d(e,e'p)n$ and $d(e,e'n)p$ reactions. The program calculates the ratio of the cross section to the PWIA result in a particular bin in Q^2, W, $\cos\theta_{ps}^{cm}$, and ϕ_{pq} where θ_{pq}^{cm} is the angle between the transferred 3-momentum $\vec{q}$ and the ejected hadron in the center-of-mass of the final hadrons, and ϕ_{pq} is the angle between the scattering plane defined by the 3-momenta of the incoming and outgoing electron and the reaction plane formed by the 3-momenta of the final hadrons. See Figure 1.

The original pion electroproduction version of EXCLURAD has been modified for deuteron electrodisintegration by, first, modifying the masses of the target and final hadrons in the subroutine SETCON. Next, a new way to calculate the response functions is required. The program DEEP was developed by W. van Orden, *et al.* is being used to calculate these response functions [6]. This code uses the covariant spectator theory and the transversity formalism to calculate the unpolarized, coincidence cross section for $d(e,e'p)n$. Note, DEEP does not calculate the structure functions associated with polarized leptons, so there will be no cross section for out-of-plane hadrons. DEEP was modified from its original form so it could be called from a subroutine within EXCLURAD and its output was changed to be consistent with the formalism used by Afanasev, *et al.* in EXCLURAD. Radiative corrections for the $d(e,e'n)p$ reaction are required for some of the analysis of the deuteron electrodisintegration data (*e.g.* the G_m^n measurement) so another version of the program was developed [7]. Here the neutron and proton masses in the subroutine SETCON are reversed and in the calculation of the response functions, the angle θ_{pq}^{cm} is changed to $\pi - \theta_{pq}^{cm}$. The DEEP program was written for $d(e,e'p)n$ so in the center-of-mass, a proton emitted at θ_{pq}^{cm} corresponds to a neutron emitted at $\pi - \theta_{pq}^{cm}$. This change only has to be made in one place in the routine `PHY_MODEL` which is part of EXCLURAD.

3 The v_{cut} Parameter

We describe here how to relate one of the common parameters of the Schwinger/Mo and Tsai calculation with the approach used in EXCLURAD. In the Schwinger method one calculates the radiative correction for the scattering of an electron in a Coulomb field (inclusive electron scattering). An essential step in the calculation is to integrate over the radiative tail of the energy of a scattered electron to arrive at a correction factor for the yield lost to the emission of photons in a particular kinematic bin. The parameters of that integration are defined in Figure 3 [9]. The parameter ΔE is the energy range over which the integral is performed

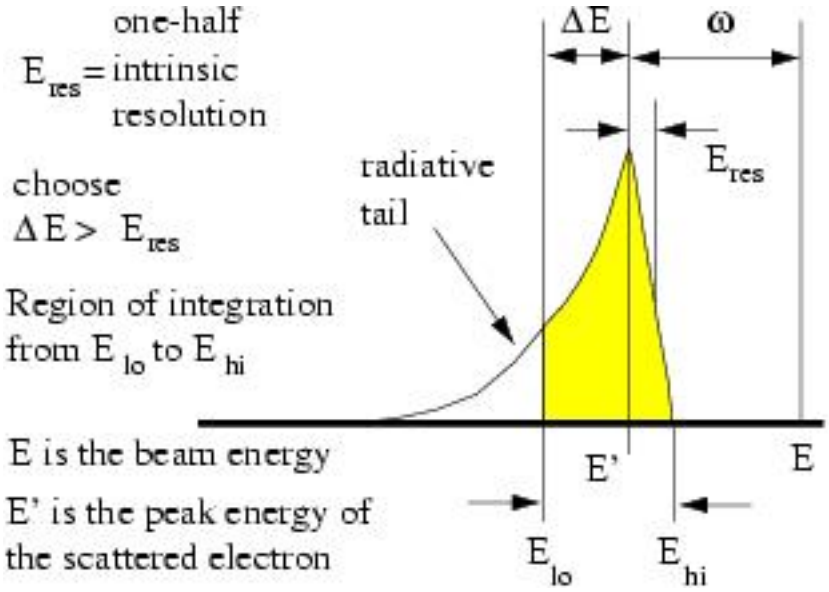

Figure 3: Energy spectrum of scattered electron showing definitions of quantities used in Schwinger radiative correction calculation.

(starting at the unradiated energy of the electron) to estimate the yield lost to radiated photons.

Afanasev, *at al.* follow an analogous procedure in their more sophisticated approach [3]. They integrate over the radiative tail of the scattered electron, but they perform the integration in terms of the covariant 'inelasticity' v defined as

$$v = \Lambda^2 - m_u^2 \tag{1}$$

where m_u is the mass of the undetected hadron and Λ is the four-momentum of the missing or undetected particles. The quantity v describes the missing mass due to the emission of a bremsstrahlung photon and can be rewritten as

$$v = W^2 + m_h^2 - m_u^2 - 2WE_h \tag{2}$$

where W is the mass of the system recoiling against the electron, m_h is the mass of the detected hadron, and E_h is the center-of-mass energy of the detected hadron. To determine

the relationship between ΔE and v consider the usual expression for W^2

$$W^2 = M^2 + 2M(E - E') - Q^2 \tag{3}$$

where

$$Q^2 \approx 4EE' \sin^2 \frac{\theta}{2} \tag{4}$$

M is the target mass, and θ is the electron scattering angle. However, for an event with a radiated photon, the measured energy of the scattered electron is not E', but some lower energy

$$E_{lo} = E' - \Delta E \tag{5}$$

so W for this event will not be 'correct'. The new value we call is

$$W^2_{rad} = M^2 + 2M(E - E_{lo}) - 4EE_{lo} \sin^2 \frac{\theta}{2} \tag{6}$$

Using Equations 5 and 6 to replace W with W_{rad} in the expression for v (Equation 2) one obtains the following function of ΔE.

$$\begin{aligned} v = {} & M^2 + 2M(E - E' + \Delta E) - 4E(E' + \Delta E) \sin^2 \frac{\theta}{2} \\ & + m_h^2 - m_u^2 - 2E_h \sqrt{M^2 + 2M(E - E' + \Delta E) - 4E(E' + \Delta E) \sin^2 \frac{\theta}{2}} \end{aligned} \tag{7}$$

This expression can be re-arranged so

$$v = W_0^2 + m_h^2 - m_u^2 + 2\Delta E(M + 2E \sin^2 \frac{\theta}{2}) - 2E_h \sqrt{W_0^2 + 2\Delta E(M + 2E \sin^2 \frac{\theta}{2})} \tag{8}$$

where

$$W_0^2 = M^2 + 2M(E - E') - 4EE' \sin^2 \frac{\theta}{2} \tag{9}$$

and the quantities E, E', and θ are determined by the electron kinematics. The hadron energy E_h is determined by the choice of the angle of the outgoing hadron relative to $\vec{q}$, the three-vector of the momentum transfer. The masses M, m_h, and m_u are all known.

As an example of applying Equation 8 consider the following CLAS kinematics. The

$E = 2.558$ GeV	$E' = 2.345$ GeV	$\theta = 14.84°$
$m_h = 0.938$ GeV	$m_u = 0.940$ GeV	$\theta_h^{cm} = 45°$
$M = 1.876$ GeV	$Q^2 = 0.52$ (GeV/c)^2	$W = 1.93$ GeV

Table 1: Kinematics for calculating $v(\Delta E)$.

results of the calculation are shown in Figure 4. The dependence of v on ΔE is almost linear implying the importance of the fourth term in Equation 8 over the sum of all the other terms.

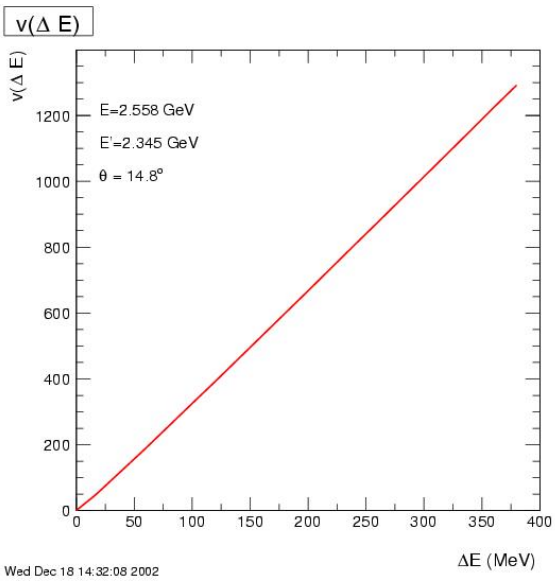

Figure 4: Dependence of v on ΔE for the kinematics listed in Table 1.

4 Results

We discuss here some of the preliminary results we have obtained for radiative corrections calculated using the program EXCLURAD modified for deuteron electrodisintegration. We first tested the code for consistency. Figure 5 shows the $\cos\theta_{pq}^{cm} - \phi_{pq}$ surface needed to calculate the average RC for our analysis for the kinematics $W = 2.55\ GeV$ and $Q^2 = 3.0\ (GeV/c)^2$. There is a large spike in the RC in the region around the point $\cos\theta_{pq}^{cm} = 0.85$ and $\phi_{pq} = 165°$. The value of RC here is inconsistent with the surrounding surface. This inconsistency occurs because the program performs a numerical integration using an adaptive step-size over the phase space of the reaction until a convergence criterion is satisfied. We have observed this integration fails occasionally and gives results inconsistent with nearby kinematics like the one shown here. This problem is easy to detect by looking at the $\cos\theta_{pq}^{cm} - \phi_{pq}$ surface. The solution to the problem is to decrease the value of the convergence parameter and redo the calculation. This will usually make the calculation take more computer time. The convergence parameter is currently hardwired into the program as the FORTRAN variable `ot` in the subroutine `qqt`. The user can also increase the limit on the number of iterations by increasing the value of the FORTRAN variable `ma` in the same subroutine `qqt`. In this case the problem was fixed by lowering the convergence parameter in EXCLURAD by a factor of 10, recalculating that single point, and editing the output file. The final result is shown in Figure 6. The spike has disappeared.

We studied the systematic behavior of the radiative corrections for $d(e, e'p)n$ as a function of Q^2. Some of those results are shown in Figure 7 where we show the radiative correction averaged over θ_{pq}^{cm} and ϕ_{pq} as a function of Q^2. The blue curve is from calculations performed with a larger number of steps in $\cos\theta_{pq}^{cm}$ than the green curve. Small bins in

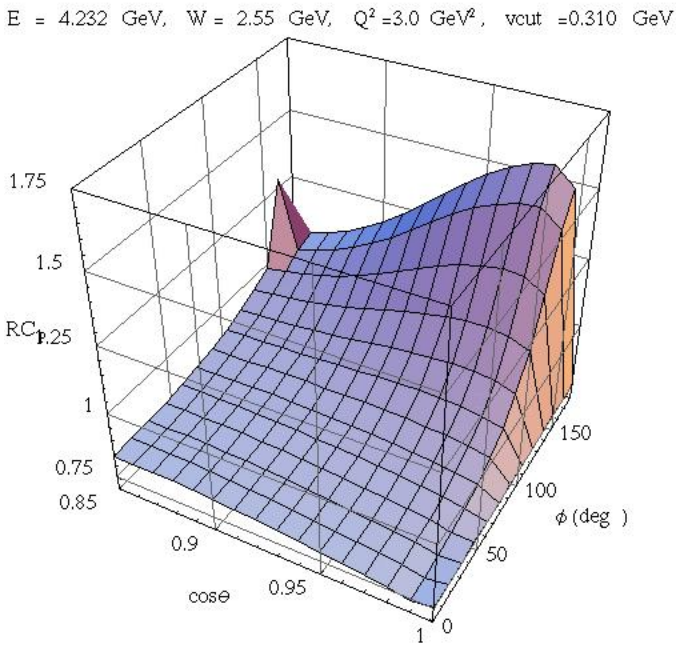

Figure 5: Calculation of RC surface. Note spike in RC.

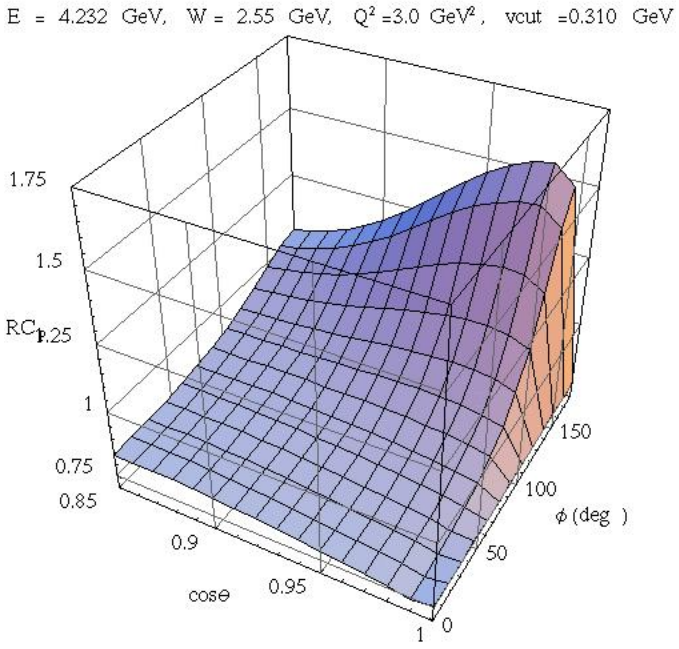

Figure 6: Calculation of RC surface. Compare with Figure 5.

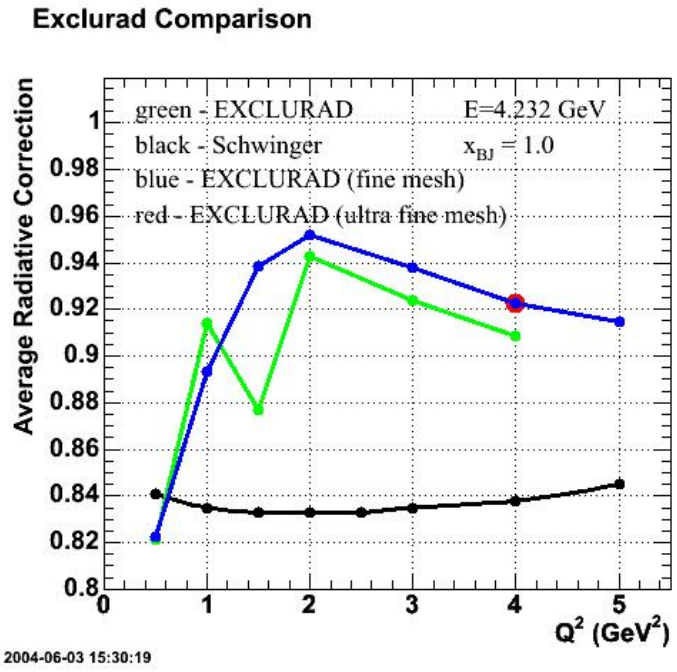

Figure 7: Systematic behavior of RC as a function of Q^2.

$\cos\theta_{pq}^{cm}$ were used near $\cos\theta_{pq}^{cm} = 1$ and larger bins used for large values of $\cos\theta_{pq}^{cm}$. There is a difference at all values of Q^2 between the blue curve (small stepsize) and the green one (large stepsize) except at the lowest value ($Q^2 = 0.5$ $(GeV/c)^2$). The large dip in the green curve near $Q^2 = 3.0$ $(GeV/c)^2$ is due to the failure of the numerical integration in the modified EXCLURAD like that discussed above and shown in Figures 5-6. The red circle at $Q^2 = 4.0$ $(GeV/c)^2$ in Figure 6 is another calculation made with non-equidistant bins that were each about half the size of the bins used for the blue curve. This calculation was done to make sure the result had converged and we are not vulnerable to uncertainties due to large stepsizes. The blue and red points sit on top of one another which shows the calculation has converged. These results imply the importance of small stepsizes in getting accurate results for the radiative corrections and the need to monitor the calculation by comparing the results at a given kinematical point with nearby ones. The black curve in Figure 6 is a Schwinger calculation performed using the website of K. Aniol [9]. We do not expect the Schwinger calculation to be the same as the one performed with EXCLURAD because we are using a different model of the response functions in EXCLURAD. Nevertheless, it is worth comparing the two results. The EXCLURAD calculation has considerably more structure than the Schwinger one and gives results that are 10-12% different from the Schwinger ones over most of the Q^2 range.

We have also compared the radiative corrections for $d(e, e'p)n$ and $d(e, e'n)p$; the results are shown in Figure 8. The difference is very small (0.2%) at small ϕ_{pq} and grows to about 1.4% at large ϕ_{pq}. The difference between the two calculations is due to real physics differences between the couplings of the neutron and proton to the virtual photon. At

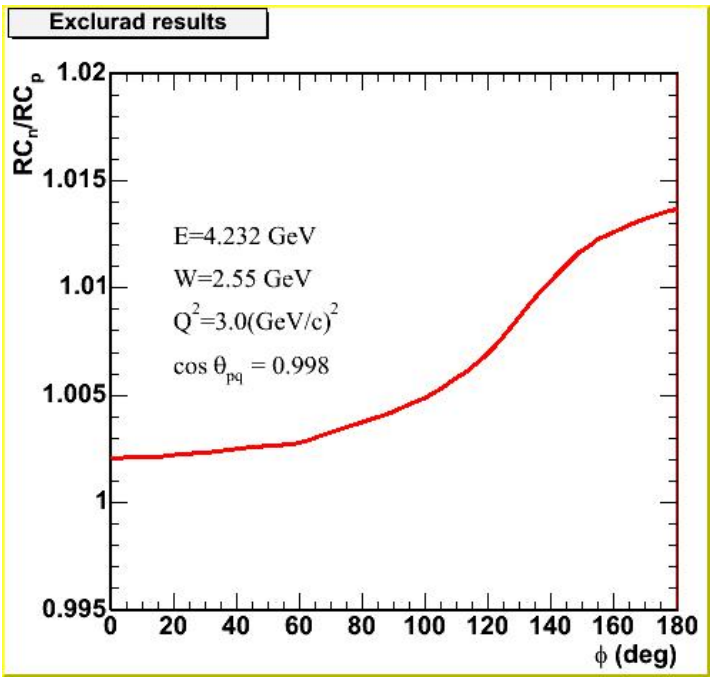

Figure 8: Comparison of radiative corrections for $d(e,e'p)n$ and $d(e,e'n)p$.

small ϕ_{pq} we are seeing the effect of a proton going forward relative to the virtual photon's 3-momentum $\vec{q}$ and the neutron going backward. For large ϕ_{pq} the situation is reversed; the neutron is now going forward and the proton backward. The difference between the couplings for neutrons and protons with the virtual photon create the asymmetry in ϕ_{pq} which manifests itself as a small difference here in the radiative correction for neutrons and protons. In other words, the difference we see in the RC for neutrons and protons is due to intrinsic differences in the interaction between them.

5 User's Guide for EXCLURAD

This section describes how to obtain the source for the modified version of EXCLURAD, build the program, run it, and interpret the results. This documentation is also available on the web [10].

5.1 Getting the Code

A tarred, zipped version of the code is available at the following site

`http://www.richmond.edu/~ggilfoyl/research/RC/exclurad-04-may-18.tar.gz`

(release date 5/18/04) and includes both proton ($d(e,e'p)n$) and neutron ($d(e,e'n)p$) versions of the E5 EXCLURAD and code for running batch jobs on the Richmond cluster. CLAS and

JLab collaborators who want to use the Richmond cluster can contact G.P.Gilfoyle (email: ggilfoyljlab.org) to get an account [8].

5.2 Building an Executable

Copy the most recent zipped, tar file to the the directory where you want to store the subdirectory containing this version of EXCLURAD. Change to that directory and unzip and untar the file.

```
cp exclurad_wvo.tar-04-may-18.gz <i>top directory</i>
cd <i>top directory</i>
gunzip exclurad_wvo-04-may-18.tar.gz
tar -xvf exclurad_wvo-04-may-18.tar
```

This will produce a new directory called exclurad that will have three subdirectories.

1. wvo_proton - contains the code for calculating the proton version ($d(e,e'p)n$).
2. wvo_neuron - code for the neutron version ($d(e,e'n)p$).
3. run - codes for using the Richmond cluster.

Now go into `wvo_proton` or `wvo_neutron` and build the program

```
cd exclurad/wvo_proton or cd exclurad/wvo_neutron
make
```

This step has been tested on RedHat linux gcc version 2.96 20000731 (Red Hat Linux 7.2 2.96-112.7.2) and gcc version 3.2.2 20030222 (Red Hat Linux 3.2.2-5) on the Richmond cluster (pscm1.richmond.edu). The first release was tested on the same Richmond machine and at jlab (gcc version 2.96 20000731 (Red Hat Linux 7.2 2.96-108.1)) on url2.jlab.org and it worked. Jeff Lachniet at CMU was able to build and run the first release of the program on RedHat 9. He had to make a change in the file *include.mk*. He changed the environment variable OSNAME to OS_NAME and set OS_NAME to LinuxRH9.

5.3 Running the Code

Once you have an executable, make any changes to 'input.dat' which is reasonably well commented (see below) and run it.

```
exclurad_deep_Linux
```

The input file is called 'input.dat' and looks like the following.

```
4       !  1: AO 2: maid98  3: maid2000 4: deep
0       !  0: Full, 1: Factorizable and Leading log
5.75    !  bmom - lepton momentum
0.0     !  tmom - momentum per nucleon
1       !  lepton - 1 electron, 2 muon
2       !  ivec - detected hadron (2) proton always
0.310   !  vcut - cut on v. (0.) if no cut, negative -- v
5
2.218       2.218       2.218       2.218       2.218
1.4         1.4         1.4         1.4         1.4
0.25        0.05        0.05        -0.5        -0.95
0.0         0.0         0.0         0.0         0.0
```

A description of the contents of each line is given below. The number in the left-hand column corresponds to the line of the input file above. The content of the line is repeated first followed by a description in the next entry in the table.

Line	Line content or explanation.
1.	`4  !  1: AO 2: maid98  3: maid2000 4: deep`
1.	Picks the source of the response functions. Always set to 4 for $d(e,e'p)n$.
2.	`0  !  0: Full, 1: Factorizable and Leading log`
2.	Chooses which level of approximation to make. Factorizable (1) is fast while the Full (0) calculation can take from one to several minutes per point.
3.	`5.75  !  bmom - lepton momentum`
3.	Sets the electron beam energy.
4.	`0.0  !  tmom - momentum per nucleon`
4.	Not used here.
5.	`1  !  lepton - 1 electron, 2 muon`
5.	Pick electron or muon beam. Always electron here.
6.	`2  !  ivec - detected hadron (2) proton always`
6.	Parameter used to handle situations where the angles definitions are reversed. Always 2 here.

Table 2: Sample input file with parameters and a description of each line following the line containing the input parameters.

Line	Line content or explanation.
7.	`0.310  !  vcut - cut on v. (0.) if no cut, negative -- v`
7.	This is used to set the inelasticity cut which determines how far to integrate to recover the lost flux due to radiation. Units are GeV. Equivalent to the ΔE parameter in more traditional Schwinger or Mo and Tsai calculations.
8.	`5`
8.	Number of cases to run. Each case is defined by W, Q^2, $\cos\theta_{pq}^{cm}$, and ϕ_{pq}.
9.	`2.218  2.218  2.218  2.218  2.218`
9.	Values of W in GeV for each of the cases. The value of W in EXCLURAD is calculated assuming the mass of the target/struck object is a deuteron. Pick out quasi-elastic scattering by require Bjorken $x = 1$ so $$x = 1 = Q^2/(2M_n\nu)$$ where M_n is the mass of a nucleon. This restricts the value of ν (the energy loss) if you have already picked Q^2 and vice versa. Then $$W^2 = M_d^2 + 2M_d\nu - Q^2$$ where M_d is the mass of the deuteron.
10.	`1.4  1.4  1.4  1.4  1.4`
10.	Values of Q^2 in $(GeV/c)^2$ for each of the cases.
11.	`0.25  0.05  0.05  -0.5  -0.95`
11.	Values of $\cos\theta_{pq}$ for each of the cases.
12.	`0.0  0.0  0.0  0.0  0.0`
12.	Values of ϕ_{pq} in degrees for each of the cases.

Table 3: Sample input file with parameters and a description of each one (continued).

5.4 Program Output

When you run EXCLURAD, it dumps lots of information to the screen. As example is shown below.

```
npoi= 5 data points
Warning!! Very small LTp structure function:  8.26599695E-28
tai:  1  1   0.9625E-02  0.0021      231  0
tai:  2  1   0.5937      0.0066     1617  0
tai:  3  1   0.5071E-01  0.0001      165  0
tai:  4  1   0.5317      0.0065     1617  0
tai:  5  1   0.7875E-02  0.0023      231  0
```

```
 tai:  1  2   0.6748E-01  0.0098     1089  0
 tai:  2  2   0.1628      0.0084      627  0
 tai:  3  2   0.1976      0.0071      759  0
 tai:  4  2   0.1462      0.0076      627  0
 tai:  5  2   0.5198E-01  0.0047     1155  0
 tai:  1  3   0.9625E-02  0.0021      231  0
 tai:  2  3   0.5937      0.0068     1617  0
 tai:  3  3   0.5071E-01  0.0001      165  0
 tai:  4  3   0.5316      0.0068     1617  0
 tai:  5  3   0.7875E-02  0.0023      231  0
 tai:  -0.2085E-14   0.2317E-02  231    0
 test :   0.922E-13 -0.632E-17 -0.146E+05
   2.218   1.400   0.967   0.250   0.000   0.811 -.15E+05   0.821
```

The first line lists the number of calculations requested. The second is a warning about extremely small response functions that are at the limit of precision. The lines that start with 'tai:' show intermediate results of the full calculation. The important ones are in columns five and six out of the seven. Column five is the result of the test of an integral EXCLURAD performs and should always be less than 0.01. Column six is the number iterations performed in calculating the same integral. The line labeled 'test' list the results of some tests on the calculation. The very last line has the results of the calculation in the following order for the example above.

```
W     Q2    epsilon cos(theta_pq) phi_pq  full result  test    leading log
2.218 1.400 0.967   0.250         0.000   0.811       -.15E+05 0.821
```

This pattern is repeated for each of the calculations requested.
The output files are described below.

1. radcor.dat - contains the primary results of the calculations.
2. all.dat - contains the monitoring information that was printed out during execution.
3. radtot.dat - actually appears to contain less information than radcor.dat.

5.5 Pitfalls

After doing many of these calculations we noticed that occasionally the calculation will 'fail'. The symptom is that the radiative correction for one or two angles out of a complete angular distribution will be noticeably inconsistent with the other values in that angular distribution. This is obvious if one plots the RC surface as a function of $\cos\theta_{pq}^{cm}$ and ϕ_{pq} for a given kinematics (Q^2 and W). See Section 4 for more details. In response we have developed code that will enable the user to perform many parallel calculations on the Richmond cluster and visualize the results with the symbolic math package *Mathematica* which is available at JLab and many universities. Once the user has identified a

problem lowering the convergence criteria and/or increasing the number of steps in the integration algorithm fixes the problem. At the moment these parameters are hardwired into the code. The convergence parameter is the FORTRAN variable `ot` and is set in the subroutine `qqt` in the file exclurad.f. The limit on the number of iterations is the FORTRAN variable `ma` and is set in the same subroutine `qqt` in the file exclurad.f. A more detailed description of the problem and the solution are available in the E5 electronic logbook at `http://clasweb.jlab.org/cgi-bin/ENOTE/enote.pl?nb=e5&action=view&page=222` and `http://clasweb.jlab.org/cgi-bin/ENOTE/enote.pl?nb=e5&action=view&page=227`. The username and password are the usual ones.

6 Summary

We have developed a computer program to calculate radiative corrections for the exclusive reactions on the deuteron $d(e,e'p)n$ and $d(e,e'n)p$ that avoids some of the restrictions of the Schwinger/Mo and Tsai method often used for inclusive electron scattering. Our initial results show significant differences with a Schwinger calculation as a function of Q^2 and reveal a dependence at the 1% level on ϕ_{pq} in the ratio of the radiative corrections for the neutron and proton exit channels. We have documented this work and include a user's guide in this CLAS-Note and are maintaining a website for supporting and distributing the program [10].

References

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