Nuclear Reactions
Some Basics

I. Reaction Cross Sections

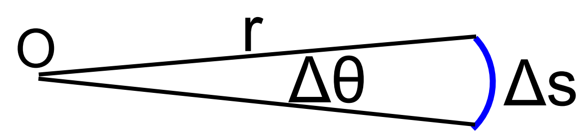
Common Units in Nuclear Physics

- Length: 1 fermi (fm) = 10^{-15} m (10^{-13} cm).
- Area (cross section): 1 barn (bn) = 10^{-24} cm^2.
- Energy: 1 electron volt (eV) = 1.6 x 10^{-19} J $\hbar$
  - Multiples: keV (10^3 eV), MeV (10^6 eV), GeV (10^9 eV)
- Mass: MeV/c^2 {from $E = mc^2$, $m = E/c^2$}
  - 1 MeV/c^2 = (10^{-13} J)/(3x10^8 m/s)^2 = 1.1 x 10^{-30} kg
- Momentum: MeV/c {from $E^2 = p^2c^2 + (m_oc^2)^2$}
  - 1 MeV/c = (10^{-13} J)/(3x10^8 m/s) = 3.3 x 10^{-22} kg·m/s
- System of units with $\hbar$ = c = 1 where $\hbar = h/2\pi$
  - h = 6.63x10-34 J·s $\hbar$ = 1.05x10-34 J · s
  - 1 Mev/c = 1 Mev/c^2 = 1 MeV
- $\hbar$c = (1.05x10^{-34} J·s/1.6x10^{-13}J/MeV)(3x10^8 m/sx10^{15} fm/m)
  = 197 MeV·fm

Solid Angle Ω

- Reminder: A "regular" angle Δθ, i.e. the angle subtended at a point O by an arc length Δs a distance r from O, is defined by:

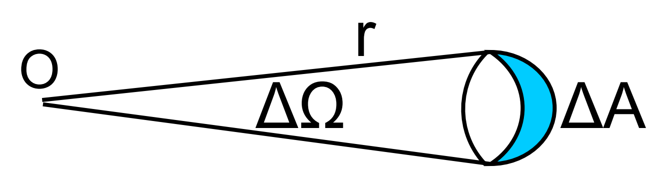
$$\Delta\theta = \frac{\Delta s}{r} \text{ radians (r)}$$



  – Note: Angle subtended by a full circle: $\theta = (2\pi r)/r = 2\pi$

- A solid angle ΔΩ (the 3-D equivalent of Δθ), the angle subtended by a surface element ΔA a distance r from O, is defined by:

$$\Delta\Omega = \frac{\Delta A}{r^2} \text{ steradians (sr)}$$



  – Note: Solid angle subtended by a full sphere: $\Omega = (4\pi r^2)/r^2 = 4\pi$

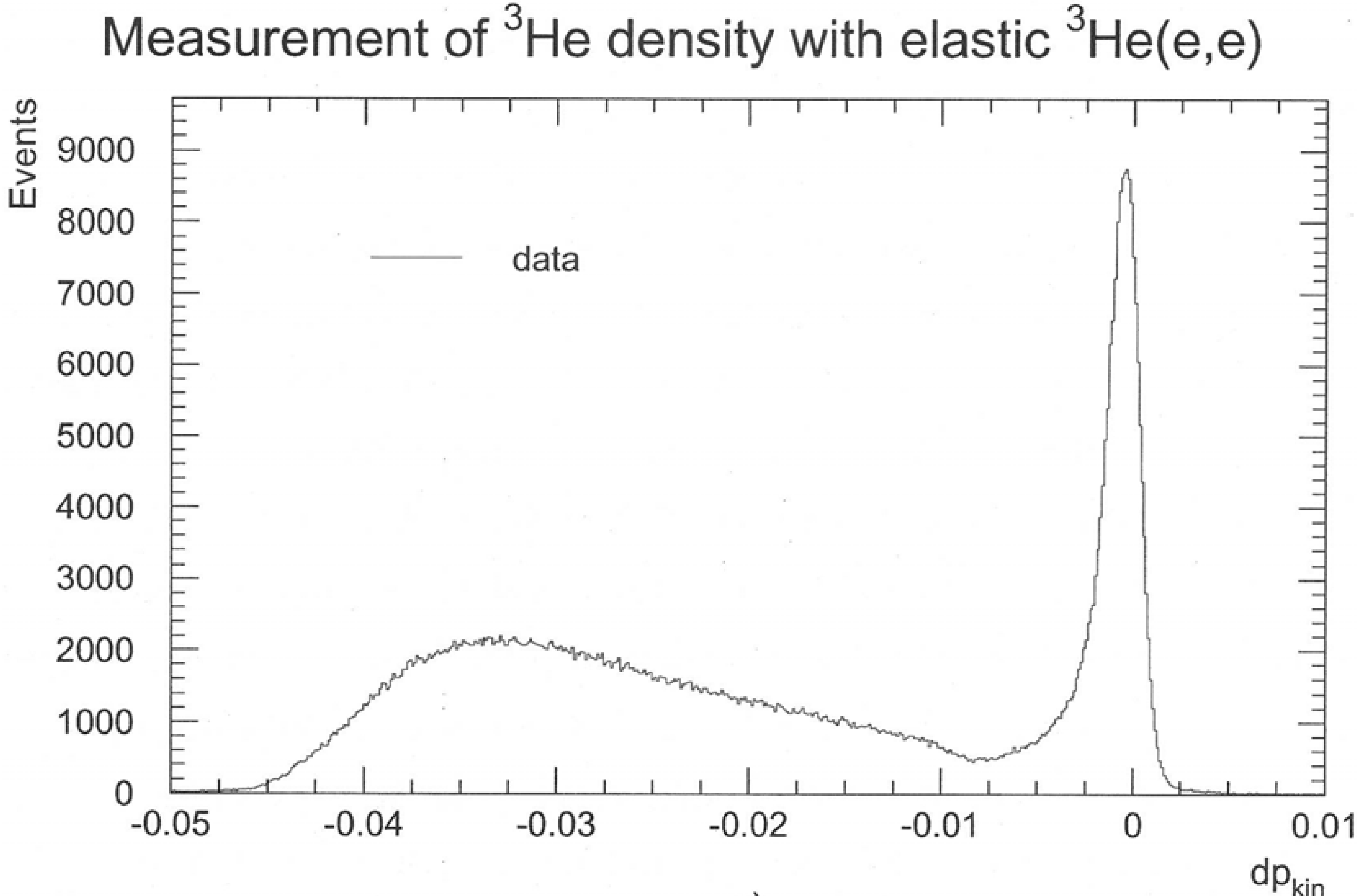
Types of Nuclear Reactions

- When a particle strikes a nucleus, the resulting interaction is referred to as a "nuclear reaction"
- Depending on its energy, the incoming particle can produce different types of "reactions" ("reaction channels"). Some examples:
  - Scattering (outgoing particle identical with the incident particle, the target nucleus doesn't break up)
    - Elastic scattering - target nucleus remains unchanged (and in ground state)
    - Inelastic scattering - target nucleus left in an excited energy state
  - (Breakup) reaction – one or more particles emitted from target nucleus, incident particle not necessarily present in the final state
  - Photodisintegration – breakup of a nucleus induced by an incident photon
- Energy release **Q** in a reaction – Definition:

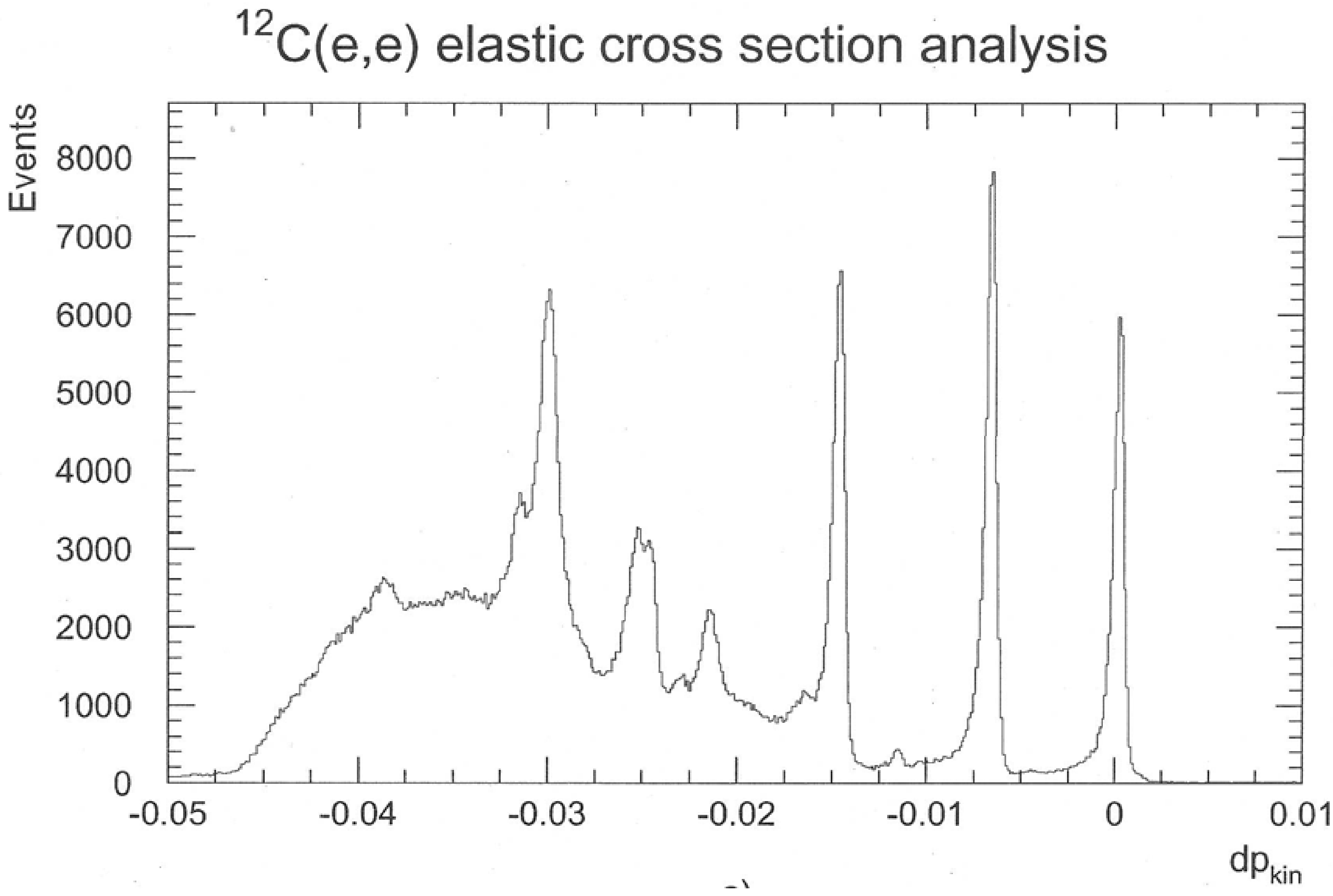
  Q = Σ masses BEFORE the reaction – Σ masses AFTER the reaction

  - Example: $e + {}^3He \rightarrow e + p + d$: $Q = (M_e + M_{3He}) - (M_e + M_p + M_d)$
  - Q may be >0 (exothermic) or <0 (endothermic)
  - Q = 0 for elastic scattering

Elastic Scattering Energy Spectrum



Inelastic Scattering Energy Spectrum



Notation for Nuclear Reactions

- A nucleus is specified by its chemical symbol (e.g. C for carbon), a superscript A (sum of Z protons and N neutrons in the nucleus) and a subscript Z (protons, often omitted)
  - Examples: ^{12}C, ^{4}He, $^{56}_{26}Fe$
- Usual shorthand for a reaction:

  a + A → b + B is also written as **A(a,b)B**

- General shorthand rule:

  Target(Incident Before,Detected After)Undetected leftovers

- Examples:
  - Elastic scattering: $^{12}C(p,p)^{12}C$
  - Inelastic scattering ("inclusive" reaction): $^{16}O(e,e')^{16}O^*$
  - Knockout ("exclusive") reaction: $^{3}He(e,e'p)^{2}H$
  - Stripping reaction: $^{7}Li(d,p)^{8}Li$
  - Photodisintegration: $^{2}H(\gamma,p)n$

Cross Section of a Nuclear Reaction

Q. What fraction of particles in a beam incident on a target nucleus participates in a particular nuclear reaction?

A. In microscopic physics we can not predict "certainties", only "probabilities".

Q. Likelihood (probability) of a dart hitting a circular dart board?

A. Proportional to the (perpendicular) area of the dart board (its cross sectional area).

Similarly in nuclear physics:

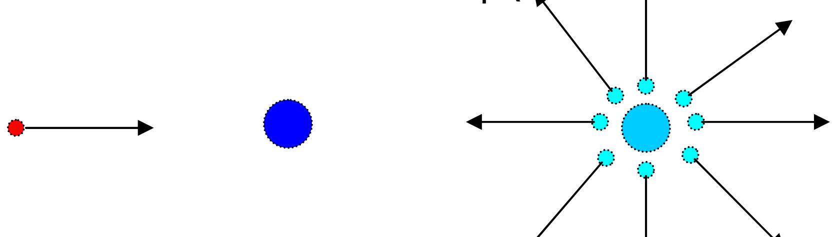
- Probability of a projectile to "hit" a target nucleus (i.e. interact with it, such as scatter from it or break it up) may be described by an analogous **"cross section"** (but not the actual, physical cross sectional area of the nucleus).
- Different processes (reaction channels) possible for a given particle incident on a nucleus have different cross sections.
- Cross sections depend on a variety of reaction variables.
- Cross section measurements are some of the most important (and most common?) measurements made in a nuclear physics lab experiment.

Formal Definition of Cross Section

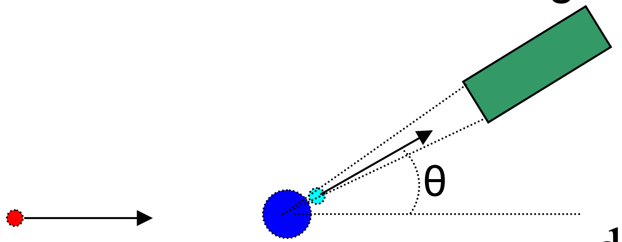
- Consider a beam of particles incident on a thin sheet of material (of **n** nuclei per unit volume, thickness **x**, area **A** hit the beam).
- There is a probability that, in passing through, some certain reaction will take place if the particle gets "close enough" to a nucleus. Let **σ** be the "effective" area of the nucleus for this particular reaction channel, i.e. if particle falls within this area, this particular reaction channel will take place.
  - Total number of nuclei in the area **A** = **n**(#/cm^3)**x**(cm)**A**(cm^2)
  - **Effective** area available for this reaction = (**nxA**)**σ** (cm^2)
  - **Probability** that this reaction will take place = **nxAσ/A** = **nxσ**
- Units: cross section (area) measured in cm^2
- Most convenient sub-multiple is 1 barn (b) = 1 x 10^{-24} cm^2
  - 1 barn is a "large" cross section for nuclear physics
  - More common: mb (10^{-3} b), μb (10^{-6} b) or nb (10^{-9} b)

Various Types of Cross Sections

- Total reaction cross section: σ_T (detect reaction products in 4π)



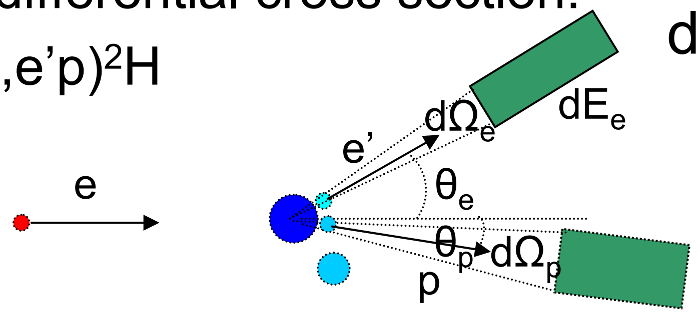
- Differential cross section (angular distribution) $\frac{d\sigma}{d\Omega}$: (detect only reaction products emitted at θ within a solid angle $d\Omega$)



- Doubly differential cross section: $$\frac{d^2\sigma}{d\Omega dE}(\text{mb/sr}\cdot\text{MeV})$$

- Triply differential cross section: $$\frac{d^5\sigma}{d\Omega_{e'} d\Omega_p dE_{e'}}(\text{mb/sr}^2\text{MeV})$$

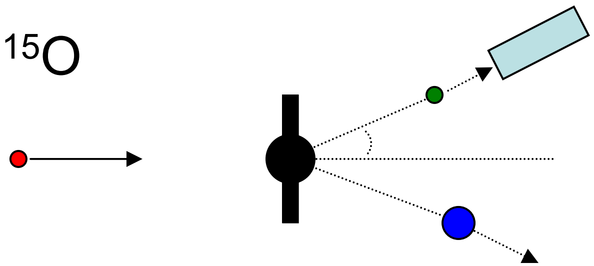
  $^3He(e,e'p)^2H$



Measurement of Cross Section

- In an actual experiment we measure the rate (#events/s) at which a certain reaction occurs under certain conditions.
  - Example: $^{14}N(d,n)^{15}O$

    Find $d\sigma/d\Omega$ at θ



- Measurable experimental parameters:
  - Incident beam current **I** (in Amperes A =Coulombs C/second s)
    - Incident beam charge Q during run of duration t: **Q = It** (C)
  - Number **n** of target nuclei per unit volume
  - Target thickness **x** (in cm)
  - Solid angle **ΔΩ** of detector (in sr)
  - Reaction events counted (**N** particles detected) during time **t**
    - Counting Statistics (uncertainty in measuring N random events): $\frac{\sqrt{N}}{N}$

Measurement of Cross Section, cont.

- Using all this info, can **calculate** fraction (N/N_o) of all incident particles (N_o) that will result in particles N scattered at θ within ΔΩ

  = Probability/incident particle that a scattered particle will be detected at angle θ by a detector subtending solid angle ΔΩ

  = nx(dσ/dΩ)ΔΩ, or:

$$\frac{N}{N_o} = nx\frac{d\sigma}{d\Omega}\Delta\Omega \quad \text{or, using} \quad n = \frac{\rho\left(\frac{g}{cm^3}\right)A_o\left(\frac{nuclei}{mol}\right)}{M\left(\frac{g}{mol}\right)} = \frac{\rho A_o}{M}\left(\frac{nuclei}{cm^3}\right)$$

  and rewriting:

$$N = N_o nx\frac{d\sigma}{d\Omega}\Delta\Omega = \frac{I(A)t(s)}{q(C)}\frac{\rho A_o}{M}\left(\frac{nuclei}{cm^3}\right)x(cm)\frac{d\sigma}{d\Omega}\left(\frac{cm^2}{sr}\right)\Delta\Omega(sr)$$

- From this, and using Q (C) = I (A) t (s), we solve for:

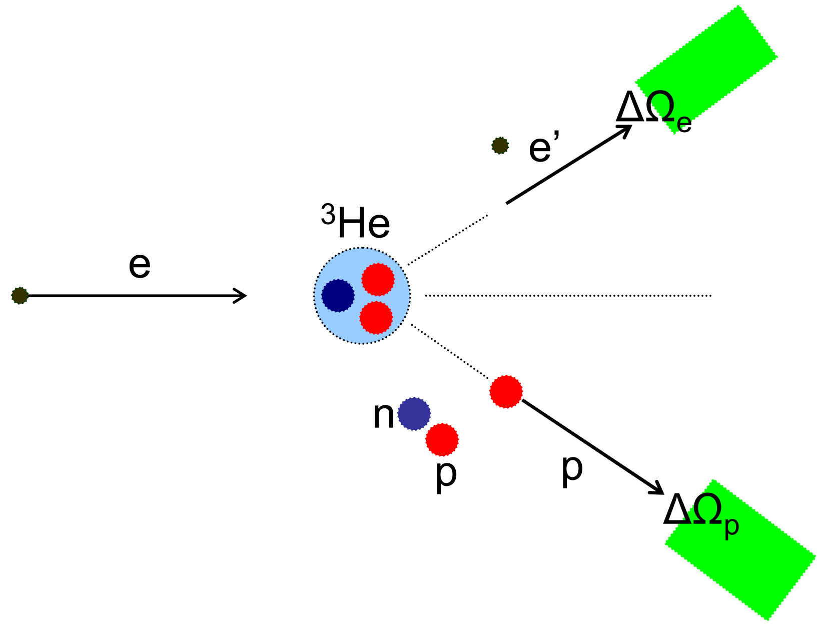
$$\frac{d\sigma}{d\Omega} = \frac{Nq}{Qnx\Delta\Omega}\left(\frac{cm^2}{sr}\right)$$

Why is a Cross Section Important?

- It is the meeting ground between theory and experiment
  - Nuclear theory, using quantum mechanics, is used to predict the probability (likelihood) that a specific nuclear process will occur under certain conditions (e.g. incident energy, angle of observation, etc.)
  - The quantitative measure of this prediction is the cross section of the process. That is, nuclear theory is used to predict the specific cross section of a process.
  - This cross section may be measured in the laboratory
  - Comparison between theoretical prediction and measurement is used to evaluate the significance of the underlying theory.
- The last bullet (above) describes the essence of “doing science”.

JLab/CSULA E89-044: $^3He(e,e'p)$ Reaction

- Two reaction channels studied:
  - Two-body breakup: $^3He(e,e'p)^2H$
  - Three-body breakup: $^3He(e,e'p)pn$
  - Continuum (above three-body breakup)
- Experimental setup:



$$\frac{d^5\sigma}{d\Omega_{e'} d\Omega_p dE_e} = \frac{Ne}{Qnx\Delta\Omega_e\Delta\Omega_p\Delta E_e}$$

Q=total beam charge of run
N=events counted during Q
e=electron electric charge
n=nuclei/volume of target
x=target thickness
ΔΩ=detector solid angle
ΔE=detector energy bin

$^3He(e,e'p)^2H$ Cross Section (E89-044)

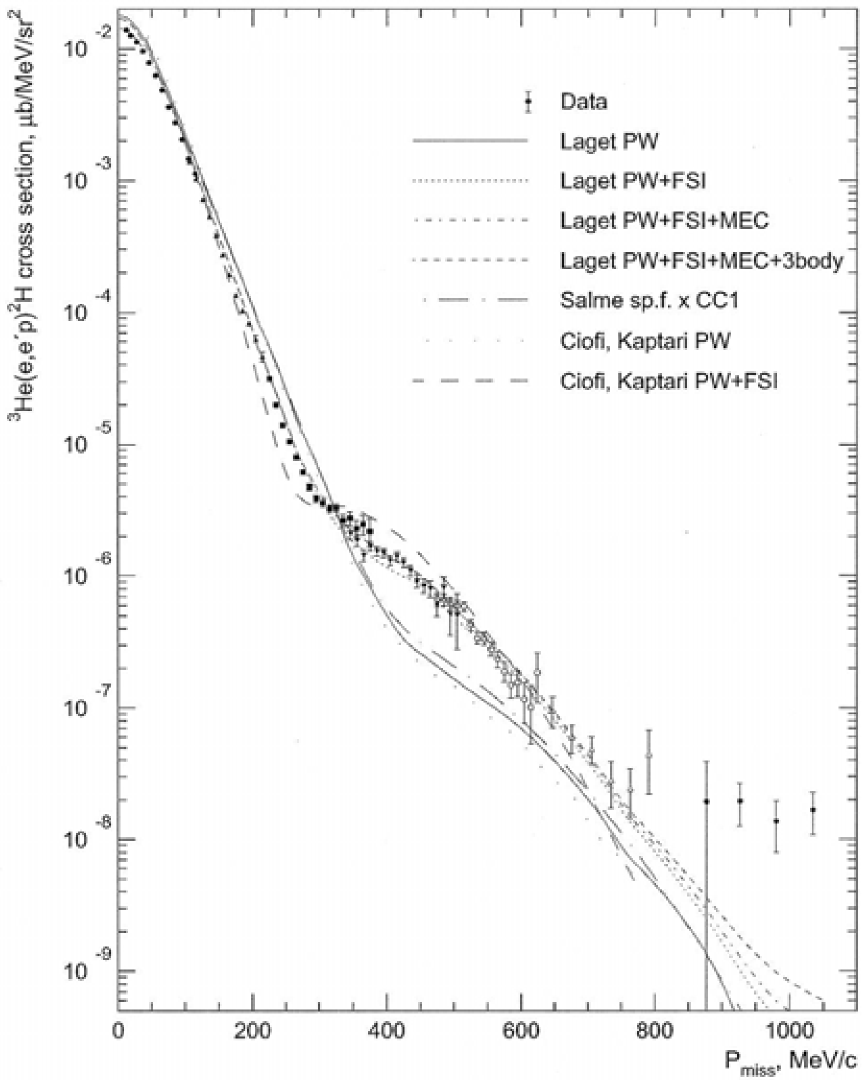


Figure 6-1: ^{3}He(e,e′p)D cross sections extracted at incident electron energy 4805.5 MeV. The detected proton is at angles back of $\vec{q}$.

Response Function Separation - $^3He(e,e'p)^2H$ Reaction - (The End Game)

- From nuclear theory (one-photon-exchange-approximation) the cross section is calculated to be:

$$\frac{d^5\sigma}{d\Omega_e d\Omega_p dE} = K\sigma_M\left(V_L R_L + V_T R_T + V_{TL} R_{TL}\cos\varphi + V_{TT} R_{TT}\cos 2\varphi\right)$$

  where:
  - K, V_L, V_T, V_{TL} and V_{TT} are (calculable) factors dependent on kinematic variables
  - σ_M is the (calculable) scattering cross section of an e from a **point** nucleus (rather the extended size ^3He nucleus)
  - R_L, R_T, R_{TL}, R_{TT} are called "nuclear response functions" and contain all the information about ^3He that can be extracted using (e,e'p)
- The main objective of measuring the cross section of this reaction is to extract the response functions R_L, R_T, R_{TL}, R_{TT}