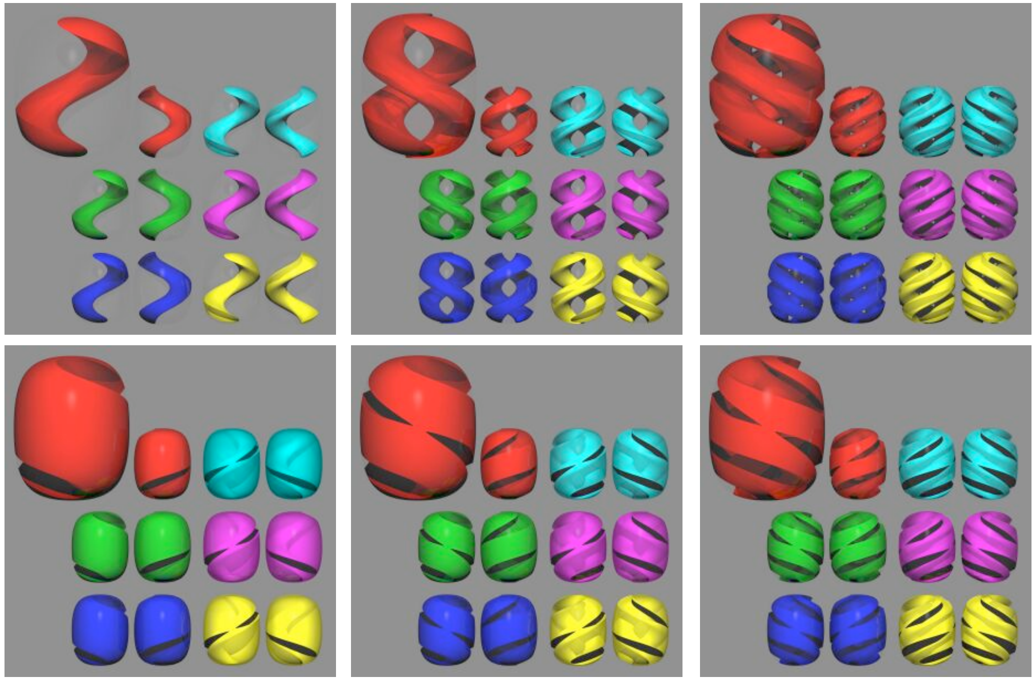
Physics 3:
Particle Physics

Lecture 5: Quarks & Leptons, Mesons & Baryons

February 25th 2008



- **Leptons**
  - Quantum Numbers

- **Quarks**
  - Quantum Numbers
  - Isospin
  - Quark Model and a little History
  - Baryons, Mesons and colour charge

Leptons

- Six leptons: e^- μ^- τ^- ν_e ν_μ ν_τ
- Six anti-leptons: e^+ μ^+ τ^+ $\bar{\nu}_e$ $\bar{\nu}_\mu$ $\bar{\nu}_\tau$

- Four quantum numbers used to characterise leptons:
  - Electron number, L_e, muon number, L_μ, tau number L_τ
  - Total Lepton number: $L = L_e + L_\mu + L_\tau$
  - L_e, L_μ, L_τ & L are conserved in all interactions

Lepton		L_e	L_μ	L_τ	$Q(e)$
electron	e^-	+1	0	0	−1
muon	μ^-	0	+1	0	−1
tau	τ^-	0	0	+1	−1
electron neutrino	ν_e	+1	0	0	0
muon neutrino	ν_μ	0	+1	0	0
tau neutrino	ν_τ	0	0	+1	0
anti-electron	e^+	−1	0	0	+1
anti-muon	μ^+	0	−1	0	+1
anti-tau	τ^+	0	0	−1	+1
electron anti-neutrino	$\bar{\nu}_e$	−1	0	0	0
muon anti-neutrino	$\bar{\nu}_\mu$	0	−1	0	0
tau anti-neutrino	$\bar{\nu}_\tau$	0	0	−1	0

Think of L_e, L_μ and L_τ like electric charge:

- They have to be conserved at every vertex.
- They are conserved in every decay and scattering

Parity: intrinsic quantum number.
$\pi=+1$ for lepton
$\pi=-1$ for anti-leptons

Introduction to Quarks

- Six quarks: **d u s c t b**
- Six anti-quarks: $\bar{\mathrm{d}}\ \bar{\mathrm{u}}\ \bar{\mathrm{s}}\ \bar{\mathrm{c}}\ \bar{\mathrm{t}}\ \bar{\mathrm{b}}$

Parity: intrinsic quantum number
$\pi=+1$ for quarks
$\pi=-1$ for anti-quarks

- Lots of quantum numbers used to describe quarks:
  - Baryon Number, $\mathcal{B}$ - (total number of quarks)/3
    - $\mathcal{B}=+1/3$ for quarks, $\mathcal{B}=-1/3$ for anti-quarks
  - Strangeness: S, Charm: C, Bottomness: B, Topness: T - number of **s, c, b, t**
    - $S=N(\bar{\mathrm{s}})-N(\mathrm{s})$ $C=N(\mathrm{c})-N(\bar{\mathrm{c}})$ $B=N(\bar{\mathrm{b}})-N(\mathrm{b})$ $T=N(\mathrm{t})-N(\bar{\mathrm{t}})$
  - Isospin: I, I_Z - describe up and down quarks

Quark		I	I_Z	S	C	B	T	$Q(e)$
down	d	1/2	−1/2	0	0	0	0	−1/3
up	u	1/2	1/2	0	0	0	0	+2/3
strange	s	0	0	−1	0	0	0	−1/3
charm	c	0	0	0	+1	0	0	+2/3
bottom	b	0	0	0	0	−1	0	−1/3
top	t	0	0	0	0	0	+1	+2/3

- $\mathcal{B}$ conserved in **all** interactions
- S, C, B, T conserved in **strong** and **electromagnetic**
- I, I_Z conserved in **strong** interactions only

Much Ado about Isospin

- **Isospin** was introduced as a quantum number before it was known that hadrons are composed of quarks.
- Hadrons with the same isospin, I, exhibit a symmetry: they have roughly equal mass and the strong force between the constituent quarks is equal.
- Now we know it describes the number of up and down quarks in hadrons.
- As for (normal) spin, use two quantum numbers to describe it:
  - **Total isospin, I**
    - same for up and down quarks, $I=½$
  - **Third component of isospin, I_Z**
    - distinguishes between up and down quarks
    - $I_Z = \frac{1}{2}\left[N(\mathrm{u}) - N(\mathrm{d}) + N(\bar{\mathrm{d}}) - N(\bar{\mathrm{u}})\right]$

Quark	I	I_Z
u	$1/2$	$+1/2$
d	$1/2$	$-1/2$
$\bar{\mathrm{u}}$	$1/2$	$-1/2$
$\bar{\mathrm{d}}$	$1/2$	$+1/2$

Example: Pions: π^+, π^0 and π^- have $\mathbf{m(\pi^\pm)=139.6\ MeV}/c^2$, $\mathbf{m(\pi^0)=135.0\ MeV}/c^2$.

- π^+ is $\mathrm{u\bar{d}}$: $I_Z = ½+½=1$
- π^0 is $\mathrm{u\bar{u}}$ or $\mathrm{d\bar{d}}$: $I_Z = ½+(-½)=0$
- π^- is $\mathrm{d\bar{u}}$: $I_Z = (-½)+(-½)=-1$

Total isospin is the highest value of the I_Z. π^+, π^0, π^- all have $I=1$

Conservation Laws

Noether's Theorem: Every symmetry of nature has a conservation law associated with it, and vice-versa.

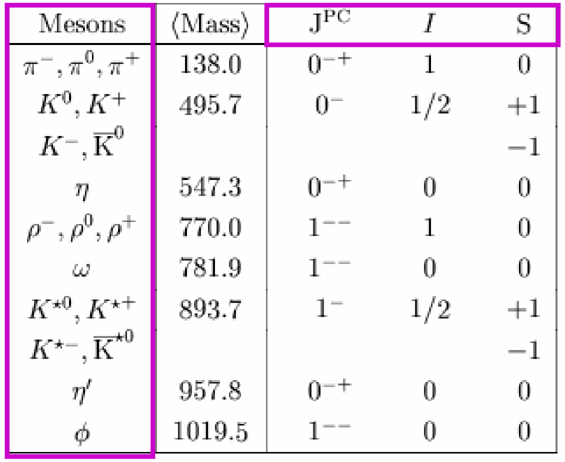
Emmy Noether
1882-1935

- **Energy, Momentum and Angular Momentum**
  - ➡ Conserved in all interactions
  - ➡ Symmetry: translations in time and space; rotations in space
- **Charge conservation Q**
  - ➡ Conserved in all interactions
  - ➡ Symmetry: gauge transformation - underlying symmetry in QM description of electromagnetism
- **Lepton Number L_e, L_μ, L_τ and Baryon Number $\mathcal{B}$ symmetry**
  - ➡ Conserved in all interactions
  - ➡ Symmetry: mystery!
- **Quark Flavour S, C, B, T; Isospin, I; Parity, π**
  - ➡ Conserved in strong and electromagnetic interactions
  - ➡ Violated in weak interactions
  - ➡ Symmetry: unknown!

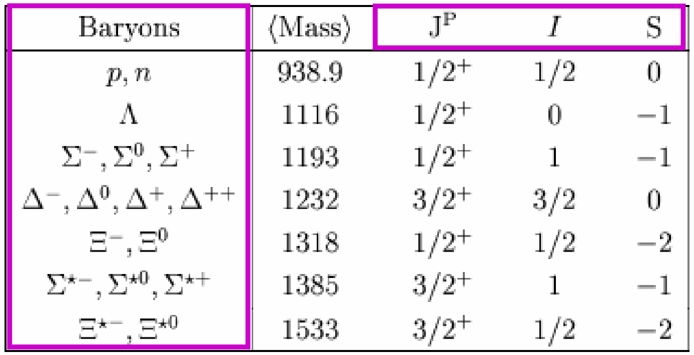
A bit of History: the Quark Model

By the mid-1960's many particles had been discovered no one really understood why there were so many.

- These could not all be fundamental particles...



Mesons	⟨Mass⟩	J^{PC}	I	S
π^-, π^0, π^+	138.0	0^{-+}	1	0
K^0, K^+	495.7	0^-	1/2	+1
$K^-, \overline{K}^0$				−1
η	547.3	0^{-+}	0	0
ρ^-, ρ^0, ρ^+	770.0	1^{--}	1	0
ω	781.9	1^{--}	0	0
$K^{\star 0}, K^{\star +}$	893.7	1^-	1/2	+1
$K^{\star -}, \overline{K}^{\star 0}$				−1
η'	957.8	0^{-+}	0	0
ϕ	1019.5	1^{--}	0	0

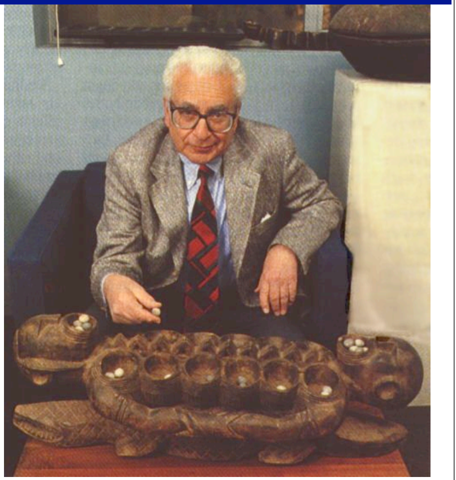


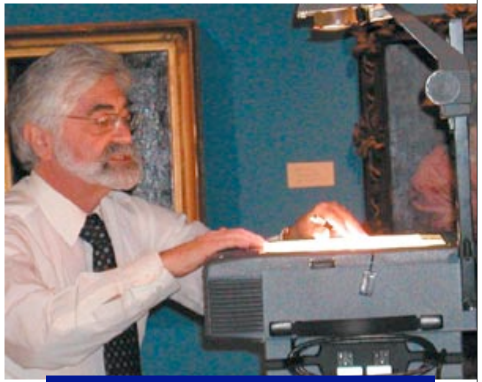
Baryons	⟨Mass⟩	J^P	I	S
p, n	938.9	$1/2^+$	1/2	0
Λ	1116	$1/2^+$	0	−1
$\Sigma^-, \Sigma^0, \Sigma^+$	1193	$1/2^+$	1	−1
$\Delta^-, \Delta^0, \Delta^+, \Delta^{++}$	1232	$3/2^+$	3/2	0
Ξ^-, Ξ^0	1318	$1/2^+$	1/2	−2
$\Sigma^{\star -}, \Sigma^{\star 0}, \Sigma^{\star +}$	1385	$3/2^+$	1	−1
$\Xi^{\star -}, \Xi^{\star 0}$	1533	$3/2^+$	1/2	−2

- In 1964 Gell-Mann and Zweig proposed the idea of ***quarks*** to explain the observed isospin and strangeness of the particles.

Quark	Charge, e	Isospin $\|I, I_Z>$	Strange-ness, S
Up	+2/3	$\|½, +½>$	0
Down	−1/3	$\|½, −½>$	0
Strange	−1/3	$\|0, 0>$	-1

Murray Gell-Man, 1969
Nobel Prize in Physics

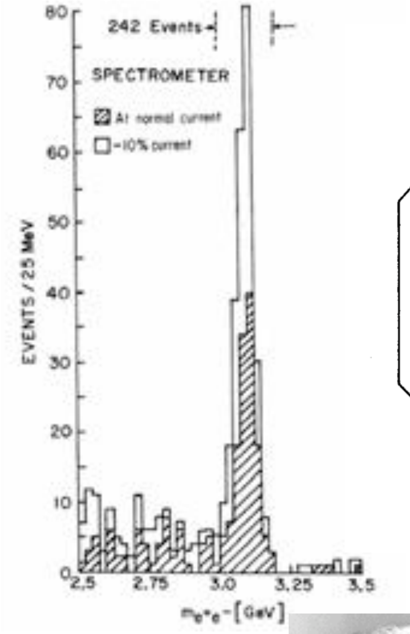


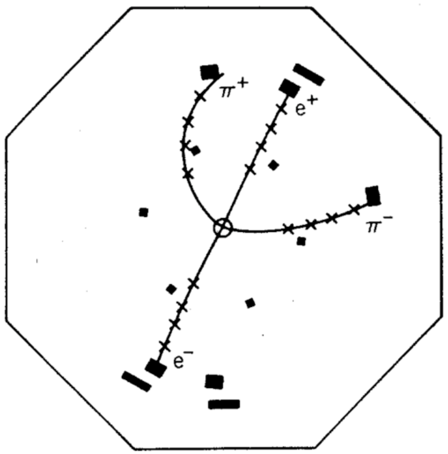


George Zweig

Discovery of the J/ψ and charm quarks

- In 1974 a new resonance state found in strong interactions: $p+\mathrm{Be}\rightarrow J/\psi+X$
- Lifetime $\tau(J/\psi) = 7.6\times10^{-21}$ s - decays by both strong and electromagnetic force.
- Mass $m(J/\psi) = 3.1$ GeV/c^2 – too heavy to be described in terms of **u, d, s** quarks.
- Instead must be a made of a new type of quark; interpretation: J/ψ is composed of **c** and $\bar{\mathbf{c}}$.
- Discovery of J/ψ caused a revolution in particle physics. Lead to acceptation of Gell-Mann and Zweig’s idea that hadrons were composed of quarks.

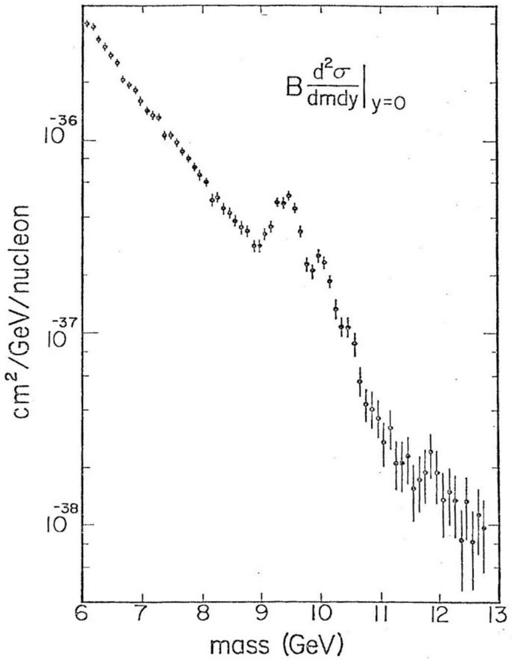


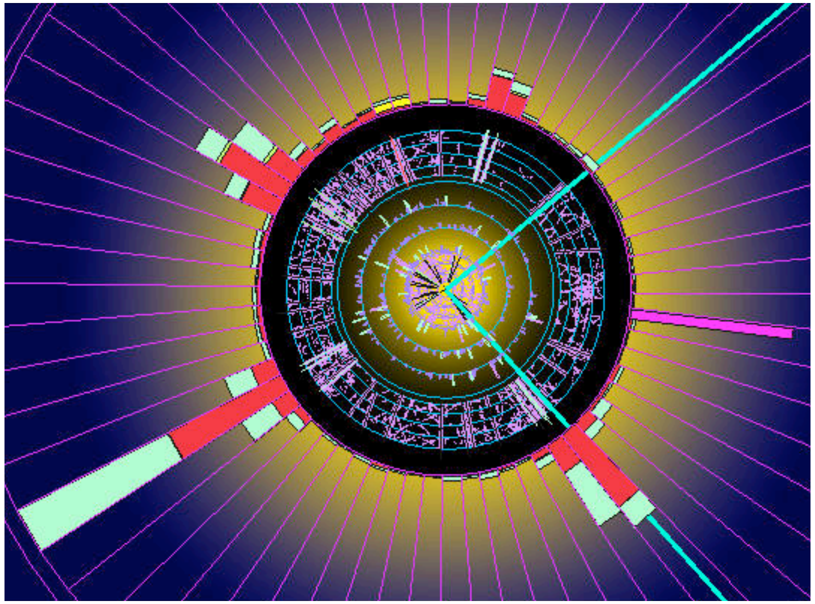


Richter and Ting independently discovered the J/ψ.
1976 Noble prize in physics

Discovery of Bottom and Top Quarks

- Bottom quark was discovered in 1977 at Fermilab in p+Be scattering
- New resonance state, called the Upsilon, ϒ, was found. $\Upsilon = b\bar{b}$
- $m(\Upsilon) = 9.64\ \mathrm{GeV}/c^2$, $\tau(\Upsilon)=1.3\times10^{-20}$ s
- Like J/ψ decays via both strong and electromagnetic force.





- The top quark was discovered in 1995 at CDF and DØ at the Tevatron.
- Production: $p\bar{p}\to t\,\bar{t}$
- The top is so heavy it decays via weak force before it forms hadrons: $t\to Wb$
- $m(t)\approx170\ \mathrm{GeV}/c^2$
- Why the top so much heavier than all the other quarks is still a mystery...

Baryons

Spin statistics theorem:
Quantum Lecture 16

- Baryons are bound state of three quarks. $\mathcal{B} = +1/3 + 1/3 + 1/3 = 1$
- **Spin statistics theorem**: No two identical fermions can be in the same quantum state.
- How do we explain a baryon such as $\Delta^{++} = uuu$?
  - Introduce a new quantum number, colour. Each quark carries a colour charge: either **red**, **green** or **blue**. A baryon contains one quark of each colour: e.g. $\Delta^{++} = u_r u_g u_b$
  - Anti-quarks carry opposite colour charge: anti-red $\bar{r}$, anti-green $\bar{g}$, anti-blue $\bar{b}$

Quantum number of Baryons:

$J^{\pi} = (\text{Total angular mom}=L+S)^{\text{parity}}$

- Only states with angular momentum $L=0$ are observed.
  - Spins aligned: ↑↑↑ or ↓↓↓ $S=+3/2$
  - Not all spins aligned: ↑↑↓ ↑↓↑ ↓↑↑ $S=+1/2$

Parity of baryon with $L=0$
$\pi(q_1q_2q_3) = \pi(q_1)\pi(q_2)\pi(q_3)$
$= (+1)(+1)(+1) = +1$

Total angular momentum $J=L+S=S$

Baryons are fermions, they have half-integer spin: $1/2\hbar$, $3/2\hbar$, ...

Mesons

Mesons: bound state of a quark and an anti-quark. They have:

- Zero net colour charge: $|\psi\rangle = \frac{1}{\sqrt{3}}|r\bar{r} + g\bar{g} + b\bar{b}\rangle$
- Zero net baryon number: $\mathcal{B} = +1/3 + (-1/3) = 0$

J^{π} = (Total angular mom=L+S) ^{parity}

Quantum number of Mesons:

- **Psudeo-scalar mesons:** $J^{\pi}=0^{-}$ Ground state of $q\bar{q}$ combination
    - Angular momentum, $L=0$
    - Spin of quark and antiquark anti-aligned ↑↓ or ↓↑ $S=0$
    - Total angular momentum $J=L+S=0$

- **Vector Mesons:** $J^{\pi}=1^{-}$ First excited state of $q\bar{q}$ combination.
    - Angular momentum, $L=0$
    - Spin of quark and antiquark aligned ↑↑ or ↓↓ $S=1$
    - Total angular momentum $J=L+S=1$

Parity of a meson:

$$\pi(q\bar{q}) = \pi(q)\pi(\bar{q})(-1)^{L}$$
$$=(+1)(-1)(-1)^{L} = -1^{L+1}$$

Mesons are bosons, they have integer spin: 0, 1ℏ, 2ℏ, ...

Hadron Masses

Hard to measure the quark mass, as we can never isolate a quark.

- At high energy density, the masses of the quarks appear very light:
  - $m_u < m_d \sim 5$ MeV/c^2, $m_s \sim 100$ MeV/c^2
- At low energy density, we form hadrons, **constituent mass** is relevant:
  - $m_u = m_d \sim 300$ MeV/c^2, $m_s \sim 500$ MeV/c^2

- Mass is due to mass of quarks and spin couplings:

$$m(q\bar{q}) = m_1 + m_2 + A\frac{\vec{S}_1 \cdot \vec{S}_2}{m_1 m_2}$$

$$\begin{aligned}\vec{S}_1 \cdot \vec{S}_2 &= 1/2\,(\vec{S}^2 - \vec{S}_1^2 - \vec{S}_2^2) \\ &= 1/2\,[S(S+1) - S_1(S_1+1) - S_2(S_2+1)] \\ &= \begin{cases} +1/4 & \text{for } S = 1 \\ -3/4 & \text{for } S = 0 \end{cases}\end{aligned}$$

  - $m_u = m_d = 310$ MeV/c^2
  - $m_s = 483$ MeV/c^2
  - $A = (2m_u)^2 \times 160$ MeV/c^2
    - Excellent agreement! →

	Mass [MeV/c^2]	
Mesons	Prediction	Experiment
π	140	138
K	484	496
ρ	780	770
ω	780	782
K^*	896	894
ϕ	1032	1019

Summary

Six leptons: e^- μ^- τ^- ν_e ν_μ ν_τ

Quantum numbers: L_e, L_μ, L_τ & L

L_e, L_μ, L_τ & L conserved in all interactions

Six quarks: **d u s c t b**

Quantum numbers: $\mathcal{B}$, S, C, B, T, I, I_Z

$\mathcal{B}$ conserved in all interactions.

S, C, B, T, conserved in strong and electromagnetic interactions

I, I_Z conserved in strong interactions only.

Total isospin is equal for up and down quarks, $I=½$

Third component of isospin, I_Z distinguishes between up and down quarks

$$I_Z = \frac{1}{2}\left[N(\mathrm{u}) - N(\mathrm{d}) + N(\bar{\mathrm{d}}) - N(\bar{\mathrm{u}})\right]$$

Isospin adds like spin

Conservation of quantum numbers are related to the underlying symmetries of the theory

The proliferation of, and properties of, baryons and mesons may be described in terms of the consistent quarks.

Baryons wave function must be anti-symmetric; leads to the prediction of the **colour quantum number** for quarks.

Each quark carries a colour charge. Baryons and mesons are colour neutral: baryons has one red, one blue & one green quark; mesons are red – anti-red, blue – anti-blue or green–anti-green combinations.