Hi Sebastian,

Responses below.

1) Could you make a figure like Fig. 2 in your response, but with the angle theta nq (Lab) on the vertical axis? In other words, plot the distribution of events in both angle (relative to q) and magnitude of the missing momentum in the lab frame, after all your cuts (the ones you propose to use, which I assume does NOT include the W cut anymore, just on  $W_n$ . I just have a very hard time relating the mix of lab and cm variables you plotted to something my brain can grasp. Ideally I'd like to see 2 versions of the plots - with and without the cut on  $p(e')$ . It might be helpful (but not necessary) if you could also produce the plots showing lines of constant  $xBj$  on the same 2-D space (p miss vs. theta nq) that you described to me in words.

The first two plots below shows the effect of the  $\Delta p_e < 0.015 \text{ GeV/c}$  cut on the  $\theta_{pq}$ versus  $p_m$  distribution (where  $\theta_{pq}$  is in the lab) for the reversed torus polarity data. The  $\Delta p_e$ 



Figure 1: Comparison of  $\theta_{pq}$  versus  $p_m$  distribution with (left-hand panel) and without (right-hand panel) the  $\Delta p_e$  cut for the reversed torus polarity data.

cut removes electrons with momenta below the value expected with no radiative effects so the measured momentum transfer  $\vec{q} = \vec{p}_{beam} - \vec{p}_e$  is greater than expected with no radiation. The missing momentum is  $\vec{p}_m = \vec{q} - \vec{p}_p$  where  $\vec{p}_p$  is the proton 3-momentum and  $p_m$  tends to have a greater magnitude than expected with no radiation. In the left-hand panel of Fig 1, the high- $p_m$  side of the distribution for a particular value of  $\theta_{pq}$  is reduced when the  $\Delta p_e$ cut is used. Fig. 2 shows the same attributes for the normal torus polarity data.

To demonstrate the kinematic range of our data we plotted lines of constant  $x_{Bj}$  on the 2-dimensional,  $\theta_{pq}$ - $p_m$  distributions with the full set of cuts (Figs 3-4. The curves in the left-hand panel of Fig. 3, for example, mark the low- $p_m$  side of the main 'ridge' in the  $\theta_{pq}$ - $p_m$  distribution. The  $x_{Bj} = 1$  curve (solid) starts at  $\theta_{pq} = 0$ °, follows the low- $p_m$  limit of the measured distribution up to  $\theta_{pq} \approx 50^{\circ}$  and then the cross section for these kinematics



Figure 2: Same as previous plot except for the normal torus polarity data.



Figure 3: Distribution of  $\theta_{pq}$  versus  $p_m$  showing kinematic relationship at the  $Q^2$  and  $x_{Bj}$ limits of the 2.6-GeV, reversed torus polarity data

goes away. The  $x_{Bj} = 0.75$  curve (dashed) marks the high- $p_m$  limit for this value of  $Q^2$ . Kinematics with  $x_{Bi} > 1.0$  lie in between the the solid and dashed curves in each panel. The high-Q<sup>2</sup> behavior of the kinematics is shown in the right-hand panel. The  $x_{Bj} = 1$ curve (solid) starts at  $\theta_{pq} = 0^{\circ}$  and passes through the high- $p_m$  tail at large  $\theta_{pq}$  and  $p_m$ . The  $x_{Bj} = 0.6$  curve roughly marks out the high- $p_m$  limit of the distribution for a particular value of  $\theta_{pq}$ . Fig. 4 shows the same attributes for the normal torus polarity data. In this plot we did choose different values of  $Q^2$  and  $x_{Bj}$  because those distributions are different for the kinematics here.

2) Only if it is not too much work: As I said, I would prefer a calculation where both the numerator and the denominator of the Born asymmetry as well as of the radiated asymmetry are simulated as a function of ALL variables  $(Q2, p_{\text{miss, cos\_th, phi}})$  over a 4-D grid within the envelope of your cuts, and then averaged for each of your bins (weighted by data) to



Figure 4: Same as the previous plot except normal torus polarity data and different kinematic curves.

calculate the model Born Asymmetry and the model radiated asymmetry for each of your p m bins (you obviously already do this for the Born asymmetry, using WvO's model). Then, I would apply just the difference between the two (born - radiated) to your measured asymmetry to get the Born asymmetry. This way, you avoid possible divisions by zero and also the unequal weighting of events which is an unavoidable consequence of event-byevent corrections. As a minimum, it would be very illustrative to see if the answer comes out differently than with your method - such a difference could be a good estimate of this particular systematic uncertainty.

The first step in doing the radiative corrections (RCs) this way is to essentially change variables. The RC code we use called EXCLURAD calculates the correction as a function of  $W, Q^2$ , cos  $\theta_{pq}$ , and  $\phi_{pq}$ . I think you want that calculation to be a function of  $p_m, Q^2$ , cos  $\theta_{pq}$ , and  $\phi_{pq}$ . To do that we need to calculate W as a function of  $p_m$ ,  $Q^2$ ,  $\cos \theta_{pq}$ , and  $\phi_{pq}$ . I have included an appendix below that shows the relevant equations taken mostly from Ref. 11 of the analysis note along with a plot of the functions relating  $p_m$  to W for different choices of  $\cos \theta_{pq}$  and  $Q^2 = 0.2 \text{ GeV}^2$ .

If I understand what you are asking, you would like to see a GSIM simulation using an event generator based on (1) WVO's model and (2) WVO's model modified by radiation calculated in EXCLURAD. The output of each simulation would be passed through our analysis chain to produce  $A'_{LT}$  and the difference between the two would be added to our measured  $A'_{LT}$ . It's worth pointing out that the radiative corrections as calculated now are small compared to the statistical uncertainty of our data. Figure 5 shows the difference between  $A'_{LT}$  extracted with and without radiative corrections and then divided by the statistical uncertainty on the measured  $A'_{LT}$ . The ratio is small across the full range of  $p_m$ even at low missing momenta where the measured  $A'_{LT}$  and the statistical uncertainty are typically small. If the plan I have outlined above is correct, that will take some time to do, but if you and the committee think it's needed, then we will do it.



Figure 5: Ratio of the difference between  $A'_{LT}$  with and without radiative corrections divided by the statistical uncertainty of the measured  $A'_{LT}$ .

Beyond that, I am still unconvinced that you need the cut on  $p(e')$  - if radiative corrections are small and well done, why avoid that region? On the other hand, I would think that the measured asymmetry might well depend on cos(theta nq) quite significantly (which is what we found for  $A\rightarrow$  - ideally, I would have preferred a binning of the data in three  $\cos(\theta)$  bins (backward, e.g.  $\lt$  -0.3; sideways, -0.3  $\lt$  cos  $\lt$  0.3; forward,  $\gt$  0.3). But I understand I may be asking for too much here

For the first question: The long tail in Fig 9 of the analysis note (and reproduced below) can come from events with low momentum AND low angle. Collisions where photons are radiated after the collision (so the electron is going at least roughly in the unradiated electron direction) will have lower-than-expected momentum. Events that radiate a photon before the



Figure 6: Scattered electron momentum  $p_e$  versus scattering angle  $\theta_e$  for QE events and reversed torus polarity. The black curve is for elastic scattering off the proton.

collision will produce electrons with lower-than-expected momenta AND distorted angles. The EXCLURAD calculation corrects for both cases including the interference between them. If we took off the  $\Delta p_e$  cut and integrated over all  $p_e$  we would be mixing in events from the wrong momentum or angle bins due to events where the photon was radiated before the collision with the target. As the photon energies get larger, we have to rely more on the calculations of the radiative corrections which rely, in turn, on our understanding of the nuclear physics at increasingly different kinematics from quasielastic scattering. The  $\Delta p_e$  cut reduces ambiguities in the interpretation of the results at kinematics far from the quasielastic peak and has limited effect on the quality of our statistics. In other words, your suggestion may work for events where the photon was radiated after the collision with the target so the electron angle is closer to the one expected for a scattered electron with no radiated photons. However, things are different when the photon is radiated before the collision with the target because both angle and momentum are now different from their values for unradiated events.

For the second question: I agree that we should see a strong dependence in the asymmetry on  $\theta_{pq}$ . However, for a quasielastic event with a given  $Q^2$ ,  $x_{Bj}$  and  $p_m$  there is only a small range of  $\theta_{pq}$  that is allowed. Consider the solid curves in Figures 3 and 4. They show the value of  $\theta_{pq}$  as a function of  $p_m$  for a particular value of  $Q^2$  and two choices of  $x_{Bj}$ . Nearly all of the quasielastic events for each  $Q^2$  lie in the region between those two curves. Consider, for example, the right-hand panel of Figure 4. Events with  $Q^2 = 0.2 \text{ GeV}^2$  and missing momentum in the range  $p_e = 0.3 - 0.4$  GeV/c have to come in the range  $\theta_{pq} = 8^\circ - 12^\circ$ . Once you have the missing momentum at some  $Q^2$ , there is not much choice in what you have for  $\theta_{pq}$ . To build a bit more on this point consider Fig. 7. We extracted  $A'_{LT}$  with cuts on the value of  $\cos \theta_{pq}$ . For  $\cos \theta_{pq} > 0.3$  as recommended in the comment there was no change in  $A'_{LT}$ . We did not seen any significant impact until  $\cos \theta_{pq}$  was close to one, the effect of a  $\cos \theta_{pq} > 0.95$  cut is shown. The missing momentum and  $\cos \theta_{pq}$  (and  $\theta_{pq}$ ) are tightly correlated with one another see Figs 3-4). As you remove events with 'small'  $\cos \theta_{pq}$ (corresponding to large  $\theta_{pq}$  and  $p_m$ ) you are removing only large  $p_m$  events. The low- $p_m$   $A'_{LT}$ in Fig. 7 is unchanged while the large- $p_m A'_{LT}$  changes significantly and loses events making



Figure 7: Impact of cuts on  $\cos \theta_{pq}$  on  $A'_{LT}$ .

the statistical uncertainties much larger.

APPENDIX: Getting W as a function of  $p_m$ ,  $Q^2$ ,  $\cos \theta_{pq}$ , and  $\phi_{pq}$ .

At the risk of repeating ourselves we start with the equation for  $p_m$  (Equation 2.19 in Ref. 11)

$$
p_m = \sqrt{\left(\frac{q_L}{2} + p\frac{E_W}{W}\cos\theta_{pq}^{cm}\right)^2 + p^2\sin^2\theta_{pq}^{cm}} \tag{1}
$$

The components of  $p_m$  are  $p$  (proton/neutron 3-momentum in the center of mass)

$$
p = \sqrt{\frac{[W^2 - (m_p + m_n)^2][W^2 - (m_p - m_n)^2]}{4W^2}},\tag{2}
$$

where  $m_p$  and  $m_n$  are the proton and neutron masses respectively and

$$
E_W = M_d + \nu = M_d + \frac{W^2 + Q^2 - M_d^2}{2M_d} \tag{3}
$$

where  $M_d$  is the deuteron mass and

$$
q_L = \sqrt{E_W^2 - W^2} \tag{4}
$$

We want the angles in the lab so we start with

$$
\tan \theta_{pq}^{lab} = \frac{\sin \theta_{pq}^{cm}}{\gamma(\frac{v_{cm}}{v_p^{cm}} + \cos \theta_{pq}^{cm})}
$$
\n(5)

and invert this to obtain a quadratic equation in  $\sin \theta_{pq}^{cm}$  which yields

$$
\sin \theta_{pq}^{cm} = \frac{\frac{2v_{cm}}{\gamma v_p^{cm} \tan \theta_{pq}} \pm \sqrt{\left(\frac{2v_{cm}}{\gamma v_p^{cm} \tan \theta_{pq}}\right)^2 - 4\left(1 + \frac{1}{\gamma^2 \tan^2 \theta_{pq}}\right)\left(\left(\frac{v_{cm}}{v_p^{cm}}\right)^2 - 1\right)}}{2\left(1 + \frac{1}{\gamma^2 \tan^2 \theta_{pq}}\right)}
$$
(6)

where

$$
\gamma = \frac{E_W}{W} \qquad v_p^{cm} = \frac{p}{\sqrt{p^2 + m_p^2}} \qquad v_{cm} = \frac{q_L}{E_W} \tag{7}
$$



Figure 8: Plot of curves showing relationship between W,  $\cos \theta_{pq}$ , and  $p_m$  for  $Q^2 = 0.2 \text{ GeV}^2$ .