CLAS12, Track-Based, SVT Alignment with Millepede

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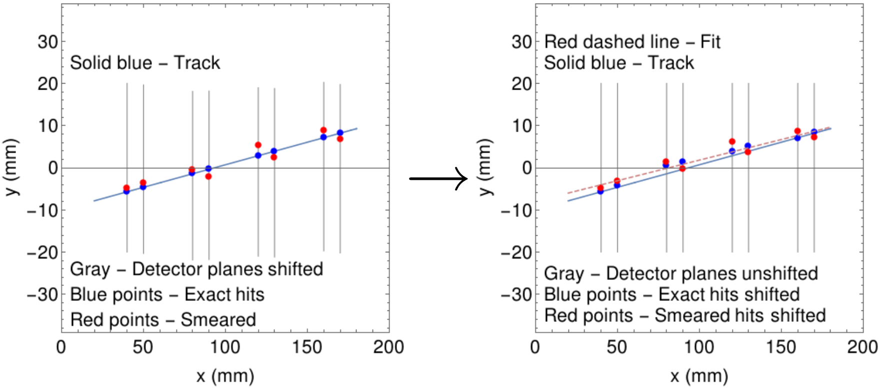
The Problem

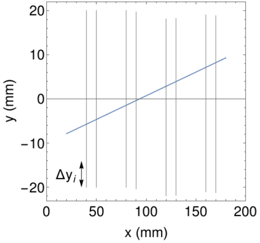


Toy model:

- Straight tracks.
- Planar detectors.
- Shift detectors only in y.

`millepede`: Linear Least Squares with Many Parameters

- In some least squares fit problems with many parameters those parameters can be divided into two classes.
  - Global - *i.e.* geometry.
  - Local - only present in subsets of the data, *i.e.* slope of a track.
- The code uses methods to solve the linear least square problem, irrespective of the number of local parameters.
- Up to ten thousand global parameters can be fitted.
- A simple test case:



Typically we fit the track with $y(x) = a + bx$.

In `millepede` use

$$y_{fit} = f(x, \vec{q}, \vec{p}) = \underbrace{\Delta y_1 + \Delta y_2 + ... + \Delta y_8}_{\text{global, } \vec{p}} + \underbrace{a + bx}_{\text{local, } \vec{q}} \quad .$$

Assume the initial fit with $\Delta y_i = 0$ is close to the final one so you can use the partial derivatives.

$$\frac{\partial z}{\partial \Delta y_i} = 1 \quad \frac{\partial z}{\partial a} = 1 \quad \frac{\partial z}{\partial b} = x$$

And use the residual $z = y_{meas} - f(x, \vec{q}, \vec{p})$.

CLAS12 `millepede`: First Results

- Test 1
  - Set $\Delta y_i = 0.0$ for all detector planes, simulate 26,000 straight tracks.
  - Fix $\Delta y_i = 0$ for planes 1-2 and 5-8 and let Δy_i vary for planes 3-4.
  - Simulation Input: `millepede` Output:

$$\Delta y_3 = 0.0\ mm \quad \longrightarrow \quad \Delta y_3 = -0.0041 \pm 0.0067\ mm$$

$$\Delta y_4 = 0.0\ mm \qquad\qquad \Delta y_4 = -0.0018 \pm 0.0066\ mm$$

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- Test 2
  - Set $\Delta y_i = 0.0$ for planes 1-2, 5-8, $\Delta y_i = -2.0\ mm$ for planes 3-4, and simulate 26,000 straight tracks.
  - Fix $\Delta y_i = 0$ for planes 1-2 and 5-8 and let Δy_i vary for planes 3-4.
  - Simulation Input: → `millepede` Output:

$$\Delta y_3 = -2.0\ mm \longrightarrow \Delta y_3 = -1.999 \pm 0.007\ mm$$

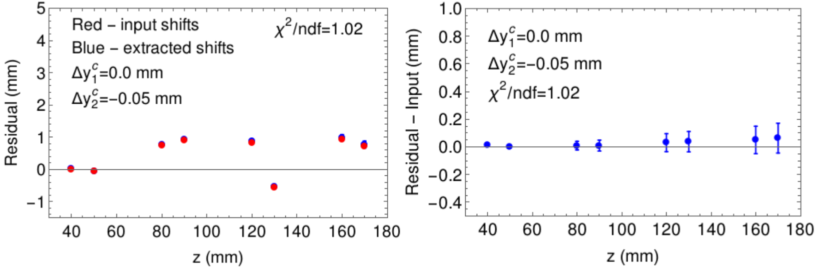
$$\Delta y_4 = -2.0\ mm \qquad \Delta y_4 = -2.012 \pm 0.007\ mm$$

CLAS12 `millepede`: Testing with 2D model - 1

- Randomly select misalignments Δy_i uniformly over the range $\Delta y_i = -1\ mm \rightarrow 1\ mm$ and get the following.

$$\Delta y_i = \{0.0, -0.05, 0.75294, 0.90532, 0.83337, -0.571876, 0.934063, 0.710517\}$$

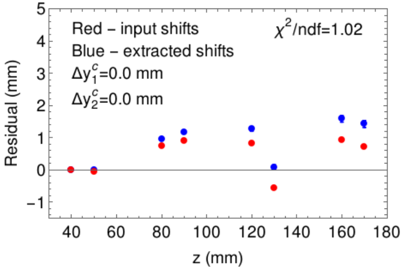
- Two constraints are needed so set the constraints on planes 1-2 to the input values $\Delta y_1 = 0\ mm$ and $\Delta y_2 = -0.05\ mm$.

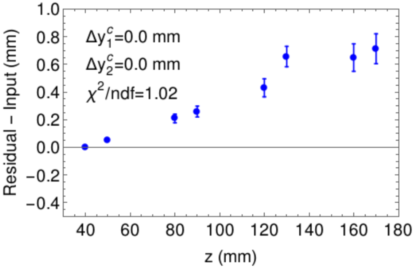


- Excellent agreement with the inputs.

CLAS12 `millepede`: Testing with 2D model - 2

- How sensitive is the `millepede` fit to the accuracy of the constraints?
- Use same set of misalignments as before.
- Two constraints are still needed. Set the constraint on plane 1 as before to $\Delta y_1 = 0\ mm$, but use the 'wrong' value for plane 2 $\Delta y_2 = 0\ mm$.

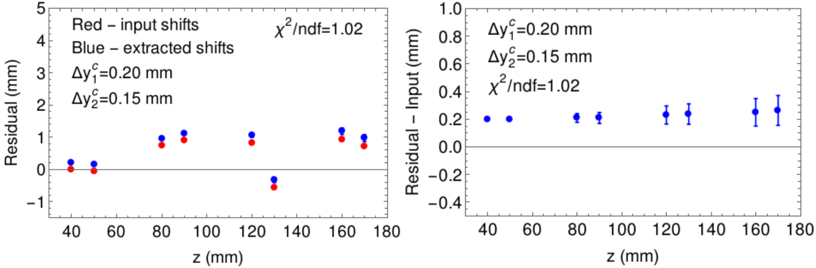




- Significant disagreement with the inputs especially as large z.

CLAS12 `millepede`: Testing with 2D model - 3

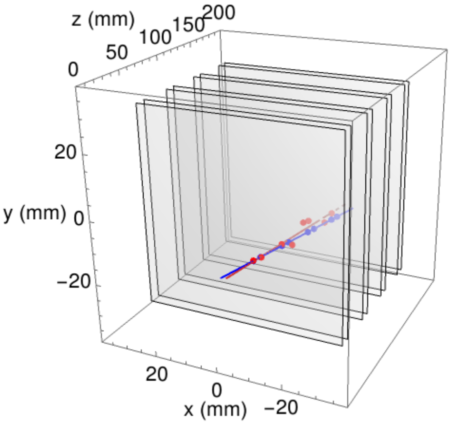
- How sensitive is the `millepede` fit to a shift in plane 1?
- Use same set of misalignments as before.
- Two constraints are still needed. Set the constraint on plane 1 to $\Delta y_1 = 0.20\ mm$, and for plane 2 $\Delta y_2 = \Delta y_1 - 0.05\ mm = 0.15\ mm$.



- Relative fit results are the same with overall shift added.

CLAS12 millepede: 3D Simulation

- Simulated a single track in an idealized planar detector.



- Working on approach to fitting the track, getting derivatives, ...

CLAS12 `millepede`: 3D Distance equation

$$
\begin{aligned}
d^2 = &\left(-\frac{(x_2-x_1)\left((x_1-x_0)(x_2-x_1)+(y_1-y_0)(y_2-y_1)+(z_1-z_0)(z_2-z_1)\right)}{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}-x_0+x_1\right)^2+ \\
&\left(-\frac{(y_2-y_1)\left((x_1-x_0)(x_2-x_1)+(y_1-y_0)(y_2-y_1)+(z_1-z_0)(z_2-z_1)\right)}{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}-y_0+y_1\right)^2+ \\
&\left(-\frac{(z_2-z_1)\left((x_1-x_0)(x_2-x_1)+(y_1-y_0)(y_2-y_1)+(z_1-z_0)(z_2-z_1)\right)}{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}-z_0+z_1\right)^2
\end{aligned} \tag{1}
$$

CLAS12 millepede: 3D Distance equation derivative

$$\frac{\partial d^2}{\partial x_1} =$$

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$$
\begin{aligned}
\frac{\partial d^2}{\partial x_1} = \; & 2 \Bigg(1 - \Big(2 \, (-x_1 + x_2)^2 \, \big((-x_0 + x_1)(-x_1 + x_2) + (-y_0 + y_1)(-y_1 + y_2) + (-z_0 + z_1)(-z_1 + z_2) \big) \Big) \Big/ \\
& \quad \Big((-x_1 + x_2)^2 + (-y_1 + y_2)^2 + (-z_1 + z_2)^2 \Big)^2 - \frac{(x_0 - 2 x_1 + x_2)(-x_1 + x_2)}{(-x_1 + x_2)^2 + (-y_1 + y_2)^2 + (-z_1 + z_2)^2} + \\
& \quad \frac{(-x_0 + x_1)(-x_1 + x_2) + (-y_0 + y_1)(-y_1 + y_2) + (-z_0 + z_1)(-z_1 + z_2)}{(-x_1 + x_2)^2 + (-y_1 + y_2)^2 + (-z_1 + z_2)^2} \Bigg) \\
& \Big(-x_0 + x_1 - \big((-x_1 + x_2) \, \big((-x_0 + x_1)(-x_1 + x_2) + (-y_0 + y_1)(-y_1 + y_2) + (-z_0 + z_1)(-z_1 + z_2) \big) \big) \Big/ \\
& \quad \big((-x_1 + x_2)^2 + (-y_1 + y_2)^2 + (-z_1 + z_2)^2 \big) \Big) + 2 \Bigg(- \Big(\big(2 \, (-x_1 + x_2)(-y_1 + y_2) \\
& \qquad \big((-x_0 + x_1)(-x_1 + x_2) + (-y_0 + y_1)(-y_1 + y_2) + (-z_0 + z_1)(-z_1 + z_2) \big) \big) \Big/ \\
& \qquad \big((-x_1 + x_2)^2 + (-y_1 + y_2)^2 + (-z_1 + z_2)^2 \big)^2 \Big) - \frac{(x_0 - 2 x_1 + x_2)(-y_1 + y_2)}{(-x_1 + x_2)^2 + (-y_1 + y_2)^2 + (-z_1 + z_2)^2} \Bigg) \\
& \Big(-y_0 + y_1 - \big((-y_1 + y_2) \, \big((-x_0 + x_1)(-x_1 + x_2) + (-y_0 + y_1)(-y_1 + y_2) + (-z_0 + z_1)(-z_1 + z_2) \big) \big) \Big/ \\
& \quad \big((-x_1 + x_2)^2 + (-y_1 + y_2)^2 + (-z_1 + z_2)^2 \big) \Big) + 2 \Bigg(- \Big(\big(2 \, (-x_1 + x_2)(-z_1 + z_2) \\
& \qquad \big((-x_0 + x_1)(-x_1 + x_2) + (-y_0 + y_1)(-y_1 + y_2) + (-z_0 + z_1)(-z_1 + z_2) \big) \big) \Big/ \\
& \qquad \big((-x_1 + x_2)^2 + (-y_1 + y_2)^2 + (-z_1 + z_2)^2 \big)^2 \Big) - \frac{(x_0 - 2 x_1 + x_2)(-z_1 + z_2)}{(-x_1 + x_2)^2 + (-y_1 + y_2)^2 + (-z_1 + z_2)^2} \Bigg) \\
& \Big(-z_0 + z_1 - \big((-z_1 + z_2) \, \big((-x_0 + x_1)(-x_1 + x_2) + (-y_0 + y_1)(-y_1 + y_2) + (-z_0 + z_1)(-z_1 + z_2) \big) \big) \Big/ \\
& \quad \big((-x_1 + x_2)^2 + (-y_1 + y_2)^2 + (-z_1 + z_2)^2 \big) \Big)
\end{aligned}
$$

CLAS12 `millepede`: Status

- Use the toy model described above as a tutorial.
- The code is running on the farm thanks to Mike Staib (CMU).
- Being used for HPS (Pelle Hansson and Alessandre Filippe) and GlueX (Mike Staib).
- A `millepede` event for the toy model.

Label	Measurement (mm)	Uncertainty (mm)	local derivatives		global derivatives
1	0.4378	1.4250	1.0	60.0	1.0
		...			
i	z_i	σ_i	1.0	x_i	1.0

- `myMille`
  - Running `millepede` requires two stages - (1) prepare a binary file with the data and (2) run the code that does the fitting called `pede`.
  - A C^{++} code to create the input binary called `myMille` has been written and tested with local tools.
- The code `pede` runs, reads the binary input file, and with the 'proper' constraints appears to work - thanks to Mike Staib, Alessandre Filippe, and Pele Hansson.

CLAS12 `millepede`: First Tests with `millepede`

- Generate a sample of straight tracks in our 2D, toy detector model.
- Limit the sample to events with hits in all eight detector planes.
- Generated over 26,000 events that satisfy this criteria.