Physics 309 Test 1

I pledge that I have given nor received unauthorized assistance during the completion of this work.
Signature: ____________________

Questions (10 pts. apiece) Answer questions in complete, well-written sentences WITHIN the spaces provided.

1. Cite at least two observations or experimental results that motivated the development of quantum mechanics.

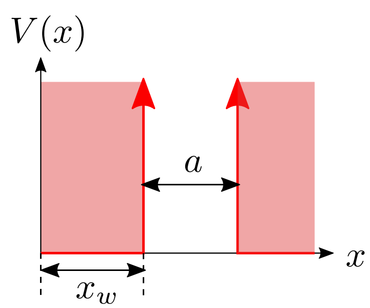
2. What is a quantum mechanical operator?

3. What is the full, time-DEPENDENT solution $|\psi(x,t)\rangle$ for a bound quantum system in terms of the eigenfunctions $|\phi_n\rangle$, eigenvalues E_n, and Fourier coefficients b_n. Any other parameters or constants should be related to the quantities above.

4. The eigenfunctions and eigenvalues of the particle in a box we studied in class are

$$|\phi\rangle = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a} \qquad E_n = n^2 \frac{\hbar^2 \pi^2}{2m_p a^2}$$

for $0 < x < a$ and particle mass m_p. The eigenfunctions are zero outside the box. Consider a new well shown in the figure. The edge of the new well is a distance x_w from the origin. What would be the new eigenfunctions? Explain.



Problems. Clearly show all work on a separate piece of paper for full credit except for problem 2. See note on that problem.

Note: If you encounter integrals that you are unable to solve label them I_1, I_2, ... and so on and carry that notation through to the solution.

1. (15 pts.) The eigenfunctions and eigenvalues of the particle in a box are the following

$$\begin{aligned} \phi(x) &= \sqrt{\tfrac{2}{a}} \sin\left[\tfrac{n\pi x}{a}\right] \quad 0 < x < a \\ &= 0 \qquad\qquad\qquad \text{otherwise} \end{aligned} \qquad E_n = \frac{n^2\hbar^2\pi^2}{2m_p a^2}$$

where m_p is the particle mass. These eigenfunctions are supposed to be orthogonal so $\langle\phi_m|\phi_n\rangle = \delta_{mn}$ where δ_{mn} is the Kronecker delta defined as

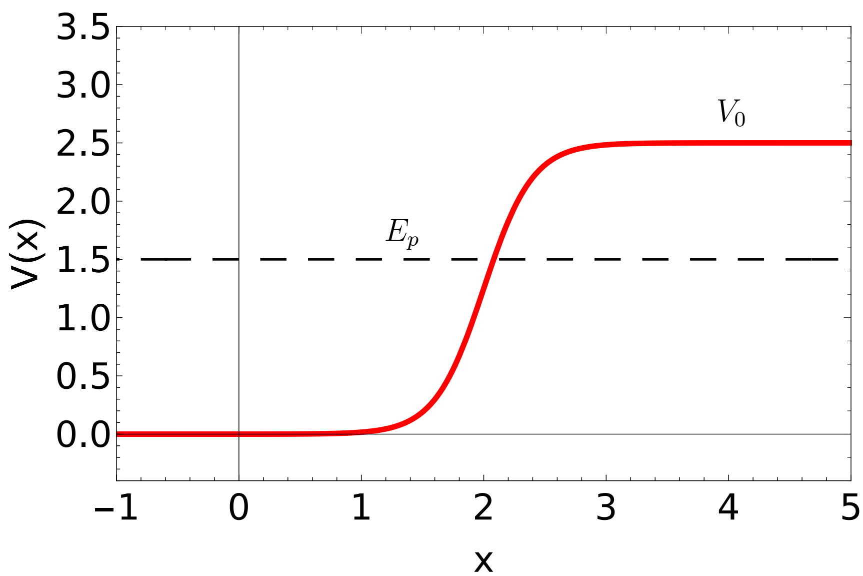
$$\begin{aligned} \delta_{mn} &= 1 \quad m = n \\ &= 0 \quad m \neq n \end{aligned}$$

Calculate the inner product $\langle\phi_m|\phi_n\rangle$ and show this is true for $m \neq n$ only. Clearly show all work.

2. (20 pts.) A particle constrained to move in one dimension x is in the potential field

$$V(x) = \frac{V_0}{1 + \exp\left[-\frac{x - x_0}{a}\right]}$$

shown in the figure. Discuss the possible motions, classically forbidden domains and turning points. In addition, consider a particle with $E_p < V_0$ where V_0 is the height of the plateau. At what value does it reflect? Note: You should annotate the figure below, but any other work should be on a separate piece of paper.



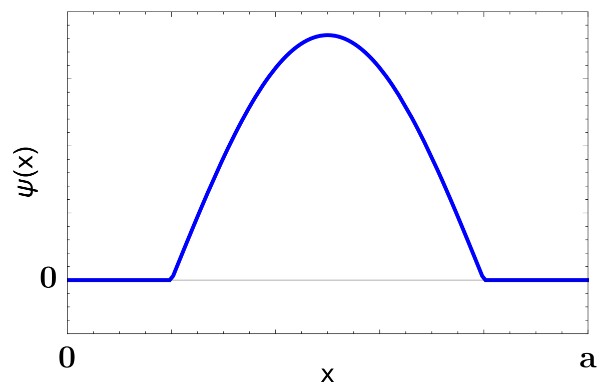
3. (25 pts.) A particle is in a square well with infinitely high walls at $x = 0$ and $x = a$. It has the following initial wave function as shown in the figure.

$$\begin{aligned} \psi(x) &= A_0 \sin\left[k_0(x - x_0)\right] \quad x_0 < x < x_1 \\ &= 0 \qquad\qquad\qquad\qquad \text{otherwise} \end{aligned}$$

The eigenfunctions and eigenvalues are the following.

$$\begin{aligned} \phi(x) &= \sqrt{\tfrac{2}{a}} \sin\left[\tfrac{n\pi x}{a}\right] \quad 0 < x < 1 \\ &= 0 \qquad\qquad\qquad \text{otherwise} \end{aligned} \qquad\qquad E_n = \tfrac{n^2\hbar^2\pi^2}{2m_p a^2}$$

What is the spectrum of wave numbers (the spectral distribution) necessary to produce the wave packet in terms of A_0, k_0, x_0, x_1, a, and the mass of the particle m_p? Clearly show all work.



Physics 309 Equations

$$R_T(\nu) = \frac{Energy}{time \times area} \quad E = h\nu = \hbar\omega \quad v_{wave} = \lambda\nu \quad I \propto |\vec{E}|^2 \quad K_{max} = h\nu - W \quad K = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}$$

$$\lambda = \frac{h}{p} \quad p = \hbar k \quad -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t) + V(x)\Psi(x,t) = i\hbar\frac{\partial}{\partial t}\Psi(x,t) \quad \hat{p}\,_x = -i\hbar\frac{\partial}{\partial x}$$

$$\hat{A}\,|\phi\rangle = a|\phi\rangle \quad \langle\hat{A}\,\rangle = \int_{-\infty}^{\infty} \psi^* \hat{A}\,\psi dx \quad |\phi\rangle = \frac{e^{\pm ikx}}{\sqrt{2\pi}} \quad E = \frac{\hbar^2 k^2}{2m} \quad |\phi\rangle = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right) \quad E_n = \frac{n^2\hbar^2\pi^2}{2ma^2}$$

$$\langle\phi_{n'}|\phi_n\rangle = \int_{-\infty}^{\infty} \phi_{n'}^*\phi_n\;dx = \delta_{n',n} \quad \langle\phi(k')|\phi(k)\rangle = \int_{-\infty}^{\infty} \phi_{k'}^*\phi_k\;dx = \delta(k-k')$$

$$|\psi\rangle = \sum b_n|\phi_n\rangle \to b_n = \langle\phi_n|\psi\rangle \quad |\psi\rangle = \int b(k)|\phi(k)\rangle dk \to b(k) = \langle\phi(k)|\psi\rangle \quad |\psi(t)\rangle = \sum b_n|\phi_n\rangle e^{-i\omega_n t}$$

$$\Delta p \Delta x \propto \hbar \quad (\Delta x)^2 = \langle x^2\rangle - \langle x\rangle^2 \quad \langle f(x)\rangle = \int_{-\infty}^{\infty} f(x)P(x)dx \quad \langle f_n\rangle = \sum_{n=0}^{\infty} f_n P_n$$

$$\text{If } f(x) = \sqrt{\frac{1}{2\pi\sigma^2}}\;e^{-x^2/2\sigma^2}, \text{ then } \Delta x = \sigma$$

$$e^{ix} = \cos x + i\sin x \quad \sin x = \frac{e^{ix} - e^{-ix}}{2i} \quad \cos x = \frac{e^{ix} + e^{-ix}}{2} \quad \lim_{x\to c}\frac{f(x)}{g(x)} = \lim_{x\to c}\frac{f'(x)}{g'(x)}$$

The wave function, $\psi(\vec{r},t)$, contains all we know of a system and $|\psi|^2$ is the probability of finding it in the region $\vec{r}$ to $\vec{r} + d\vec{r}$. The wave function and its derivative are (1) *finite*, (2) *continuous*, and (3) *single-valued*.

$$\ln(ab) = \ln a + \ln b \quad \ln\left(\frac{a}{b}\right) = \ln a - \ln b \quad \ln x^n = n\ln x \quad \ln(e^a) = a \quad e^{\ln a} = a$$

$$\frac{df}{du} = \frac{df}{dx}\frac{du}{dx} \quad \frac{d}{dx}(x^n) = nx^{n-1} \quad \frac{d}{dx}(\sin x) = \cos x \quad \frac{d}{dx}(\cos x) = -\sin x \quad \frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x} \quad \int x^n dx = \frac{x^{n+1}}{n+1} \quad \int e^{ax}dx = \frac{e^{ax}}{a} \quad \int \frac{1}{x} = \ln x$$

$$\int \frac{1}{\sqrt{x^2+a^2}}dx = \ln\left[x + \sqrt{x^2+a^2}\right] \quad \int \frac{x}{\sqrt{x^2+a^2}}dx = \sqrt{x^2+a^2}$$

$$\int \frac{x^2}{\sqrt{x^2+a^2}}dx = \frac{1}{2}x\sqrt{x^2+a^2} - \frac{1}{2}a^2\ln\left[x + \sqrt{x^2+a^2}\right] \quad \int \frac{x^3}{\sqrt{x^2+a^2}}dx = \frac{1}{3}(-2a^2+x^2)\sqrt{x^2+a^2}$$

$$\int \cos(Ax)dx = \frac{\sin(Ax)}{A} \quad \int \cos(Ax)x dx = \frac{\cos(Ax)}{A^2} + \frac{x\sin(Ax)}{A}$$

More Physics 309 Equations

$$\int \sin(Ax)dx = -\frac{\cos(Ax)}{A} \qquad \int \sin(Ax)x dx = \frac{\sin(Ax)}{A^2} - \frac{x\cos(Ax)}{A}$$

$$\int \sin(Ax)x^2 dx = \frac{2x\sin(Ax)}{A^2} - \frac{(A^2x^2-2)\cos(Ax)}{A^3} \qquad \int \cos(Ax)x^2 dx = \frac{2x\cos(Ax)}{A^2} + \frac{(A^2x^2-2)\sin(Ax)}{A^3}$$

$$\int_a^b \sin(Ax)dx = \frac{\cos(aA)-\cos(Ab)}{A} \qquad \int_a^b \sin(Ax)x dx = \frac{-\sin(aA)+aA\cos(aA)+\sin(Ab)-Ab\cos(Ab)}{A^2}$$

$$\int_a^b \cos(Ax)dx = \frac{\sin(Ab)-\sin(aA)}{A} \qquad \int_a^b \cos(Ax)x dx = \frac{-aA\sin(aA)-\cos(aA)+Ab\sin(Ab)+\cos(Ab)}{A^2}$$

$$\int \sin(Ax)\sin(Bx)dx = \frac{\sin(x(A-B))}{2(A-B)} - \frac{\sin(x(A+B))}{2(A+B)}$$

$$\int \cos(Ax)\cos(Bx)dx = \frac{\sin(x(A-B))}{2(A-B)} + \frac{\sin(x(A+B))}{2(A+B)}$$

$$\int \sin(Ax)\cos(Bx)dx = -\frac{\cos(x(A-B))}{2(A-B)} - \frac{\cos(x(A+B))}{2(A+B)}$$

Physics 309 Conversions, and Constants

Speed of light (c)	$2.9979 \times 10^8\ m/s$	fermi (fm)	$10^{-15}\ m$
Boltzmann constant (k_B)	$1.381 \times 10^{-23}\ J/K$	angstrom (Å)	$10^{-10}\ m$
	$8.62 \times 10^{-5}\ eV/K$	electron-volt (eV)	$1.6 \times 10^{-19}\ J$
Planck constant (h)	$6.621 \times 10^{-34}\ J-s$	MeV	$10^6\ eV$
	$4.1357 \times 10^{-15}\ eV-s$	GeV	$10^9\ eV$
Planck constant ($\hbar$)	$1.0546 \times 10^{-34}\ J-s$	Electron charge (e)	$1.6 \times 10^{-19}\ C$
	$6.5821 \times 10^{-16}\ eV-s$	e^2	$\hbar c/137$
Planck constant ($\hbar c$)	$197\ MeV-fm$	Electron mass (m_e)	$9.11 \times 10^{-31}\ kg$
	$1970\ eV-$Å		$0.511\ MeV/c^2$
Proton mass (m_p)	$1.67 \times 10^{-27} kg$	atomic mass unit (u)	$1.66 \times 10^{-27}\ kg$
	$938\ MeV/c^2$		$931.5\ MeV/c^2$
Neutron mass (m_n)	$1.68 \times 10^{-27}\ kg$		
	$939\ MeV/c^2$		