Physics 309 Test 1

I pledge that I have given nor received unauthorized assistance during the completion of this work.
Signature: ____

Questions (6 pts. apiece) Answer questions in complete, well-written sentences WITHIN the spaces provided. For multiple-choice questions circle the correct answer.

1. Cite at least two experimental results that motivated the development of quantum mechanics.

2. The eigenfunctions and eigenvalues of the particle in a box are

$$|\phi\rangle = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a} \qquad E_n = n^2 \frac{\hbar^2 \pi^2}{2ma^2}$$

   for $0 < x < a$. The eigenfunctions are zero outside the box. Why?

3. What does the orthonormality of the eigenfunctions mean?

4. In the photoelectric effect what is the work function of a metal?

DO NOT WRITE BELOW THIS LINE.

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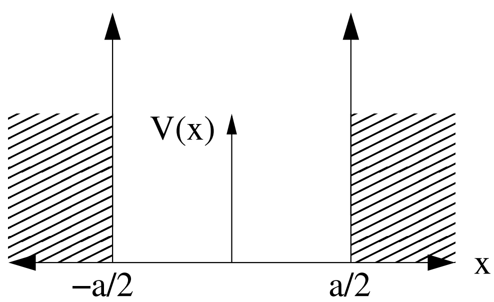
5. What is the value of $f(x) = \sin x/x$ at $x = 0$? Clearly show all your steps.

6. What are the energies for the infinite square well shown in the figure? Explain your reasoning. The eigenfunctions and eigenvalues of the particle-in-a-box we studied in class are listed in Question 2.



Problems. Clearly show all work on a separate piece of paper for full credit.

Note: If you encounter integrals that you are unable to solve label them I_1, I_2, ... and so on and carry that notation through to the solution.

1. (14 pts.) In a double-slit experiment a detector traces across a screen along a straight line with coordinate x. If one slit is closed, the amplitude

$$\psi_1 = e^{-x^2} e^{i\omega t}$$

is measured. If the other slit is closed, the amplitude

$$\psi_2 = e^{-x^2} e^{i(\omega t - ax)}$$

is measured. What is the intensity pattern along the x axis when both slits are open?

2. (20 pts.) A criterion which discerns if a given configuration is classical or quantum mechanical may be stated in terms of the de Broglie wavelength λ. Namely, if L is the scale length characteristic of the configuration at hand, then one has the following criteria.

$$\begin{array}{lll} \lambda \ll L & : & \text{Classical} \\ \lambda \gtrsim L & : & \text{QuantumMechanical} \end{array}$$

Consider a rubidium atom of mass $m = 102\ u$ (where u is an atomic mass unit) in a magnetic trap with an active region about 1 μm (a μm is $10^{-6} m$) across. The active region has been cooled to a temperature $T = 1.7 \times 10^{-7}\ K$. The average energy of the rubidium atom can be determined from the temperature using the expression $\langle E \rangle = \langle p^2 \rangle / 2m = \frac{3}{2} k_B T$ where k_B is Boltzmann's constant. Is this system quantum mechanical or classical? Explain your reasoning.

3. (30 pts.) The initial wave packet for a particle in an infinite square well of width a like the one we studied in class is

$$\begin{aligned} \psi(x) &= -\alpha x \quad x_0 < x < x_1 \\ &= 0 \qquad \text{otherwise} \end{aligned}$$

where $\alpha = \sqrt{3/(x_1^3 - x_0^3)}$. What is the spectrum of wave numbers (the spectral distribution) necessary to produce this wave packet in terms of a, m, x_0, x_1? The eigenfunctions and eigenvalues of the particle in a box are

$$|\phi_n\rangle = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a} \qquad E_n = n^2 \frac{\hbar^2 \pi^2}{2ma^2}$$

for $0 < x < a$. The eigenfunctions are zero outside the box.

Physics 309 Equations

$$R_T(\nu) = \frac{Energy}{time \times area} \quad E = h\nu = \hbar\omega \quad v_{wave} = \lambda\nu \quad I \propto |\vec{E}|^2 \quad K_{max} = h\nu - W \quad K = \frac{p^2}{2m}$$

$$\lambda = \frac{h}{p} \quad p = \hbar k \quad -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t) + V(x)\Psi(x,t) = i\hbar\frac{\partial}{\partial t}\Psi(x,t) \quad \hat{p}_{\ x} = -i\hbar\frac{\partial}{\partial x}$$

$$\hat{A}\ |\phi\rangle = a|\phi\rangle \quad \langle \hat{A}\ \rangle = \int_{-\infty}^{\infty} \psi^* \hat{A}\ \psi dx$$

$$\langle \phi_{n'}|\phi_n\rangle = \int_{-\infty}^{\infty} \phi_n^* \phi\ dx = \delta_{n',n} \quad \langle \phi(k')|\phi(k)\rangle = \int_{-\infty}^{\infty} \phi_{k'}^* \phi_k\ dx = \delta(k-k')$$

$$|\psi\rangle = \sum b_n|\phi_n\rangle \to b_n = \langle\phi_n|\psi\rangle \quad |\psi\rangle = \int b(k)|\phi(k)\rangle dk \to b(k) = \langle\phi(k)|\psi\rangle \quad |\psi(t)\rangle = \sum b_n|\phi_n\rangle e^{-i\omega_n t}$$

$$\Delta p \Delta x \propto \hbar \quad (\Delta x)^2 = \langle x^2\rangle - \langle x\rangle^2 \quad \langle f(x)\rangle = \int_{-\infty}^{\infty} f(x)P(x)dx \quad \langle f_n\rangle = \sum_{n=0}^{\infty} f_n P_n$$

$$\text{If } f(x) = \sqrt{\frac{1}{2\pi\sigma^2}}\ e^{-x^2/2\sigma^2},\ \text{then } \Delta x = \sigma \quad \langle\phi|\hat{B}\phi\rangle = \langle\hat{B}\phi|\phi\rangle \text{ (Hermiticity)}$$

$$e^{ix} = \cos x + i\sin x \quad \sin x = \frac{e^{ix} - e^{-ix}}{2i} \quad \cos x = \frac{e^{ix} + e^{-ix}}{2} \quad \lim_{x\to c}\frac{f(x)}{g(x)} = \lim_{x\to c}\frac{f'(x)}{g'(x)}$$

The wave function, $\psi(\vec{r},t)$, contains all we know of a system and $|\psi|^2$ is the probability of finding it in the region $\vec{r}$ to $\vec{r} + d\vec{r}$. The wave function and its derivative are (1) *finite*, (2) *continuous*, and (3) *single-valued.*

$$\frac{df}{du} = \frac{df}{dx}\frac{du}{dx} \quad \frac{d}{dx}(x^n) = nx^{n-1} \quad \frac{d}{dx}(\sin x) = \cos x \quad \frac{d}{dx}(\cos x) = -\sin x \quad \frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x} \quad \int x^n dx = \frac{x^{n+1}}{n+1} \quad \int e^{ax}dx = \frac{e^{ax}}{a} \quad \int \frac{1}{x} = \ln x$$

$$\int \frac{1}{\sqrt{x^2+a^2}}dx = \ln\left[x + \sqrt{x^2+a^2}\right] \quad \int \frac{x}{\sqrt{x^2+a^2}}dx = \sqrt{x^2+a^2}$$

$$\int \frac{x^2}{\sqrt{x^2+a^2}}dx = \frac{1}{2}x\sqrt{x^2+a^2} - \frac{1}{2}a^2\ln\left[x + \sqrt{x^2+a^2}\right] \quad \int \frac{x^3}{\sqrt{x^2+a^2}}dx = \frac{1}{3}(-2a^2+x^2)\sqrt{x^2+a^2}$$

$$\int \cos(Ax)dx = \frac{\sin(Ax)}{A} \quad \int \cos(Ax)x dx = \frac{\cos(Ax)}{A^2} + \frac{x\sin(Ax)}{A}$$

More Physics 309 Equations

$$\int \sin(Ax)dx = -\frac{\cos(Ax)}{A} \qquad \int \sin(Ax)x dx = \frac{\sin(Ax)}{A^2} - \frac{x\cos(Ax)}{A}$$

$$\int \sin(Ax)x^2dx = \frac{2x\sin(Ax)}{A^2} - \frac{(A^2x^2-2)\cos(Ax)}{A^3} \qquad \int \cos(Ax)x^2dx = \frac{2x\cos(Ax)}{A^2} + \frac{(A^2x^2-2)\sin(Ax)}{A^3}$$

$$\int_a^b \sin(Ax)dx = \frac{\cos(aA)-\cos(Ab)}{A} \qquad \int_a^b \sin(Ax)x dx = \frac{-\sin(aA)+aA\cos(aA)+\sin(Ab)-Ab\cos(Ab)}{A^2}$$

$$\int_a^b \cos(Ax)dx = \frac{\sin(Ab)-\sin(aA)}{A} \qquad \int_a^b \cos(Ax)x dx = \frac{-aA\sin(aA)-\cos(aA)+Ab\sin(Ab)+\cos(Ab)}{A^2}$$

$$\int \sin(Ax)\sin(Bx)dx = \frac{\sin(x(A-B))}{2(A-B)} - \frac{\sin(x(A+B))}{2(A+B)}$$

$$\int \cos(Ax)\cos(Bx)dx = \frac{\sin(x(A-B))}{2(A-B)} + \frac{\sin(x(A+B))}{2(A+B)}$$

$$\int \sin(Ax)\cos(Bx)dx = -\frac{\cos(x(A-B))}{2(A-B)} - \frac{\cos(x(A+B))}{2(A+B)}$$

Physics 309 Conversions, and Constants

Speed of light (c)	$2.9979 \times 10^8\ m/s$	fermi (fm)	$10^{-15}\ m$
Boltzmann constant (k_B)	$1.381 \times 10^{-23}\ J/K$	angstrom (Å)	$10^{-10}\ m$
	$8.62 \times 10^{-5}\ eV/k$	electron-volt (eV)	$1.6 \times 10^{-19}\ J$
Planck constant (h)	$6.621 \times 10^{-34}\ J-s$	MeV	$10^6\ eV$
	$4.1357 \times 10^{-15}\ eV-s$	GeV	$10^9\ eV$
Planck constant ($\hbar$)	$1.0546 \times 10^{-34}\ J-s$	Electron charge (e)	$1.6 \times 10^{-19}\ C$
	$6.5821 \times 10^{-16}\ eV-s$	e^2	$\hbar c/137$
Planck constant ($\hbar c$)	$197\ MeV-fm$	Electron mass (m_e)	$9.11 \times 10^{-31}\ kg$
	$1970\ eV-$Å		$0.511\ MeV/c^2$
Proton mass (m_p)	$1.67 \times 10^{-27} kg$	atomic mass unit (u)	$1.66 \times 10^{-27}\ kg$
	$938\ MeV/c^2$		$931.5\ MeV/c^2$
Neutron mass (m_n)	$1.68 \times 10^{-27}\ kg$		
	$939\ MeV/c^2$		