Physics 309 Final

I pledge that I have given nor received unauthorized assistance during the completion of this work.

Name ______________________ Signature ______________________

Questions (3 pts. apiece) Answer questions in complete, well-written sentences WITHIN the spaces provided.

1. The CO molecule can be represented by quantum numbers n, l, and m. Describe in words the meaning of each quantum number.

2. In our study of the CO molecule we used the quantum number ℓ to terminate the series for the rotational (θ) part of the CO eigenfunctions. What physical quantity is ℓ related to and how did we connect it to that quantity?

3. What is a solid angle?

4. What is the quantum program?

Do not write below this line.

5. Cite at least two experimental measurements that required quantum mechanics to explain.

6. The eigenfunctions and eigenvalues of the particle in a box are

$$|\phi\rangle = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a} \qquad E_n = n^2 \frac{\hbar^2 \pi^2}{2ma^2}$$

for $0 < x < a$. The eigenfunctions are zero outside the box. Consider the following sequence of measurements of a particle in a box.

(a) The energy of the particle is measured. A value E_1 is obtained.

(b) The position of the particle is measured and a value x_2 is obtained.

(c) The energy of the particle is measured again.

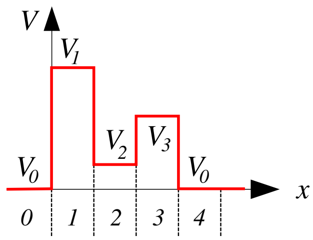
What possible values of the energy can you obtain in step 6.c? Explain.

7. Why does the Sun shine? Your answer should be descriptive and qualitative - not quantitative.

8. Why do we express the wave function in terms of energy eigenstates?

Do not write below this line.

9. Consider the potential barrier shown below. How would you use the transfer-matrix approach to connect the wave function $\underset{\sim}{\psi_0}$ in region 0 to the wave function $\underset{\sim}{\psi_4}$ in region 4? Give your answer in the appropriate notation used in class for problems like this one. What is the form of the wave number k_i in each region?

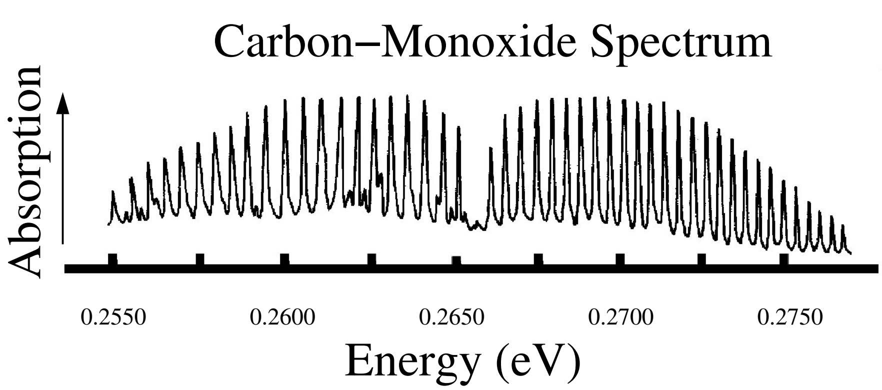


10. A proton and electron are each trapped in their own infinite square well which covers the same range $0 < x < a$. Both particles are in the ground state. At the center of the well is the probability density of the proton greater than, less than, or equal to the probability density of the electron? Explain.

Problems. Clearly show all work for full credit on a separate piece of paper.

1. (10 pts.) The work function of zinc is $\Phi = 3.6\ eV$. What is the energy of the most energetic photoelectron emitted by ultraviolet light of wavelength $\lambda = 2500$ Å?

2. (10 pts.) Recall the vibration-rotation spectrum of carbon monoxide shown in the figure. The peaks are separated by constant energy except at the center of the spectrum where the separation is larger (the 'hole'). The energy levels of the carbon monoxide are the sum of the harmonic oscillator energies E_n and the rotational ones E_ℓ. Starting from the expression for the energy levels in CO calculate an expression for the size of the hole.



3. (10 pts.) Legendre's differential equation determines Θ the solution of the polar angle part θ of the CO rotator Schoedinger equation

$$(1-z^2)\frac{d^2\Theta}{dz^2} - 2z\frac{d\Theta}{dz} + \left(A - \frac{m^2}{1-z^2}\right)\Theta = 0$$

where m is an integer, A is the separation constant, and $z = \cos\theta$. For the case $m = 0$ what is the recursion relationship for the series solution to Legendre's differential equation? In other words, let $\Theta = \sum a_k z^k$, set $m = 0$, and show that Legendre's differential equation leads to a relationship between the coefficients in the sum. What must the constant A equal if we want to terminate the series at some arbitrary value of $k = l$?

4. (10 pts.) A hypernucleus is an atomic nucleus which contains hyperons, particles that contain a strange quark replacing one of the u or d quarks in a nucleon. Suppose a hyperon is confined in a nucleus of diameter a and has the following initial wave function.

$$\psi(x,0) = \sqrt{\frac{2}{a}}\sin\frac{4\pi x}{a} \qquad 0 \le x \le a$$
$$= 0 \qquad \text{otherwise}$$

Treat the system as a one dimensional infinite rectangular well. The eigenfunctions and eigenvalues are

$$E_n = \frac{n^2\hbar^2\pi^2}{2ma^2} \qquad \phi_n = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right) \qquad 0 \le x \le a$$
$$= 0 \qquad x < 0 \text{ and } x > a \quad .$$

The mass of the hyperon in energy units is $m_h c^2 = 1405\ MeV$.

1. What are the coefficients of the Fourier series describing the initial wave function?

2. If $a = 1.0\ fm$, what is the probability of the hyperon being in the ground state ($n = 1$)? What is the probability of the hyperon being in the third excited state?

5. (15 pts.) For a particle-in-a-box (see Prob. 4 for eigenfunctions and eigenvalues) with initial state

$$\begin{aligned} \psi(x,0) &= A_1 \sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{3\pi x}{a}\right) & 0 \le x \le a \\ &= 0 & x < 0 \text{ and } x > a \end{aligned}$$

what are A_1, $\psi(x,t)$ and $P(E_n)$ at $t > 0$ in terms of m, a, and any other constants?

6. (15 pts.) Cold emission is a process where electrons are drawn from a metal at room temperature by an external electric field. The potential of the electrons in the metal without the external field is shown in the left-hand panel below. The electrons fill all available energy states (the Fermi sea) up to a maximum value E_F. The potential with with the field $\mathcal{E}$ on is shown in the right-hand panel.

$$V(x) = \Phi + E_F - e\mathcal{E}x$$

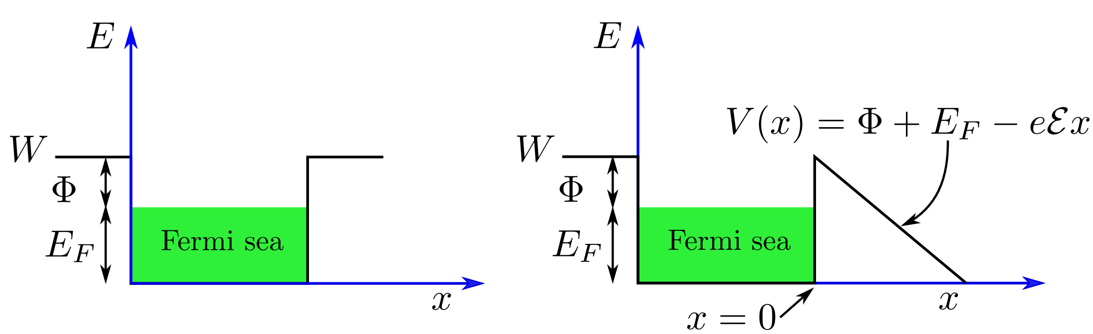
where E_F is the Fermi energy, Φ is the work function, $\mathcal{E}$ is the applied electric field, e is the electronic charge, and x is the position. See the figure for more information. Electrons can 'tunnel' through this barrier.

1. Use the WKB approximation to calculate the transmission coefficient

$$T = \exp\left[-2\int_{x_1}^{x_2}\sqrt{\frac{2m(V(x)-E)}{\hbar^2}}dx\right]$$

where x_1 and x_2 are values of the position x where the energy E_F equals $V(x)$ (see the figure). Get your answer in terms of the electron mass m, Φ, e, $\mathcal{E}$, and any other necessary constants.

2. The electric current inside the metal is described by $J_{inc} = env$ where n is the electron density and v is the electron speed in the Fermi sea. Consider a current coming out of the metal. The most likely electrons to tunnel through the barrier are the ones at the Fermi energy E_F (the top of the Fermi sea). Calculate an expression for the electric field $\mathcal{E}$ needed to reach a current J_0 through the barrier from the Fermi sea in terms of m, e, n, E_F, Φ, and J_0.



Physics 309 Equations

$$R_T(\nu) = \frac{Energy}{time \times area} \quad E = h\nu = \hbar\omega \quad v_{wave} = \lambda\nu \quad I \propto |\vec{E}|^2 \quad \lambda = \frac{h}{p} \quad p = \hbar k \quad K = \frac{p^2}{2m} \quad K_{max} = h\nu - \Phi$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t) + V(x)\Psi(x,t) = i\hbar\frac{\partial}{\partial t}\Psi(x,t) \quad \hat{p}_{\ x} = -i\hbar\frac{\partial}{\partial x} \quad \hat{A}\ |\phi\rangle = a|\phi\rangle \quad \langle \hat{A}\ \rangle = \int_{-\infty}^{\infty} \psi^* \hat{A}\ \psi dx$$

$$\langle \phi_{n'}|\phi_n\rangle = \int_{-\infty}^{\infty} \phi^*_{n'}\phi_n dx = \delta_{n',n} \quad \langle \phi(k')|\phi(k)\rangle = \int_{-\infty}^{\infty} \phi^*_{k'}\phi_k\ dx = \delta(k-k') \quad e^{i\phi} = \cos\phi + i\sin\phi$$

$$|\psi\rangle = \sum b_n|\phi_n\rangle \to b_n = \langle\phi_n|\psi\rangle \qquad |\phi\rangle = e^{\pm ikx} \qquad |\psi\rangle = \int b(k)|\phi(k)\rangle dk \to b(k) = \langle\phi(k)|\psi\rangle$$

$$|\psi(x,t)\rangle = \sum b_n|\phi_n\rangle e^{-i\omega_n t} \quad |\psi(x,t)\rangle = \int b(k)|\phi(k)\rangle e^{-i\omega(k)t}dk \quad \Delta p\Delta x \geq \frac{\hbar}{2} \qquad (\Delta x)^2 = \langle x^2\rangle - \langle x\rangle^2$$

The wave function, $\Psi(\vec{r},t)$, contains all we know of a system and its square is the probability of finding the system in the region $\vec{r}$ to $\vec{r}+d\vec{r}$. The wave function and its derivative are (1) *finite*, (2) *continuous*, and (3) *single-valued* ($\psi_1(a) = \psi_2(a)$ and $\psi_1'(a) = \psi_2'(a)$) .

$$V_{HO} = \frac{\kappa x^2}{2} \quad \omega = 2\pi\nu = \sqrt{\frac{\kappa}{m}} \quad E_n = (n+\frac{1}{2})\hbar\omega_0 = \hbar\omega \quad |\phi_n\rangle = e^{-\xi^2/2}H_n(\xi) \quad \xi = \beta x \quad \beta^2 = \frac{m\omega_0}{\hbar}$$

$$\psi_1 = \mathbf{t}\psi_3 = \mathbf{d_{12}p_2d_{21}p_1^{-1}}\psi_3 \qquad T = \frac{1}{|t_{11}|^2} \qquad R + T = 1$$

$$\mathbf{d_{ij}} = \frac{1}{2}\begin{pmatrix} 1+\frac{k_j}{k_i} & 1-\frac{k_j}{k_i} \\ 1-\frac{k_j}{k_i} & 1+\frac{k_j}{k_i} \end{pmatrix} \quad \mathbf{p_i} = \begin{pmatrix} e^{-ik_i 2a} & 0 \\ 0 & e^{ik_i 2a} \end{pmatrix} \quad \mathbf{p_i^{-1}} = \begin{pmatrix} e^{ik_i 2a} & 0 \\ 0 & e^{-ik_i 2a} \end{pmatrix}$$

$$E = \frac{\hbar^2 k^2}{2m} \quad k = \sqrt{\frac{2m(E-V)}{\hbar^2}} \quad T = \frac{\text{transmitted flux}}{\text{incident flux}} \quad R = \frac{\text{reflected flux}}{\text{incident flux}} \quad \text{flux} = |\psi|^2 v$$

$$V(r) = \frac{Z_1 Z_2 e^2}{r} \quad E = \frac{1}{2}\mu v^2 + V(r) \quad \vec{R}_{cm} = \frac{\sum_i m_i\vec{r}_i}{\sum_i m_i} \quad \mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$\psi(x) = \sum_{n=1}^{\infty} a_n x^n \quad \langle K\rangle = \frac{3}{2}kT \qquad n(v) = 4\pi N\left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 e^{-mv^2/2k_BT} \quad \vec{L} = \vec{r}\times\vec{p} = \mathcal{I}\vec{\omega}$$

$$\mathcal{I} = \sum_i m_i r_1^2 = \int r^2 dm \quad KE_{rot} = \frac{L^2}{2\mathcal{I}} \quad E_\ell = \frac{\ell(\ell+1)\hbar^2}{2\mathcal{I}} \quad V_{coul} = \frac{Z_1 Z_2 e^2}{r} \quad ME = \frac{p_r^2}{2\mu} + \frac{L^2}{2\mu r^2} + V(r)$$

$$L_z|nlm\rangle = m\hbar|nlm\rangle \quad L^2|nlm\rangle = \ell(\ell+1)\hbar^2|nlm\rangle$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right] \quad \cos A\cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A-B) + \sin(A+B)\right] \quad \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

Constants

Speed of light (c)	$2.9979 \times 10^8\ m/s$	fermi (fm)	$10^{-15}\ m$
Boltzmann constant (k_B)	$1.381 \times 10^{-23}\ J/K$	angstrom (Å)	$10^{-10}\ m$
	$8.62 \times 10^{-5}\ eV/k$	electron-volt (eV)	$1.6 \times 10^{-19}\ J$
Planck constant (h)	$6.621 \times 10^{-34}\ J-s$	MeV	$10^6\ eV$
	$4.1357 \times 10^{-15}\ eV-s$	GeV	$10^9\ eV$
Planck constant ($\hbar$)	$1.0546 \times 10^{-34}\ J-s$	Electron charge (e)	$1.6 \times 10^{-19}\ C$
	$6.5821 \times 10^{-16}\ eV-s$	e^2	$\hbar c/137$
Planck constant ($\hbar c$)	$197\ MeV-fm$	Electron mass (m_e)	$9.11 \times 10^{-31}\ kg$
	$1970\ eV-$Å		$0.511\ MeV/c^2$
Proton mass (m_p)	$1.67 \times 10^{-27} kg$	atomic mass unit (u)	$1.66 \times 10^{-27}\ kg$
	$938\ MeV/c^2$		$931.5\ MeV/c^2$
Neutron mass (m_n)	$1.68 \times 10^{-27}\ kg$		
	$939\ MeV/c^2$		

Integrals and Derivatives

$$\frac{df}{du} = \frac{df}{dx}\frac{du}{dx} \quad \frac{d}{dx}(x^n) = nx^{n-1} \quad \frac{d}{dx}(\sin x) = \cos x \quad \frac{d}{dx}(\cos x) = -\sin x \quad \frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\ln ax) = \frac{1}{x} \quad \int x^n dx = \frac{x^{n+1}}{n+1} \quad \int e^{ax} dx = \frac{e^{ax}}{a} \quad \int \frac{1}{x} = \ln x \quad \int \frac{1}{\sqrt{x^2+a^2}} dx = \ln\left[x + \sqrt{x^2+a^2}\right]$$

$$\int \frac{x}{\sqrt{x^2+a^2}} dx = \sqrt{x^2+a^2} \quad \int \frac{x^2}{\sqrt{x^2+a^2}} dx = \frac{1}{2} x\sqrt{x^2+a^2} - \frac{1}{2} a^2 \ln\left[x + \sqrt{x^2+a^2}\right]$$

$$\int \frac{x^3}{\sqrt{x^2+a^2}} dx = \frac{1}{3}(-2a^2+x^2)\sqrt{x^2+a^2} \quad \int x^2 \sin(ax) dx = \frac{2x\sin(ax)}{a^2} - \frac{\left(a^2x^2-2\right)\cos(ax)}{a^3}$$

$$\int x\sin(ax) dx = \frac{\sin(ax)}{a^2} - \frac{x\cos(ax)}{a} \quad \int x^3 \sin ax dx = \frac{3\left(a^2x^2-2\right)\sin(ax)}{a^4} - \frac{x\left(a^2x^2-6\right)\cos(ax)}{a^3}$$

Hermite polynomials ($H_n(\xi)$)

$$H_0(\xi) = \frac{1}{\sqrt{\sqrt{\pi}}}$$

$$H_1(\xi) = \frac{1}{\sqrt{2\sqrt{\pi}}} 2\xi$$

$$H_2(\xi) = \frac{1}{\sqrt{8\sqrt{\pi}}}(4\xi^2 - 2)$$

$$H_3(\xi) = \frac{1}{\sqrt{48\sqrt{\pi}}}(8\xi^3 - 12\xi)$$

$$H_4(\xi) = \frac{1}{\sqrt{384\sqrt{\pi}}}(16\xi^4 - 48\xi^2 + 12)$$

$$H_5(\xi) = \frac{1}{\sqrt{3840\sqrt{\pi}}}(32\xi^5 - 160\xi^3 + 120\xi)$$

$$H_6(\xi) = \frac{1}{\sqrt{46080\sqrt{\pi}}}(64\xi^6 - 480\xi^4 + 720\xi^2 - 120)$$

$$H_7(\xi) = \frac{1}{\sqrt{645120\sqrt{\pi}}}(128\xi^7 - 1344\xi^5 + 3360\xi^3 - 1680\xi)$$

$$H_8(\xi) = \frac{1}{\sqrt{10321920\sqrt{\pi}}}(256\xi^8 - 3584\xi^6 + 13440\xi^4 - 13440\xi^2 + 1680)$$

$$H_9(\xi) = \frac{1}{\sqrt{185794560\sqrt{\pi}}}(512\xi^9 - 9216\xi^7 + 48384\xi^5 - 80640\xi^3 + 30240\xi)$$

hydrogen 1 **H** 1.0079																		helium 2 **He** 4.0026
lithium 3 **Li** 6.941	beryllium 4 **Be** 9.0122												boron 5 **B** 10.811	carbon 6 **C** 12.011	nitrogen 7 **N** 14.007	oxygen 8 **O** 15.999	fluorine 9 **F** 18.998	neon 10 **Ne** 20.180
sodium 11 **Na** 22.990	magnesium 12 **Mg** 24.305												aluminium 13 **Al** 26.982	silicon 14 **Si** 28.086	phosphorus 15 **P** 30.974	sulfur 16 **S** 32.065	chlorine 17 **Cl** 35.453	argon 18 **Ar** 39.948
potassium 19 **K** 39.098	calcium 20 **Ca** 40.078		scandium 21 **Sc** 44.956	titanium 22 **Ti** 47.867	vanadium 23 **V** 50.942	chromium 24 **Cr** 51.996	manganese 25 **Mn** 54.938	iron 26 **Fe** 55.845	cobalt 27 **Co** 58.933	nickel 28 **Ni** 58.693	copper 29 **Cu** 63.546	zinc 30 **Zn** 65.39	gallium 31 **Ga** 69.723	germanium 32 **Ge** 72.61	arsenic 33 **As** 74.922	selenium 34 **Se** 78.96	bromine 35 **Br** 79.904	krypton 36 **Kr** 83.80
rubidium 37 **Rb** 85.468	strontium 38 **Sr** 87.62		yttrium 39 **Y** 88.906	zirconium 40 **Zr** 91.224	niobium 41 **Nb** 92.906	molybdenum 42 **Mo** 95.94	technetium 43 **Tc** [98]	ruthenium 44 **Ru** 101.07	rhodium 45 **Rh** 102.91	palladium 46 **Pd** 106.42	silver 47 **Ag** 107.87	cadmium 48 **Cd** 112.41	indium 49 **In** 114.82	tin 50 **Sn** 118.71	antimony 51 **Sb** 121.76	tellurium 52 **Te** 127.60	iodine 53 **I** 126.90	xenon 54 **Xe** 131.29
caesium 55 **Cs** 132.91	barium 56 **Ba** 137.33	57-70 *	lutetium 71 **Lu** 174.97	hafnium 72 **Hf** 178.49	tantalum 73 **Ta** 180.95	tungsten 74 **W** 183.84	rhenium 75 **Re** 186.21	osmium 76 **Os** 190.23	iridium 77 **Ir** 192.22	platinum 78 **Pt** 195.08	gold 79 **Au** 196.97	mercury 80 **Hg** 200.59	thallium 81 **Tl** 204.38	lead 82 **Pb** 207.2	bismuth 83 **Bi** 208.98	polonium 84 **Po** [209]	astatine 85 **At** [210]	radon 86 **Rn** [222]
francium 87 **Fr** [223]	radium 88 **Ra** [226]	89-102 **	lawrencium 103 **Lr** [262]	rutherfordium 104 **Rf** [261]	dubnium 105 **Db** [262]	seaborgium 106 **Sg** [266]	bohrium 107 **Bh** [264]	hassium 108 **Hs** [269]	meitnerium 109 **Mt** [268]	ununnilium 110 **Uun** [271]	unununium 111 **Uuu** [272]	ununbium 112 **Uub** [277]		ununquadium 114 **Uuq** [289]				

*Lanthanide series	lanthanum 57 **La** 138.91	cerium 58 **Ce** 140.12	praseodymium 59 **Pr** 140.91	neodymium 60 **Nd** 144.24	promethium 61 **Pm** [145]	samarium 62 **Sm** 150.36	europium 63 **Eu** 151.96	gadolinium 64 **Gd** 157.25	terbium 65 **Tb** 158.93	dysprosium 66 **Dy** 162.50	holmium 67 **Ho** 164.93	erbium 68 **Er** 167.26	thulium 69 **Tm** 168.93	ytterbium 70 **Yb** 173.04
**Actinide series	actinium 89 **Ac** [227]	thorium 90 **Th** 232.04	protactinium 91 **Pa** 231.04	uranium 92 **U** 238.03	neptunium 93 **Np** [237]	plutonium 94 **Pu** [244]	americium 95 **Am** [243]	curium 96 **Cm** [247]	berkelium 97 **Bk** [247]	californium 98 **Cf** [251]	einsteinium 99 **Es** [252]	fermium 100 **Fm** [257]	mendelevium 101 **Md** [258]	nobelium 102 **No** [259]