Physics 309 Final

I pledge that I have given nor received unauthorized assistance during the completion of this work.

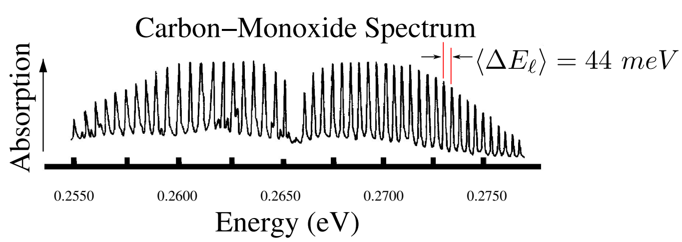
Name ______________________ Signature ______________________

Questions (5 pts. apiece) Answer questions in complete, well-written sentences WITHIN the spaces provided.

1. Describe the model we used to explain α decay.

2. What is Rutherford scattering?

3. Recall how the we explained the vibration-rotation spectrum of the carbon monoxide molecule (see figure). Suppose that when the molecule absorbed a photon it was constrained to change the value of the angular momentum quantum number by $\Delta l = \pm 1$ units AND $\Delta l = 0$. How would the spectrum change? Explain.

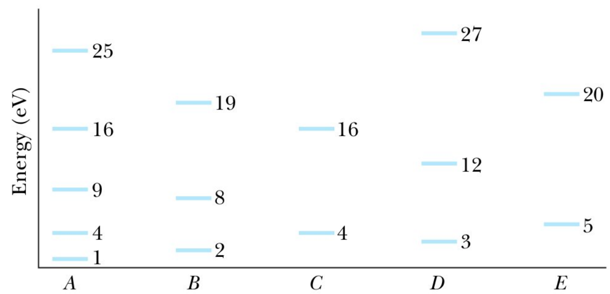


4. What is the quantum program?

DO NOT WRITE BELOW THIS LINE.

5. Cite at least three experimental measurements that required quantum mechanics to explain.

6. The figure shows the lowest energy levels in eV for five different potential wells trapping a single electron in each. In wells B, C, D, and E the electron is in the ground state. The electron in well A is excited to the fourth state at 25 eV and then de-excites by emitting one of more photons corresponding to a single long jump or several smaller jumps. What photon emission energies of the de-excitation of the electron in well A match a photon absorption transition from the ground state for the other four wells (B-E)? Give the corresponding quantum numbers for the transitions in each well. Clearly label which atom you reference.



7. When we solved the rectangular barrier problem we required the wave function to continuous across the boundary between different potential energy regions. Why?

8. What is the CLASSICAL expectation for the transmission coefficient of a particle of energy E striking a one-dimensional rectangular barrier of height V_0? Explain.

DO NOT WRITE BELOW THIS LINE.

Problems. Clearly show all work for full credit on a separate piece of paper.

1. (8 pts.) A car with bad shock absorbers bounces up and down with a period of 1.5 s after hitting a bump. The car has a mass of 1500 kg and is supported by four springs of equal force constant k. What is k?

2. (8 pts.) A particle detector has an active volume in the shape of a right circular cylinder. The endcaps of the cylinder each have a diameter $d = 6\ mm$. If the cylinder is oriented so one endcap faces a target that is a distance $R = 1.0\ m$ away, what is the solid angle Ω of the detector?

3. (10 pts.) In solving the Schroedinger equation for the harmonic oscillator potential we rewrote the Schroedinger equation in the form

$$\frac{d^2\phi}{d\xi^2} + \left(\frac{\alpha}{\beta^2} - \xi^2\right)\phi = 0$$

where $\xi = \beta x$, $\alpha = 2mE/\hbar^2$ and $\beta = \sqrt{m\omega_0/\hbar}$. What is the asymptotic form of this differential equation? In other words, what does it look like for large ξ? Show the asymptotic solution is

$$|\phi_{asymp}\rangle = A_{asymp}e^{-\xi^2/2} + B_{asymp}e^{\xi^2/2} \quad .$$

4. (10 pts.) A beam of light P has twice the wavelength, but the same intensity as beam Q. How is the number of photons that hit a given area in a given time when it is illuminated by beam P related to the number that hit when the area is illuminated by beam Q?

5. (12 pts.) A pulse of protons is $L = 1\ m$ long and contains $N = 10000$ particles. At $t = 0$ each proton is in the state

$$\begin{aligned} \psi(x,0) &= \frac{1}{100}e^{ik_0x} \quad & |x| < 0.5\ m \\ &= 0 & elsewhere \end{aligned}$$

The free particle eigenfunctions are the following.

$$|\phi\rangle = \frac{e^{ikx}}{\sqrt{2\pi}}$$

   1. What are the $b(k)$'s?
   2. Would any momenta in this initial wave packet be forbidden, *i.e.* values of $p = \hbar k$ with a probability of zero?

6. (12 pts.) Suppose a rigid rotator is in the following initial state of $\hat{L}^2$ with $\ell = 1$.

$$|\psi\rangle = \frac{Y_1^1 + Y_1^{-1}}{\sqrt{3}}$$

We want to find the probability of obtaining the values of $m_x = 0, \pm 1$ from a measurement of $\hat{L}_x$. The eigenfunctions of $\hat{L}_x$ for $\ell = 1$ can be expressed in terms of spherical harmonics (Y_ℓ^m's) and are shown below along with their eigenvalues α.

$$\begin{aligned} X_1^1 &= \frac{Y_1^1 + \sqrt{2}Y_1^0 + Y_1^{-1}}{2} & \alpha = \hbar \\ X_1^0 &= \frac{Y_1^1 - Y_1^{-1}}{\sqrt{2}} & \alpha = 0 \\ X_1^{-1} &= \frac{Y_1^1 - \sqrt{2}Y_1^0 + Y_1^{-1}}{2} & \alpha = -\hbar \end{aligned}$$

What are the probabilities for obtaining $m_x = 0, \pm 1$?

Physics 309 Equations

$R_T(\nu) = \dfrac{Energy}{time \times area}$ $\quad E = h\nu = \hbar\omega$ $\quad v_{wave} = \lambda\nu$ $\quad I \propto |\vec{E}|^2$ $\quad \lambda = \dfrac{h}{p}$ $\quad p = \hbar k$ $\quad K = \dfrac{p^2}{2m}$ $\quad K_{max} = h\nu - \Phi$

$-\dfrac{\hbar^2}{2m}\dfrac{\partial^2}{\partial x^2}\Psi(x,t) + V(x)\Psi(x,t) = i\hbar\dfrac{\partial}{\partial t}\Psi(x,t)$ $\quad \hat{p}_x = -i\hbar\dfrac{\partial}{\partial x}$ $\quad \hat{A}\,|\phi\rangle = a|\phi\rangle$ $\quad \langle \hat{A}\,\rangle = \displaystyle\int_{-\infty}^{\infty} \psi^* \hat{A}\,\psi dx$

$\langle \phi_{n'}|\phi_n\rangle = \displaystyle\int_{-\infty}^{\infty} \phi_{n'}^*\phi_n dx = \delta_{n',n}$ $\quad \langle \phi(k')|\phi(k)\rangle = \displaystyle\int_{-\infty}^{\infty} \phi_{k'}^*\phi_k \, dx = \delta(k-k')$ $\quad e^{i\phi} = \cos\phi + i\sin\phi$

$|\psi\rangle = \sum b_n|\phi_n\rangle \to b_n = \langle\phi_n|\psi\rangle$ $\qquad |\phi\rangle = \dfrac{e^{\pm ikx}}{\sqrt{2\pi}}$ $\qquad |\psi\rangle = \displaystyle\int b(k)|\phi(k)\rangle dk \to b(k) = \langle\phi(k)|\psi\rangle$

$|\psi(x,t)\rangle = \sum b_n|\phi_n\rangle e^{-i\omega_n t}$ $\quad |\psi(x,t)\rangle = \displaystyle\int b(k)|\phi(k)\rangle e^{-i\omega(k)t}dk$ $\quad \Delta p \Delta x \geq \dfrac{\hbar}{2}$ $\qquad (\Delta x)^2 = \langle x^2\rangle - \langle x\rangle^2$

The wave function, $\Psi(\vec{r},t)$, contains all we know of a system and its square is the probability of finding the system in the region $\vec{r}$ to $\vec{r} + d\vec{r}$. The wave function and its derivative are (1) *finite*, (2) *continuous*, and (3) *single-valued* ($\psi_1(a) = \psi_2(a)$ and $\psi_1'(a) = \psi_2'(a)$) .

$V_{HO} = \dfrac{\kappa x^2}{2}$ $\quad \omega = 2\pi\nu = \dfrac{2\pi}{T} = \sqrt{\dfrac{\kappa}{m}}$ $\quad E_n = (n + \dfrac{1}{2})\hbar\omega_0 = \hbar\omega$ $\quad |\phi_n\rangle = e^{-\xi^2/2}H_n(\xi)$ $\quad \xi = \beta x$ $\quad \beta^2 = \dfrac{m\omega_0}{\hbar}$

$\psi_1 = \mathbf{t}\psi_3 = \mathbf{d_{12}p_2d_{21}p_1^{-1}}\psi_3$ $\qquad T = \dfrac{1}{|t_{11}|^2}$ $\qquad R + T = 1$ $\quad T_{WKB} = \exp\left[-2\displaystyle\int_{x_0}^{x_1}\sqrt{\frac{2m(V(x)-E)}{\hbar^2}}\,dx\right]$

$$\mathbf{d_{ij}} = \frac{1}{2}\begin{pmatrix} 1+\frac{k_j}{k_i} & 1-\frac{k_j}{k_i} \\ 1-\frac{k_j}{k_i} & 1+\frac{k_j}{k_i} \end{pmatrix} \quad \mathbf{p_i} = \begin{pmatrix} e^{-ik_i 2a} & 0 \\ 0 & e^{ik_i 2a} \end{pmatrix} \quad \mathbf{p_i^{-1}} = \begin{pmatrix} e^{ik_i 2a} & 0 \\ 0 & e^{-ik_i 2a} \end{pmatrix}$$

$$E = \frac{\hbar^2 k^2}{2m} \quad k = \sqrt{\frac{2m(E-V)}{\hbar^2}} \quad T = \frac{\text{transmitted flux}}{\text{incident flux}} \quad R = \frac{\text{reflected flux}}{\text{incident flux}} \quad \text{flux} = |\psi|^2 v$$

$$V(r) = \frac{Z_1 Z_2 e^2}{r} \quad E = \frac{1}{2}\mu v^2 + V(r) \quad \vec{R}_{cm} = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i} \quad \mu = \frac{m_1 m_2}{m_1 + m_2} \quad \frac{dN_s}{dt} = \frac{d\sigma}{d\Omega}\frac{dN_{inc}}{dt} n_{tgt} d\Omega$$

$$n_{tgt} = \frac{\rho_{tgt}}{A_{tgt}} N_A \frac{V_{hit}}{a_{beam}} = \frac{\rho_{tgt}}{A_{tgt}} N_A L_{tgt} \quad d\Omega = \frac{dA}{r^2} \quad \frac{dN_{inc}}{dt} = \frac{I_{beam}}{Ze} \quad \frac{d\sigma}{d\Omega} = \left(\frac{Z_1 Z_2 e^2}{4E}\right)^2 \frac{1}{\sin^4\left(\frac{\theta}{2}\right)}$$

$$\psi(x) = \sum_{n=1}^{\infty} a_n x^n \quad \langle K \rangle = \frac{3}{2}kT \qquad n(v) = 4\pi N \left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 e^{-mv^2/2k_B T} \quad \vec{L} = \vec{r} \times \vec{p} = \mathcal{I}\vec{\omega}$$

$$\mathcal{I} = \sum_i m_i r_1^2 = \int r^2 dm \quad KE_{rot} = \frac{L^2}{2\mathcal{I}} \quad E_\ell = \frac{\ell(\ell+1)\hbar^2}{2\mathcal{I}} \quad V_{coul} = \frac{Z_1 Z_2 e^2}{r} \quad ME = \frac{p_r^2}{2\mu} + \frac{L^2}{2\mu r^2} + V(r)$$

$$L_z|nlm\rangle = m\hbar|nlm\rangle \quad L^2|nlm\rangle = \ell(\ell+1)\hbar^2|nlm\rangle$$

Constants

Speed of light (c)	$2.9979 \times 10^8\ m/s$	fermi (fm)	$10^{-15}\ m$
Boltzmann constant (k_B)	$1.381 \times 10^{-23}\ J/K$	angstrom (Å)	$10^{-10}\ m$
	$8.62 \times 10^{-5}\ eV/k$	electron-volt (eV)	$1.6 \times 10^{-19}\ J$
Planck constant (h)	$6.621 \times 10^{-34}\ J-s$	MeV	$10^6\ eV$
	$4.1357 \times 10^{-15}\ eV-s$	GeV	$10^9\ eV$
Planck constant ($\hbar$)	$1.0546 \times 10^{-34}\ J-s$	Electron charge (e)	$1.6 \times 10^{-19}\ C$
	$6.5821 \times 10^{-16}\ eV-s$	e^2	$\hbar c/137$
Planck constant ($\hbar c$)	$197\ MeV-fm$	Electron mass (m_e)	$9.11 \times 10^{-31}\ kg$
	$1970\ eV-$Å		$0.511\ MeV/c^2$
Proton mass (m_p)	$1.67 \times 10^{-27} kg$	atomic mass unit (u)	$1.66 \times 10^{-27}\ kg$
	$938\ MeV/c^2$		$931.5\ MeV/c^2$
Neutron mass (m_n)	$1.68 \times 10^{-27}\ kg$		
	$939\ MeV/c^2$		

Integrals and Derivatives and other Formulae

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc \quad |B| = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a(ei - fh) - b(di - fg) + c(dh - eg)$$

$$\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right] \quad \cos A \cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$$

$$\sin A \cos B = \frac{1}{2}\left[\sin(A-B) + \sin(A+B)\right] \quad \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\frac{df}{du}=\frac{df}{dx}\frac{du}{dx} \quad \frac{d}{dx}(x^n)=nx^{n-1} \quad \frac{d}{dx}(\sin x)=\cos x \quad \frac{d}{dx}(\cos x)=-\sin x \quad \frac{d}{dx}(e^{ax})=ae^{ax}$$

$$\frac{d}{dx}(\ln ax)=\frac{1}{x} \quad \int x^n dx=\frac{x^{n+1}}{n+1} \quad \int e^{ax}dx=\frac{e^{ax}}{a} \quad \int\frac{1}{x}=\ln x \quad \int\frac{1}{\sqrt{x^2+a^2}}dx=\ln\left[x+\sqrt{x^2+a^2}\right]$$

$$\int\frac{x}{\sqrt{x^2+a^2}}dx=\sqrt{x^2+a^2} \quad \int\frac{x^2}{\sqrt{x^2+a^2}}dx=\frac{1}{2}x\sqrt{x^2+a^2}-\frac{1}{2}a^2\ln\left[x+\sqrt{x^2+a^2}\right]$$

$$\int\frac{x^3}{\sqrt{x^2+a^2}}dx=\frac{1}{3}(-2a^2+x^2)\sqrt{x^2+a^2} \quad \int x^2\sin(ax)dx=\frac{2x\sin(ax)}{a^2}-\frac{\left(a^2x^2-2\right)\cos(ax)}{a^3}$$

$$\int x\sin(ax)dx=\frac{\sin(ax)}{a^2}-\frac{x\cos(ax)}{a} \quad \int x^3\sin ax dx=\frac{3\left(a^2x^2-2\right)\sin(ax)}{a^4}-\frac{x\left(a^2x^2-6\right)\cos(ax)}{a^3}$$

Hermite polynomials ($H_n(\xi)$)

$$H_0(\xi)=\frac{1}{\sqrt{\sqrt{\pi}}} \qquad H_5(\xi)=\frac{1}{\sqrt{3840\sqrt{\pi}}}(32\xi^5-160\xi^3+120\xi)$$

$$H_1(\xi)=\frac{1}{\sqrt{2\sqrt{\pi}}}2\xi \qquad H_6(\xi)=\frac{1}{\sqrt{46080\sqrt{\pi}}}(64\xi^6-480\xi^4+720\xi^2-120)$$

$$H_2(\xi)=\frac{1}{\sqrt{8\sqrt{\pi}}}(4\xi^2-2) \qquad H_7(\xi)=\frac{1}{\sqrt{645120\sqrt{\pi}}}(128\xi^7-1344\xi^5+3360\xi^3-1680\xi)$$

$$H_3(\xi)=\frac{1}{\sqrt{48\sqrt{\pi}}}(8\xi^3-12\xi) \qquad H_8(\xi)=\frac{1}{\sqrt{10321920\sqrt{\pi}}}(256\xi^8-3584\xi^6+13440\xi^4-13440\xi^2+1680)$$

$$H_4(\xi)=\frac{1}{\sqrt{384\sqrt{\pi}}}(16\xi^4-48\xi^2+12) \qquad H_9(\xi)=\frac{1}{\sqrt{185794560\sqrt{\pi}}}(512\xi^9-9216\xi^7+48384\xi^5-80640\xi^3+30240\xi)$$

hydrogen 1 **H** 1.0079																		helium 2 **He** 4.0026
lithium 3 **Li** 6.941	beryllium 4 **Be** 9.0122												boron 5 **B** 10.811	carbon 6 **C** 12.011	nitrogen 7 **N** 14.007	oxygen 8 **O** 15.999	fluorine 9 **F** 18.998	neon 10 **Ne** 20.180
sodium 11 **Na** 22.990	magnesium 12 **Mg** 24.305												aluminium 13 **Al** 26.982	silicon 14 **Si** 28.086	phosphorus 15 **P** 30.974	sulfur 16 **S** 32.065	chlorine 17 **Cl** 35.453	argon 18 **Ar** 39.948
potassium 19 **K** 39.098	calcium 20 **Ca** 40.078		scandium 21 **Sc** 44.956	titanium 22 **Ti** 47.867	vanadium 23 **V** 50.942	chromium 24 **Cr** 51.996	manganese 25 **Mn** 54.938	iron 26 **Fe** 55.845	cobalt 27 **Co** 58.933	nickel 28 **Ni** 58.693	copper 29 **Cu** 63.546	zinc 30 **Zn** 65.39	gallium 31 **Ga** 69.723	germanium 32 **Ge** 72.61	arsenic 33 **As** 74.922	selenium 34 **Se** 78.96	bromine 35 **Br** 79.904	krypton 36 **Kr** 83.80
rubidium 37 **Rb** 85.468	strontium 38 **Sr** 87.62		yttrium 39 **Y** 88.906	zirconium 40 **Zr** 91.224	niobium 41 **Nb** 92.906	molybdenum 42 **Mo** 95.94	technetium 43 **Tc** [98]	ruthenium 44 **Ru** 101.07	rhodium 45 **Rh** 102.91	palladium 46 **Pd** 106.42	silver 47 **Ag** 107.87	cadmium 48 **Cd** 112.41	indium 49 **In** 114.82	tin 50 **Sn** 118.71	antimony 51 **Sb** 121.76	tellurium 52 **Te** 127.60	iodine 53 **I** 126.90	xenon 54 **Xe** 131.29
caesium 55 **Cs** 132.91	barium 56 **Ba** 137.33	57-70 *	lutetium 71 **Lu** 174.97	hafnium 72 **Hf** 178.49	tantalum 73 **Ta** 180.95	tungsten 74 **W** 183.84	rhenium 75 **Re** 186.21	osmium 76 **Os** 190.23	iridium 77 **Ir** 192.22	platinum 78 **Pt** 195.08	gold 79 **Au** 196.97	mercury 80 **Hg** 200.59	thallium 81 **Tl** 204.38	lead 82 **Pb** 207.2	bismuth 83 **Bi** 208.98	polonium 84 **Po** [209]	astatine 85 **At** [210]	radon 86 **Rn** [222]
francium 87 **Fr** [223]	radium 88 **Ra** [226]	89-102 * *	lawrencium 103 **Lr** [262]	rutherfordium 104 **Rf** [261]	dubnium 105 **Db** [262]	seaborgium 106 **Sg** [266]	bohrium 107 **Bh** [264]	hassium 108 **Hs** [269]	meitnerium 109 **Mt** [268]	ununnilium 110 **Uun** [271]	unununium 111 **Uuu** [272]	ununbium 112 **Uub** [277]		ununquadium 114 **Uuq** [289]				

*Lanthanide series	lanthanum 57 **La** 138.91	cerium 58 **Ce** 140.12	praseodymium 59 **Pr** 140.91	neodymium 60 **Nd** 144.24	promethium 61 **Pm** [145]	samarium 62 **Sm** 150.36	europium 63 **Eu** 151.96	gadolinium 64 **Gd** 157.25	terbium 65 **Tb** 158.93	dysprosium 66 **Dy** 162.50	holmium 67 **Ho** 164.93	erbium 68 **Er** 167.26	thulium 69 **Tm** 168.93	ytterbium 70 **Yb** 173.04
**Actinide series	actinium 89 **Ac** [227]	thorium 90 **Th** 232.04	protactinium 91 **Pa** 231.04	uranium 92 **U** 238.03	neptunium 93 **Np** [237]	plutonium 94 **Pu** [244]	americium 95 **Am** [243]	curium 96 **Cm** [247]	berkelium 97 **Bk** [247]	californium 98 **Cf** [251]	einsteinium 99 **Es** [252]	fermium 100 **Fm** [257]	mendelevium 101 **Md** [258]	nobelium 102 **No** [259]