Measurement Magic

"...the principles of quantum mechanics have not been found to fail."

Richard Feynman in
The Feynman Lectures

"On the other hand, I think I can safely say, no one understands quantum mechanics."

Richard Feynman in
The Character of Physical Law

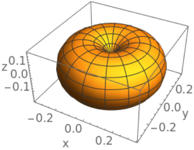
"Is the Moon there when we are not looking?"

Albert Einstein to Neils Bohr

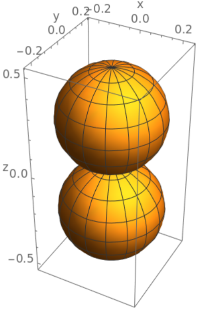
Suppose a rigid rotator is in a superposition of eigenstates with $\ell = 1$ and

$$|\psi_1\rangle = \frac{Y_1^1 + \sqrt{2}Y_1^0 + Y_1^{-1}}{2}$$

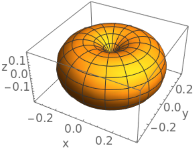
where $Y_l^{m_z}$ are the spherical harmonics that are the eigenfunctions of the angular (θ,ϕ) part of the three-dimensional Schroedinger equation. (a) What are the values and the probabilities that a measurement of $\hat{L}_z$ finds? (b) What will a subsequent measurement of $\hat{L}_z$ find and with what probability? The magnitudes of the spherical harmonics for $\ell = 1$ are shown below.



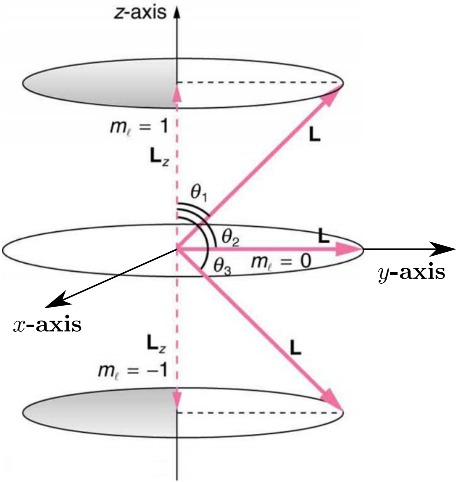
$|Y_1^1|$



$|Y_1^0|$



$|Y_1^{-1}|$



$$|\vec{L}| = \sqrt{\ell(\ell+1)} = \sqrt{2}$$

$$L_z = m_l = 0, \pm 1$$

$$\hat{L}_x = \frac{\hbar}{i}\left(y\frac{d}{dz} - z\frac{d}{dy}\right) \quad \hat{L}_y = \frac{\hbar}{i}\left(z\frac{d}{dx} - x\frac{d}{dz}\right) \quad \hat{L}_z = \frac{\hbar}{i}\left(x\frac{d}{dy} - y\frac{d}{dx}\right)$$

For the initial wave function we just saw

$$|\phi_1\rangle = \frac{Y_1^1 + \sqrt{2}Y_1^0 + Y_1^{-1}}{2}$$

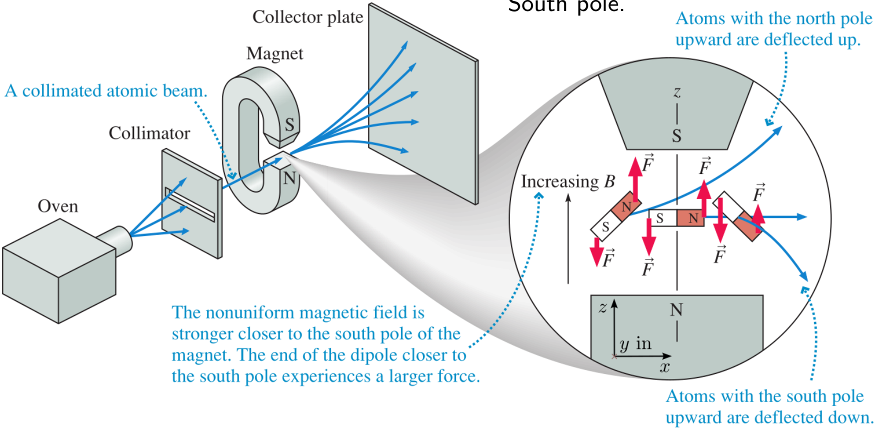
What is $\hat{L}_x|\phi_1\rangle$? You may find the list of tools below useful. What does the result say about $|\phi_1\rangle$?

$\hat{L}^2|\ell m_z\rangle = \ell(\ell+1)\hbar^2\ |\ell m_z\rangle \qquad \hat{L}_z|\ell m_z\rangle = m_z\hbar\,|\ell m_z\rangle$

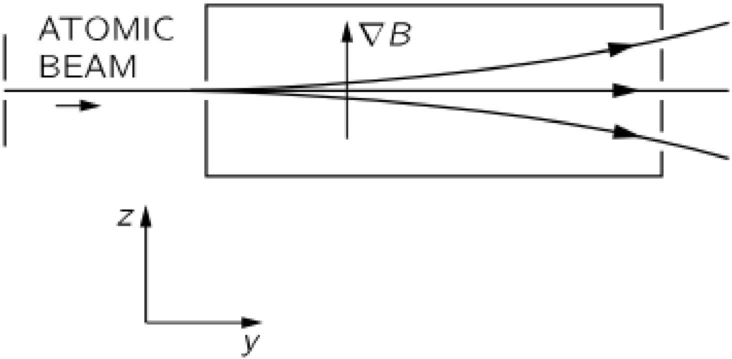
$\hat{L}_\pm|\ell m_z\rangle = \hat{L}_x \pm i\hat{L}_y = \hbar\sqrt{\ell(\ell+1) - m_z(m_z \pm 1)}\ |\ell\, m_z \pm 1\rangle$

To study quantum $\vec{L}$ consider an $\ell = 1$ atom beam sent through a non-uniform magnetic field.

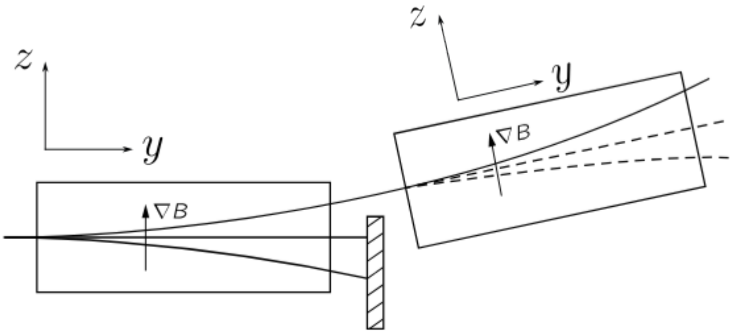
The $\vec{B}$ field is stronger near the South pole.



Trajectory of $\ell = 1$ atoms

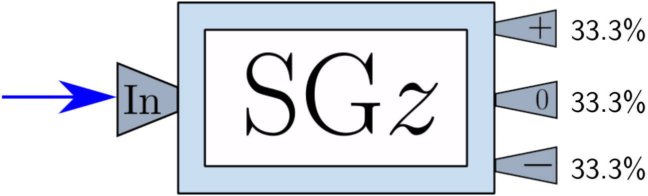


The different paths correspond to $m_\ell = 0, \pm 1$.

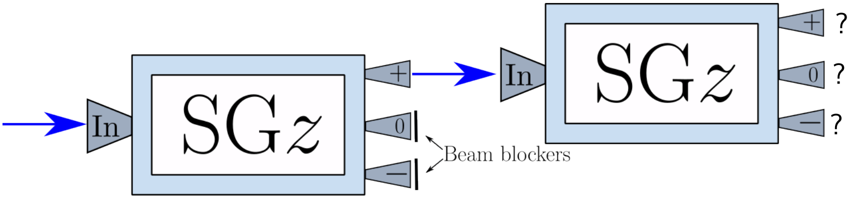


The different magnets are identical with the field pointing in the z direction in each device. Some trajectories are selected while others are blocked. The atoms selected here have $\ell = 1$, $m_z = 1$.

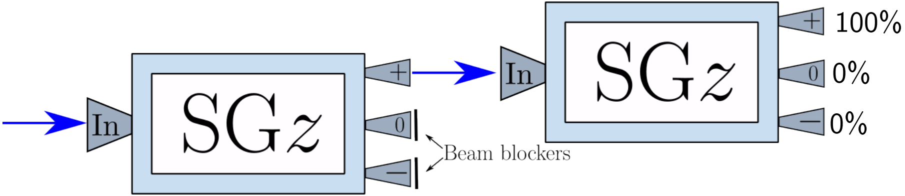
Use the symbol above to represent the Stern-Gerlach device and its effect on the atomic beam. The beam is coming out of a heated oven so the orientations of the input atoms are initially random. What fraction of the input beam exits through the three different paths?



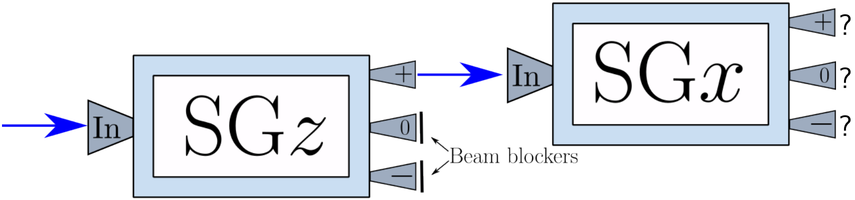
Use the symbol above to represent the Stern-Gerlach device and its effect on the atomic beam. The beam is coming out of a heated oven so the orientations of the input atoms are initially random. What fraction of the input beam exits through the three different paths?



Consider the output of the second SGz device. What do you predict for the three outputs?



Consider the output of the second SGz device. What do you predict for the three outputs?



Consider the output of the SGx device. What do you predict for the three outputs?

$$\hat{L}^2|\ell,m\rangle = \ell(\ell+1)\hbar^2\,|\ell,m\rangle$$

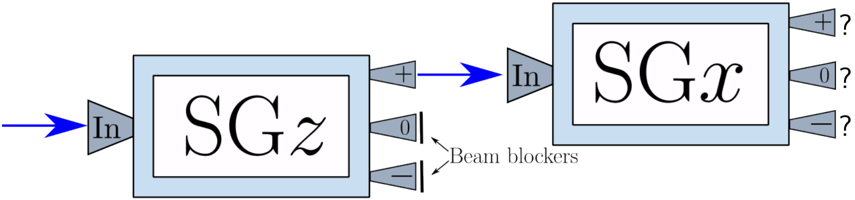
$$\hat{L}_z|\ell m\rangle = m\hbar\,|\ell m\rangle$$

$$\hat{L}_x|\ell,m\rangle = \frac{\hbar}{2}\sqrt{(\ell-m)(\ell+m+1)}\;|\ell,m+1\rangle + \frac{\hbar}{2}\sqrt{(\ell+m)(\ell-m+1)}\;|\ell,m-1\rangle$$

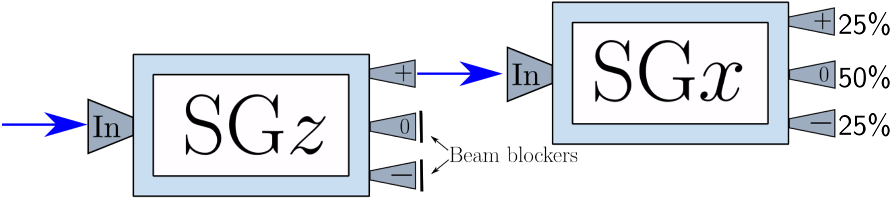
$$\hat{L}_y|\ell,m\rangle = -\frac{\hbar}{2}\sqrt{(\ell-m)(\ell+m+1)}\;|\ell,m+1\rangle + \frac{\hbar}{2}\sqrt{(\ell+m)(\ell-m+1)}\;|\ell,m-1\rangle$$

$$\hat{L}_\pm|\ell,m\rangle = \hbar\sqrt{\ell(\ell+1)-m(m\pm 1)}\;|\ell,m\pm 1\rangle$$

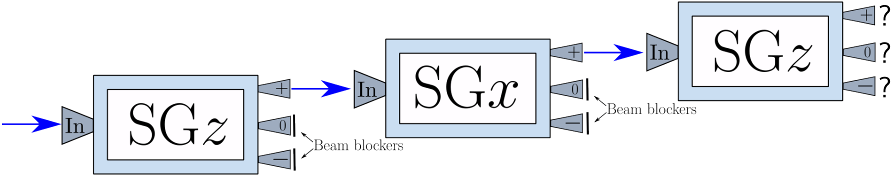
$$\langle\ell' m'|\ell m\rangle = \int_0^\pi\int_0^{2\pi} Y_{\ell'}^{m'*}\,Y_\ell^m\,d\Omega = \delta_{\ell\ell'}\delta_{mm'}$$



Consider the output of the SGx device. What do you predict for the three outputs?

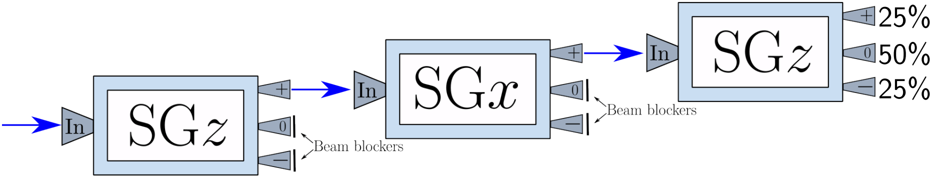


Consider the output of the SGx device. What do you predict for the three outputs?



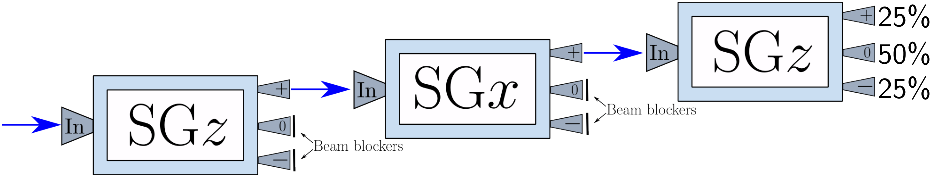
Consider the output of the second SGz device. What do you predict for the three outputs?

Checking the output of SGx Again 17



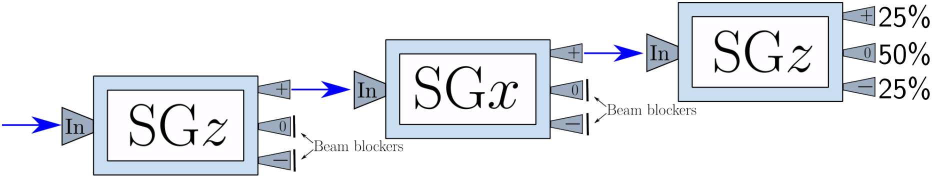
Consider the output of the second SGz device. What do you predict for the three outputs?

Checking the output of SGx Again



Consider the output of the second SGz device. What do you predict for the three outputs?

Where do the $m_z = 0$ and $m_z = -1$ atoms come from?



Consider the output of the second SGz device. What do you predict for the three outputs?

Where do the $m_z = 0$ and $m_z = -1$ atoms come from?

The act of measurement changes the beam!

1. Realism - Regularities in observed phenomena are caused by a physical reality whose existence is independent of human observers.
2. Inductive Inference - Legitimate conclusions can be drawn from consistent observations.
3. Einstein locality - No influence can propagate faster than the speed of light.

This list is often referred to as local realism.

1. The Statistics - A system is described by a wave function ψ where $|\psi|^2$ is the probability distribution of the possible results of an experiment.
2. Calculating observables - Each observable is associated with an operator $\hat{A}$ with eigenfunctions ϕ_i, eigenvalues a_i, and

$$\psi = \sum \alpha_i \phi_i \qquad .$$

3. The Measurement - Doing the experiment 'collapses' the wave function so a well-defined, single result is obtained.

This is the Copenhagen Interpretation associated with Neils Bohr.

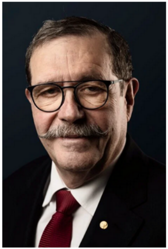
1. There are two ways for the quantum wave function to evolve in time.
2. The first is $\Psi(x,t) = \psi(x, t=0)e^{-i\omega t}$.
3. The second is the impact of a measurement. We write $\psi(x, t=0) = \sum b_n|\phi_n\rangle$ and say words like "In a measurement a single eigenfunction is picked out of the array of possible potentialities".
4. Both are radically different, but both are necessary.

Nobel Prize 2022

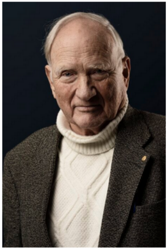
Scientific Background on the Nobel Prize in Physics 2022

"FOR EXPERIMENTS WITH ENTANGLED PHOTONS, ESTABLISHING THE VIOLATION OF BELL INEQUALITIES AND PIONEERING QUANTUM INFORMATION SCIENCE"

The Nobel Committee for Physics



Alain Aspect



John Clauser

Anton Zeilinger

Mathematica **Commands**

Mathematica calculations for $\vec{L}$ magic.

```
In[1]:= M1[alpha_] := ( √2 * alpha   -1           0
                        -1           √2 * alpha   -1
                        0            -1           √2 * alpha );

        MatrixForm[M1[alpha]]
```

Out[2]//MatrixForm=

$$\begin{pmatrix} \sqrt{2}\,\text{alpha} & -1 & 0 \\ -1 & \sqrt{2}\,\text{alpha} & -1 \\ 0 & -1 & \sqrt{2}\,\text{alpha} \end{pmatrix}$$

```
In[•]:= Solve[Det[M1[alpha]] == {0, 0, 0}, alpha]
```

Out[•]= $\{\{\text{alpha} \to -1\}, \{\text{alpha} \to 0\}, \{\text{alpha} \to 1\}\}$

```
In[•]:= Eigenvectors[M1[alpha]]
```

Out[•]= $\{\{1, \sqrt{2}, 1\}, \{-1, 0, 1\}, \{1, -\sqrt{2}, 1\}\}$